

Skill Mismatch, Job Polarization and the Great Recession*

PRELIMINARY AND INCOMPLETE

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Abstract

Job Polarization and the Great Recession have reshaped the occupational structure of the U.S. labor market. This paper investigates if this shift has determined bigger education-to-job mismatch and a rise of skill requirements across occupations in the post-recession era. Using data from CPS, I find that higher state-level polarization over the recession led to stronger downward mismatch during the recovery: (i) high-skilled workers downgraded to routine jobs; (ii) middle-skilled workers moved out of the labor force or downgraded to manual jobs; (iii) low-skilled quitted the market. Overall, job-skill requirements increased across occupations. Downward mismatch gave rise to a wage penalty, that could be partially attenuated by experience. Finally, I reconcile these results in a theoretical model of labor search and matching with skill-mismatch and skill-biased technological change.

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1 Introduction

From the 80's onward the U.S. labor market has been experiencing a strong polarization of occupations: employment is falling in jobs with a high content of *Routine tasks*, while increasing in occupations requiring *Abstract* and *Manual tasks*. Along with employment, also wages are polarizing with salaries growing relatively faster for those at the poles (Acemoglu (2002), Autor, Katz, and Kearney (2006), Goos and Manning (2007) and Acemoglu and Autor (2011)). This phenomenon is driven by two forces. One is technological progress: the rapid diffusion of new and cheaper technologies allows substitution of man-work with machines in performing *Routine* duties, while it complements high skilled individuals in performing *Abstract* ones (Autor, Levy, and Murnane (2003), Autor and Dorn (2013), Autor (2008, 2011)). The second is international trade and offshoring that allow respectively to substitute home *Routine* productions with imports and to move *Routine* activities into countries with lower labor costs (Autor et al. (2013, 2015)). Both drivers are shifting the domestic labor demand in favor of non *Routine* occupations.

The long-run trend of job polarization has also a short-run counterpart: it accelerates during recessions with *Routine* jobs more hit than others. This is because *Routine* activities are typically more procyclical and volatile so that the joint effect of polarization and economic downturn leads to higher destruction of *Routine* jobs. Moreover, once the economy recovers, employment in *Routine* occupations does not. For this reason, recent research states that polarization can account for jobless recoveries (Jaimovich and Siu (2012), Cortes et al. (2014) and Foote and Ryan (2015)) and shows how *Routine* workers are more likely to lose their job and to transition into non-labor force because neither able to upgrade to *Abstract* mansions nor willing to downgrade to *Manual* ones. All of this has traduced into sluggish labor mobility and longer unemployment spells.

In light of this, it is important to investigate what is the friction that impedes *Routine* workers to move successfully to other occupations, in particular in bad times. I argue that educational attainments and changes in skill requirements play an important role in the story: they affect the cyclical process of allocation of skills across jobs thus leading to skill-to-job mismatches, larger movements from the top to the bottom of the job ladder and therefore higher competition between skill groups when looking for the same job. Therefore, to understand the effects of polarization and skill requirements on the process of skills' sorting can help to comprehend the inner nature of jobless recoveries, non participation and human capital misallocation over the cycle. Needless to say, it has potential policy implications concerning firms' incentives to change workforce skill composition, welfare implications for optimal allocation of skills and efficiency.

With this purpose in mind, this paper contributes to the literature by connecting the strand on job polarization with research on procyclical efficiency in human capital allocation and mismatch (McLaughlin and Bils (2001), Krolkowski (2014), Carillo-Tudela and Visschers (2014), Cortes (2014) and many others).

My attention turns to the Great Recession in the U.S. I consider a time period in which the economy goes from a phase of expansion (2005:Q1-2007:Q3) to an unpredicted shock (2007:Q4-2009:Q2) after which it recovers (2009:Q3-2010:Q2) and moves back to its expansion path (2010:Q3-2013:Q4). For these time windows, I build a panel dataset for the 50 States and the District of Columbia to study (i) how an heterogeneous workforce reallocates in states that experienced a more severe polarization during the recession, (ii) which educational attainments matter most, when and to obtain which job and (iii) what are the cost of education-to-job mismatch. Thus, I obtain two classes of results: macro and micro.

On the macro side, I find that polarization characterizes not only sectors that are more procyclical and subject to automatization (typically construction and manufacture), but it is a common feature across all industries and it preserves its “cyclical” behavior across them. In fact, polarization accelerates during the Great Recession everywhere. This shift in the occupational structure leads to a larger skill mismatch and a distortion in the allocation of human capital afterwards. In particular, states that polarize more during the downturn experience larger movements from the top to the bottom of the job ladder during the recovery: HS workers climb down the ladder into Routine jobs; MS workers cannot upgrade but instead move down to Manual jobs or transition to non participation; LS workers are dismissed everywhere. However, when the economy goes back to its expansion path, only HS workers are able to climb up again the ladder and be efficiently employed in Abstract jobs. This is not true for the other skill groups for which mismatches are more persistent, if not permanent: MS workers cannot go back to Routine jobs because of polarization, while LS workers disappear mainly due to demographic dynamics. These macro facts suggests that something has changed at the micro level, precisely in the process of matching heterogeneous individuals with jobs. I show that cyclical changes in the skill requirements and demand within each job can rationalize these dynamics.

In fact, on the micro side, I find that skills matter the most during the downturn to access Abstract and Routine jobs and these occupations are experiencing an up-skilling: individuals moving first from unemployment to employment are always the most educated of the unemployment pool. This suggests a rise of skill requirements across jobs in bad times and an increase of the median skill level within occupation. For example, during

recession periods, a Master/PhD degree gives 12% more chances to get an Abstract job than a Bachelor Degree, while the difference vanishes once the recession is over. In other words, during bad times the best of the HS unemployment pool are served first in the matching process with Abstract vacancies. Similarly, during the downturn the probability to flow from unemployment to a Routine job increases for the best of the MS group only: individuals with some college or a vocational degree have almost 6% more chances to get a Routine job than individuals with a High School diploma, but transition probabilities never go back to pre-recession levels for Routine jobs. These facts shed light on the mismatch process and the importance of skills over the cycle. Moreover, it explains why a large mass of MS working population -not enough educated to get at an *Abstract* job but too much educated for a *Manual* one- prefer to move out of the labor force. Finally, I show how much individuals are penalized from being mismatched: for every HS and MS workers, moving down the job ladder leads to a wage loss that is not compensable through experience; the lowest wage available on the market is in Manual jobs and it is independent on individual skills; when downgraded to a Routine job, only HS workers benefit a significant wage premium with respect to their MS peers. These facts suggest that the demand of skills is changing across Routine and Abstract occupations, with employers willing to hire more skilled workers in bad times.

Along with the empirical analysis, I introduce a theoretical framework that captures the main dynamics of a polarizing labor market when hit by a shock. Precisely, I build a model of search and matching that defines a minimum skill requirement for each occupation that is dependent on labor markets and economic conditions. Given the minimum requirement, I show how heterogeneous agents sort into different occupations and what are the variables and mechanics that affect workforce skill composition in each job during different economic phases. In particular, the model predicts that -when an aggregate productivity shock hits the economy- switching markets widen so that workers can move from the top to the bottom of the ladder and firms can exploit more productive workers for the accomplishment of simpler tasks.

This paper is organized as follows: in Section 2, I describe the data and the main variables; Section 3 provides qualitative descriptives of polarization and employment mismatch at national level; in Section 4 I move the analysis at state level; Section 5 and 6 develops an empirical analysis on the role of educational attainment and the cost of skill mismatch; in Section 7 and 8 I introduce the theoretical model and its implications. Section 9 concludes.

2 Data Description and Variables Definition

I use monthly CPS data to investigate labor market dynamics and the quality of the labor force between 2005 and 2013, both at national and state level. The time span has several advantages. First, the negative shock represented by the 2008 recession is almost exogenous. Second, the definition and meaning of Abstract, Routine and Manual occupations and their respective task contents have not changed. This feature is absolutely not trivial: in fact in the last decades the classification of occupations has gone under several revisions and adjustments due to the rapid change of tasks and means within each job. Moreover, the meaning of education and investments in education have changed in the long run. Hence, by reducing the analysis to a narrower time window, I limit the potential bias due to endogenous adjustments of the skill composition of the labor force, but also due to the endogenous reshaping of tasks and technology in each occupations. Third, state-level data on industrial production is available at quarterly frequency for these periods, thus allowing for further controls for state-level business cycle and labor demand.

I claim that all of this helps to better decompose the effect of polarization and business cycle asymmetries on the allocation of an heterogeneous labor force in the market and to infer correctly the role of skills.

2.1 Employment Rates and Flows

The CPS is a monthly U.S. household survey and it is representative of the civilian populations of the U.S.A. In each month around 70,000 households are interviewed. More precisely, household members are surveyed in 4 consecutive months, then they leave the sample for the following 8 months, then are interviewed for 4 consecutive months again and leave forever. Thus, the CPS 4-8-4 rotating structure gives two types of information: (i) by using the cross sectional dimension of the survey, I build employment rates of each skill group into each type of occupation and industry; (ii) by using the longitudinal dimension of the survey, I match respondents in consecutive periods in order to study the flows from unemployment to employment for each type of skill group into each type of occupation and industry. Given the sampling structure and recent development of new linking algorithms¹, up to 95% of the individuals are potentially matched across consecutive months. The remaining 5% is lost due to attrition.

In order to understand the quality of the match between the demand and supply of skills within each occupation, it is first necessary to define jobs and skill groups. For jobs,

¹The CPS is an address-based survey so that households that migrate or move to another address are not perfectly followed. For more details on the matching algorithm I use in this paper, see Madrian and Lefgren (2000) and more recent Rivera Drew, Flood and Warren (2014).

I follow Acemoglu and Autor (2011) where -under ISCO-08 classification- occupations are labeled according to the main task performed and the nature of the job. Hence, occupations are defined into three broad classes:

- Abstract jobs (A): managerial and professional speciality occupations
- Routine jobs (R): technical, sales and administrative support occupations; precision production, craft, and repair occupations
- Manual jobs (M): service occupations.

For skills, agents are grouped by educational attainments² into:

- High Skilled (HS): from 3 years of college to doctorate degree
- Middle Skilled (MS): from twelfth grade to one or two years of college (but no degree) or to a vocational program
- Low Skilled (LS): from no schooling to eleventh grade.

Under this classification, I build national time series and a quarterly State-level panel for employment rates of each skill group into each occupation and industry³. Nonetheless, I extract the survey component of the CPS for those individuals flowing from unemployment to employment or non participation between two consecutive months.

2.2 Gdp and Business Cycle Dummies

I use data from the Bureau of Economic Analysis (BEA) to build a quarterly State-level panel for real Gdp in different industries for the years 2005-2013. Since the time of the Gdp peak and trough defined by the NBER to identify a recession period is not always consistent with the cyclical phases of each State economy, I define ad-hoc recession dummies for each of the 50 States. Precisely, I build an algorithm that -for each State- determines (i) the peak of Gdp closest in time to the NBER peak date and (ii) the trough of Gdp closest in time to the NBER trough date. Recoveries are instead defined as the

²ISCED-97 defines precisely the educational boundaries for each group. The ILO defines also the 1-to-1 mapping between ISCO-08 classifications of jobs and ISCED-97 skill requirements so that Abstract jobs are proper for HS worker, Routine jobs for MS workers and Manual jobs for LS ones. ILO's mapping between skills and occupations is a simplification that helps us to study mismatches in a tractable way (if I considered all possible educational attainments (12 groups) and all possible occupations (3 broad classes) I should track 36 different (mis)matches).

³I consider a sample of individuals aged between 16 and 75 years old, with a full time job. All observations related to individuals occupied in Farming, Fishing, Forestry and Military activities and individuals reporting to be self-employed are dropped from the sample. All series are seasonally adjusted.

time window necessary for Gdp to go back to pre recession level.

In this way, I add an extra source of variation to the panel without unfairly imposing that recession, recovery and expansion periods coincide across States.

3 Descriptives on Polarization and Skill Mismatches



Notice: per capita values; reference period 2005Q1.

Figure 1: Job Polarization

Between 2005 and the end of 2013, the employment stock (or employment per capita) in Abstract and Manual occupations grew respectively by the 1 and .05 percentage points, whereas Routine employment stock fell by 4.6 points. This is the baseline fact of job polarization (look at Figure 1(a)). Even though the long run trend, the loss in Routine employment is concentrated in the recession (grey-shaded area) and stops only in the middle of the recovery (blue-shaded area). During the new expansion phase, Routine employment does not ketch up. Instead, it slowly diminishes following its long run trend. On the other hand, Abstract employment shrinks only from the middle of the recession to the end of the recovery, and starts growing again afterwards. Manual employment does not seem to be affected by the Great Recession.

Jaimovich and Siu (2012) shows that such a pattern is true also for other recessions. Yet, they do not explain if the cyclicity of polarization is due to industries that are more pro-cyclical and volatile or if it is a sizable fact across sectors. For this reason, in Figure 1(b) I show the same time series now with manufacture and construction industry excluded. As it is clear, polarization happens across all industries and it is not driven by only the most automatized and cyclical sectors. Yet, the spread is smaller: the fall in Routine employment is now by 2.7 percentage points, while the change in employment in

other occupations is very close to the aggregate dynamics.

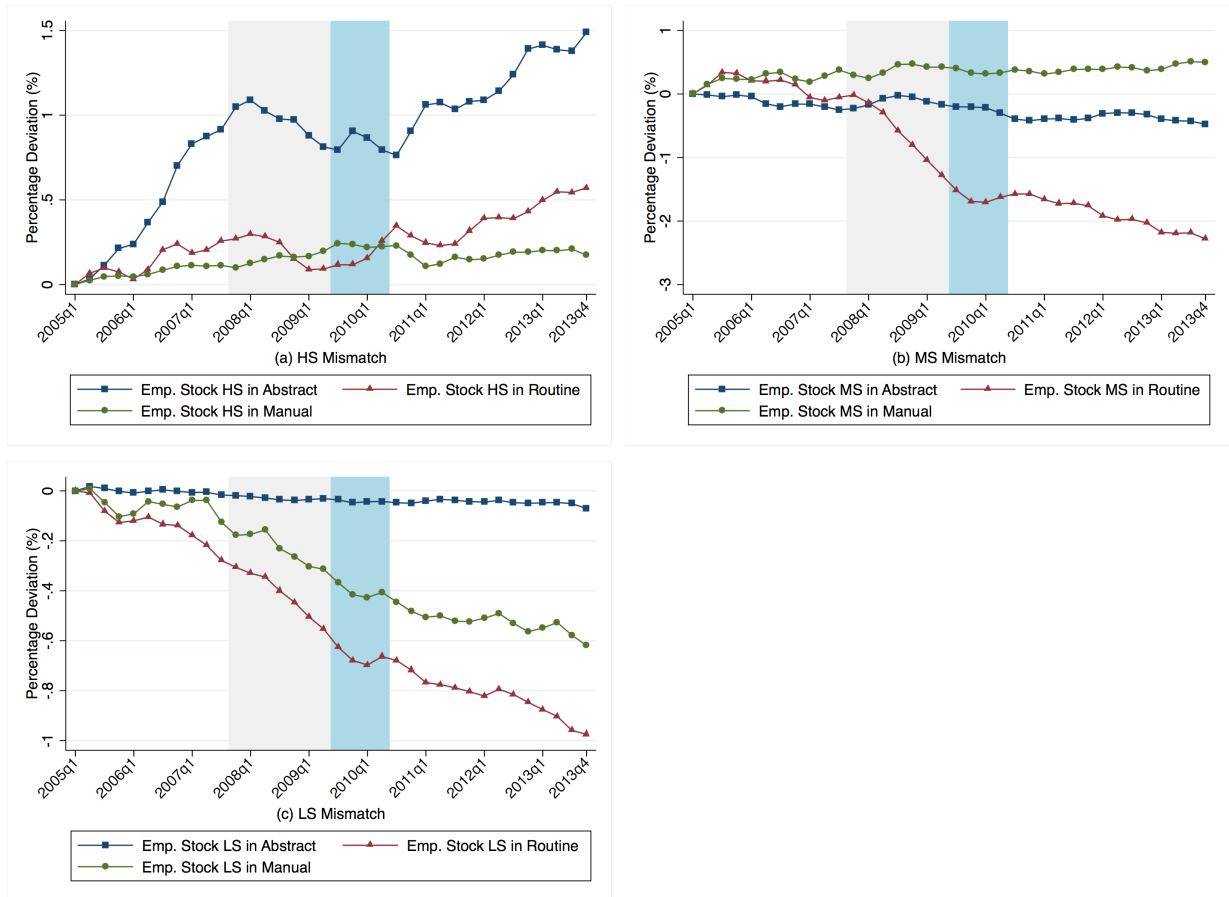
This issue, discussed also in Foote and Ryan (2014), rises some concerns. In fact, to infer correctly the role of polarization and the Great Recession on the reallocation of human resources and on the individual behavior of the unemployed, keeping into account the manufacture and construction sector bias our results. Indeed, there is large evidence that the link between employees and the firm is very strong for these industries and affects individual decisions on search intensity, participation and unemployment duration (see Elsby et al. (2011) and Loungani and Trehan (1997)). In other words, unemployed routine workers gravitate so much around these sectors that their job opportunities are strongly dependent on the life of the industry itself. Hence, they experience longer unemployment spell and transition out of the labor force more frequently when a recession hit. From the task-to-job prospective, this is also because manufacture and construction workers, whose ability and experience are more easily applicable to the reference industry but not easily exportable to others, suffer scarce inter-sectorial mobility and geographical mobility in the short run, but are also more likely to be recalled by the same firm. Even though keeping track of these workers does not change the qualitative pictures of the labor market and the (mis)match dynamics (Section 3 and 4), it creates a lot of noise when analyzing transition rates and wage equations as functions of education-to-job productivity match (Section 5 and 6)⁴ in particular during the recovery and next expansion. In light of this, from now onwards I focus on labor market dynamics outside manufacturing and constructions.

3.1 Decomposing the Employment Rate

Polarization and Great Recession affected different demographic groups with different intensity. Here I disaggregate the population by skill groups according to the classification previously introduced.

Consider Figure 2(b). The fall in Routine employment in non manufacture/construction sector affected MS workers the most, with a fall in of employment stock around 2.3 points. This skill group performed bad also in Abstract jobs, with employment decreasing by half point in the long run. In opposite direction goes MS employment in Manual jobs for which we observe an increase by 0.6 points. Figure 2(a) shows dynamics for HS workers. Even though a flection in the middle of the recession, HS employment increased by 0.6 points

⁴This is because recalls in manufacture and construction sector represent a large share of unemployment-to-employment flows. For these individuals, their work history and experience within the industry is more important than their education in the match formation. Thus, keeping them in the sample does not make clear the effect of education for some type of jobs.



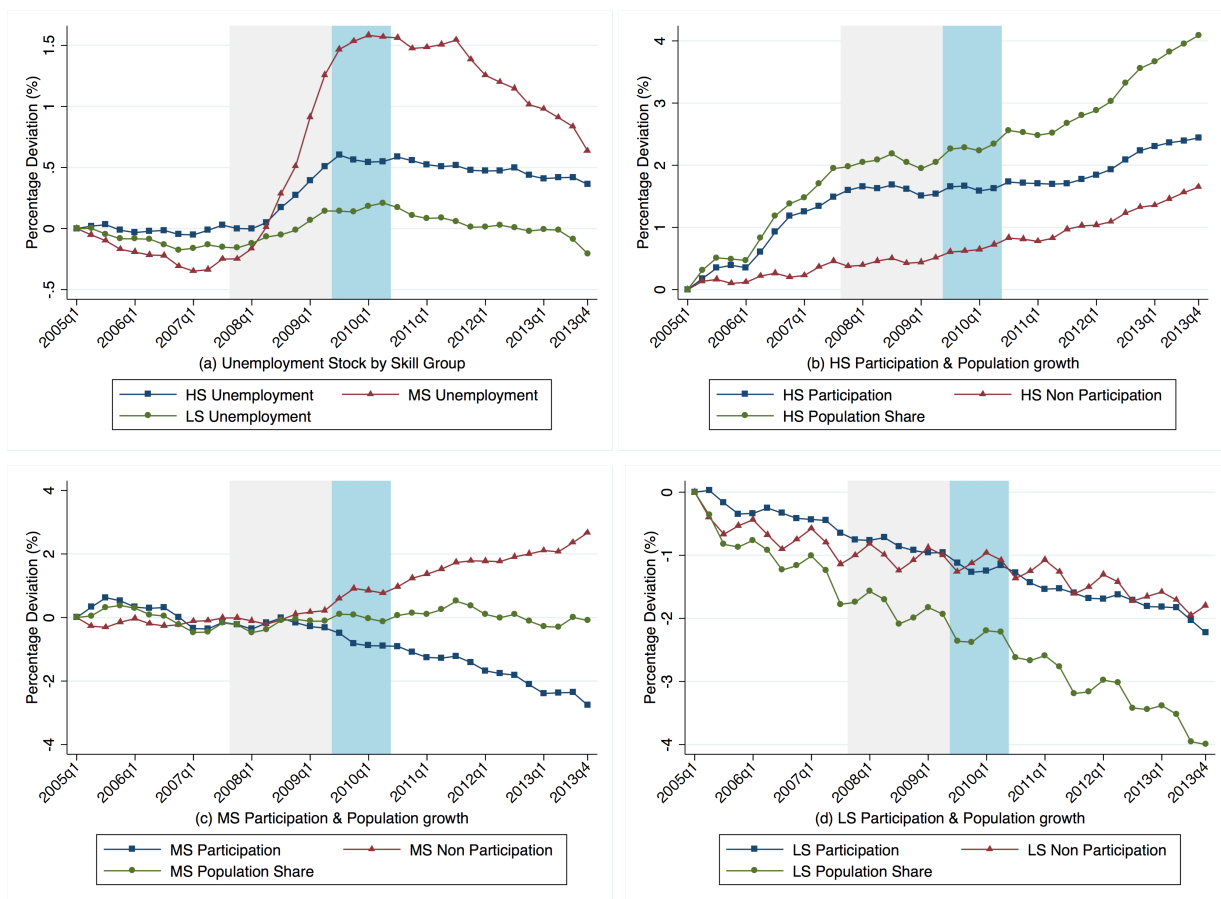
Notice: per capita values; manufacture and construction excluded; reference period 2005Q1.

Figure 2: Employment and Skill Mismatches

in the shrinking Routine segment of the job ladder. Of course, the large part of this skill group is absorbed by Abstract occupations. The increase of HS employment in Manual occupation is only by 0.3 points and it is mainly due to younger cohorts. Finally, Figure 2(c) shows the dynamics for LS workers. As it is clear, this skill group -that represents only a “dying out” 8% of the population- is always relatively less employed in every sector over time. Consistently with polarization, the fall was larger in Routine occupations (by 1 percentage point).

3.2 Unemployment, Participation and Demographic Dynamics

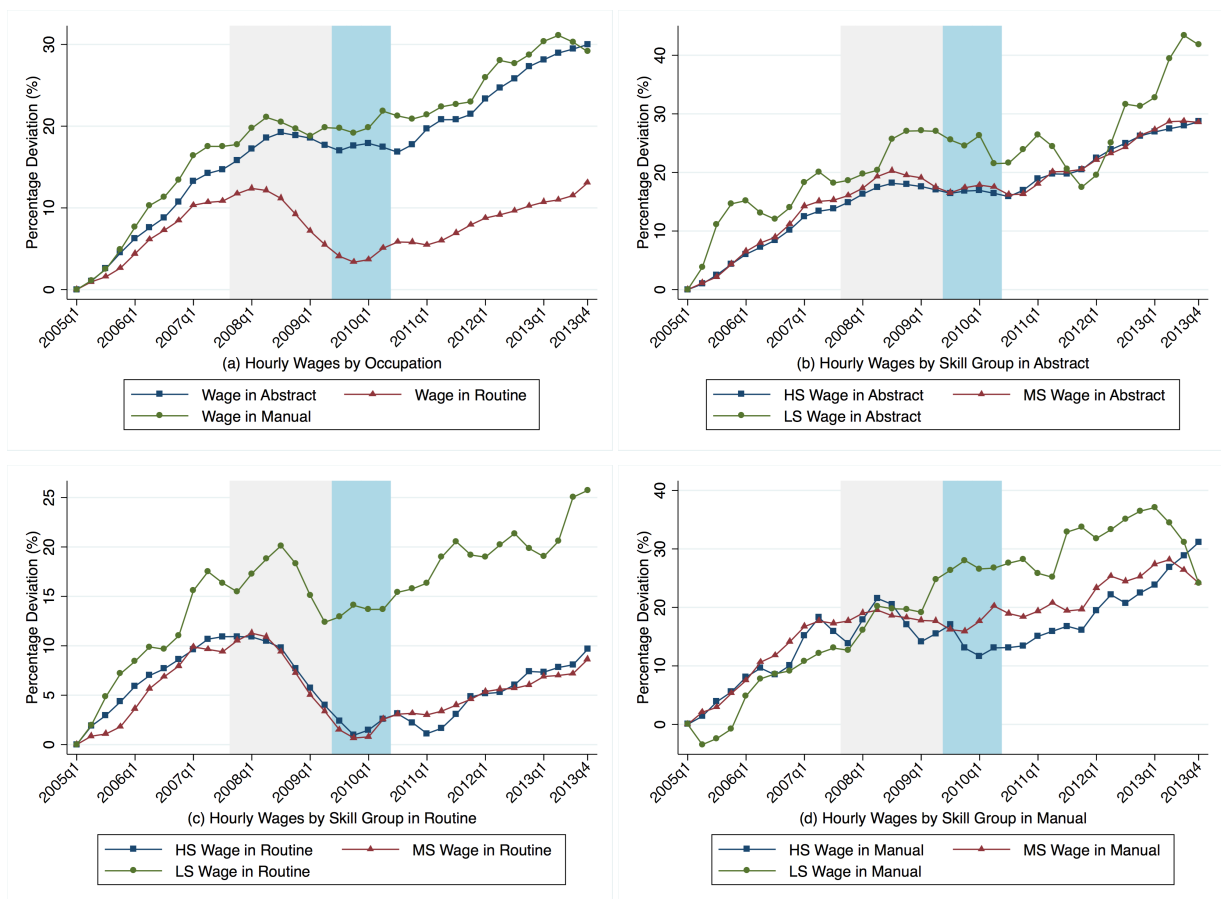
Long-run employment changes do not imply equal and opposite changes in unemployment for the three skill-groups. The difference is explained by non-participation and demographic dynamics. These two margins are important for the comprehension of changes in the labor market and the supply of skills, in particular in light of the secular growth of higher education and the progressive disappearance of very low educated population.



Notice: per capita values; manufacture and construction excluded; reference period 2005Q1.

Figure 3: Unemployment, Participation and Pop. Dynamics

For example, between 2005 and 2013, MS employment stock fell by 3.4 points. Yet, unemployment -after picking at the end of the recovery- was only 0.6 points above the initial level (look at Figure 3(a)). The difference is explained by the non participation margin. In fact, this skill group did not grow in the long run, but participation fell by 2.8 points (look at Figure 3(c)). Notice that the divergence between participation and non participation starts exactly in the middle of the recession. This suggests that, despite of population dynamics, the rate at which MS workers quitted the labor force for non participation was endogenous and not determined by demographics. This does not seem to be true for other skill groups. In fact, HS population increase by 4 points, and the dynamics seem to track well both the participation and non participation margin (Figure 3(b)). The same reasoning holds for LS population that is shrinking over time tracking both participation and non participation margin (Figure 3(d)).



Notice: average values by occupation; manufacture and construction excluded; reference period 2005Q1.

Figure 4: Wage Dynamics

3.3 Wages

Figure 4 shows hourly wage growth within each occupation. In line with the literature on polarization, Abstract and Manual wages are growing faster than Routine ones. When looking to each skill group within job, HS workers have always a larger skill premium over MS workers, whereas the wage difference between MS and LS workers is smaller (see Appendix A for levels). Despite of this, the wage dynamics across skill groups are very similar in Abstract and Manual jobs, but with LS wages always more volatile. On the other hand, in Routine occupations the wage of HS and MS workers follows the same dynamic over time while LS wages are growing faster (but starting from a lower level).

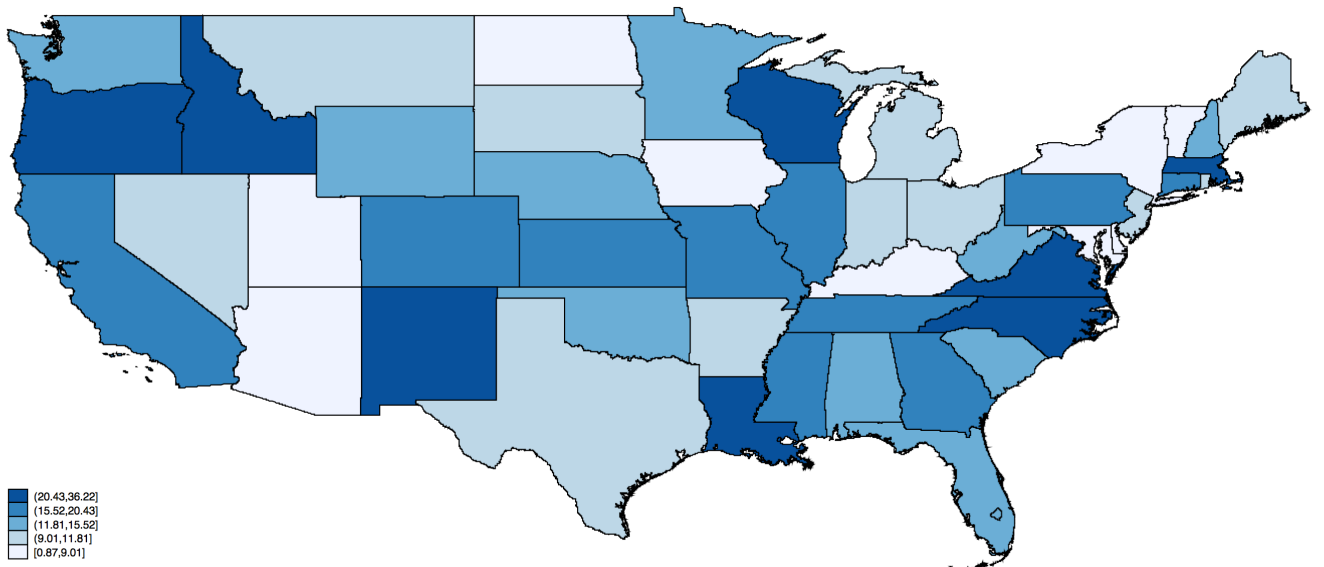
4 Local Polarization and Labor Market Outcomes

In this section, I exploit state-level variation to show how the occupational shift caused by the interacted effects of polarization and the Great Recession affected job opportunities and human capital reallocation after the crisis, which skills and demographic group were

rewarded most and in which occupations. To do so, I build a simple measure capturing state-level polarization during the recession, i.e. the degree at which the market destroyed more Routine jobs relative to non-Routine ones. Consider

$$Polarization_{GR}^s = \Delta_{s,GR} \left(\frac{E^{-R}}{E^R} \right)$$

where GR indicates that the change in the ratio of non-Routine (E^{-R}) to Routine (E^R) employment is evaluated between the beginning and the end of the Great Recession in state s ⁵. According to this definition, an increase of the measure of polarization implies a faster decline of Routine employment with respect to non-Routine one, and therefore a faster polarization. Figure 5 summarizes the degree of polarization of the States during the Great Recession by quintiles. As it is clear, the great recession accelerated the process of polarization at different intensities with Washington D.C and Vermont at the extremes. In general, compared to central states, the East and West coast experienced a faster decline in Routine employment with Respect to non-Routine one during recession periods.



Notice: the measure of polarization here is reported in quintile groups; Alaska and Hawaii (both polarizing) not reported; manufacture and construction excluded.

Figure 5: State-level Polarization

4.1 Polarization and Skill-to-Occupation Mismatch

Recent studies have explored the dynamic adjustments of local labor markets when exposed to employment structural change. Among all, Autor and Dorn (2013) and Autor,

⁵According to state s recession periods.

Dorn and Hanson (2015) use local employment specialization in Routine jobs ⁶ as a measure of exposure to technical change and trade shocks. In the spirit of this literature, here I use my measure of polarization to study how local labor markets, that experienced higher polarization during local recession, reallocated human capital afterwards, i.e. during the recovery and next economic expansion. I consider the following model:

$$\begin{aligned} \Delta E_t^{j,k,s} = & \beta_1(\delta_t^{s,Recovery} * Polarization_{GR}^s) + \beta_2(\delta_t^{s,Expansion} * Polarization_{GR}^s) \\ & + \gamma \Delta gdp_t^s + \eta(\Delta X_t^{j,s}) + \delta_t^{s,Expansion} + \varepsilon_t^{j,k,s} \end{aligned}$$

Where $\Delta E_t^{j,k,s}$ is the change in employment share of group j in job k in state s between the end of the recession and time t ; $\delta_t^{s,Recovery}$ and $\delta_t^{s,Expansion}$ are two mutually exclusive state-level dummies for state s recovery and expansion periods. Δgdp_t^s captures the change of gdp in non manufacture/construction sector in state s . $\Delta X_t^{j,s}$ controls for group j demographic dynamics and participation; $\varepsilon_t^{j,k,s}$ is the error term.

Table 1 shows results for the Middle Skill group at state level (model (1)) and for different subgroups (model (2) to (6)⁷). Panel A reports results for MS employment in Abstract jobs. As shown in model (1), higher state-level polarization did not allow MS worker to recover through Abstract occupations in the aftermath of the Great Recession. In fact, for a 1 percentage increase in polarization, MS employment share in Abstract jobs fell by almost 3 points during the Recovery; women and the younger generations performed worse. Panel B show the same but for MS employment in Routine jobs, i.e. those occupations hit the most by the joint effect of polarization and the downturn. These jobs were mostly destroyed during the recession, but -once the economy reorganizes and goes back on the expansion path- the polarization trend dominates with MS employment share declining by 7 percentage points. In the aftermath of the Great Recession, MS workers can recover only through Manual occupations, with a 3 points increase for states that polarized more (Panel 3). Worth to notice is the pattern of women: both during the recovery and the expansion, MS women do better than man. This fact suggests that men and women might have different preferences for Manual jobs, with men less willing to downgrade towards easier and less rewarding jobs. This would explain the higher non participation of men in the post recession era.

⁶Employment share in Routine jobs.

⁷Model (2) for male sample; (3) for female sample; (4) for 16-to-35 years old cohort; (5) for 36-to-55 years old cohort; (6) for 56-to-75 years old cohort.

Table 1: The effect of Polarization on Post Recession Middle-Skill Employment

	(1)	(2)	(3)	(4)	(5)	(6)
	State	Males	Females	1st cohort	2nd cohort	3rd cohort
Panel A: MS Employment in Abstract Jobs						
Polarization ^{GR} * $\delta_t^{s,Recovery}$	-0.0287*** [0.006]	-0.0221* [0.013]	-0.0288*** [0.010]	-0.0467*** [0.012]	-0.0324** [0.012]	0.0573** [0.024]
Polarization ^{GR} * $\delta_t^{s,Expansion}$	-0.0201 [0.028]	0.0129 [0.031]	-0.0423 [0.038]	0.0152 [0.035]	-0.0667** [0.032]	0.0195 [0.057]
R^2	0.261	0.123	0.164	0.169	0.156	0.155
Panel B: MS Employment in Routine Jobs						
Polarization ^{GR} * $\delta_t^{s,Recovery}$	0.00466 [0.005]	0.0185 [0.020]	-0.0134 [0.014]	0.00887 [0.014]	-0.00120 [0.016]	-0.00974 [0.030]
Polarization ^{GR} * $\delta_t^{s,Expansion}$	-0.0671*** [0.024]	-0.101** [0.045]	-0.0397 [0.042]	-0.0688 [0.041]	-0.0560 [0.035]	-0.115** [0.048]
R^2	0.667	0.340	0.431	0.368	0.276	0.120
Panel C: MS Employment in Manual Jobs						
Polarization ^{GR} * $\delta_t^{s,Recovery}$	0.0338*** [0.006]	0.0153 [0.013]	0.0489*** [0.012]	0.0503*** [0.014]	0.0440*** [0.014]	-0.0490* [0.026]
Polarization ^{GR} * $\delta_t^{s,Expansion}$	0.0590 [0.036]	0.0641 [0.056]	0.0548* [0.032]	0.0202 [0.052]	0.101* [0.054]	0.0676 [0.050]
R^2	0.130	0.061	0.122	0.068	0.191	0.033
State Level Controls	Yes	Yes	Yes	Yes	Yes	Yes
Demographic Dynamics	Yes	Yes	Yes	Yes	Yes	Yes
Expansion Dummy	Yes	Yes	Yes	Yes	Yes	Yes
Observations	919	919	919	919	919	919

Standard errors in brackets

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 2: The effect of Polarization on Post Recession High-Skill Employment

	(1)	(2)	(3)	(4)	(5)	(6)
	State	Males	Females	1st cohort	2nd cohort	3rd cohort
Panel A: HS Employment in Abstract Jobs						
Polarization ^{GR} * $\delta_t^{s,Recovery}$	-0.0351** [0.016]	-0.0301 [0.024]	-0.0381* [0.019]	-0.0880*** [0.025]	-0.0144 [0.016]	0.00613 [0.040]
Polarization ^{GR} * $\delta_t^{s,Expansion}$	-0.0513 [0.037]	0.0545 [0.041]	-0.132*** [0.041]	-0.121** [0.051]	-0.0164 [0.052]	0.00784 [0.072]
R^2	0.301	0.148	0.250	0.173	0.225	0.103
Panel B: HS Employment in Routine Jobs						
Polarization ^{GR} * $\delta_t^{s,Recovery}$	0.0236* [0.014]	0.0170 [0.021]	0.0268* [0.015]	0.0637*** [0.021]	0.0151 [0.013]	-0.0312 [0.039]
Polarization ^{GR} * $\delta_t^{s,Expansion}$	0.0240 [0.031]	-0.0554* [0.032]	0.0869** [0.040]	0.0690 [0.056]	0.00545 [0.041]	-0.0146 [0.064]
R^2	0.281	0.134	0.240	0.154	0.248	0.096
Panel C: HS Employment in Manual Jobs						
Polarization ^{GR} * $\delta_t^{s,Recovery}$	0.00526 [0.009]	0.00790 [0.015]	0.00387 [0.013]	0.0190 [0.013]	-0.00720 [0.008]	0.0169 [0.019]
Polarization ^{GR} * $\delta_t^{s,Expansion}$	-0.00552 [0.015]	-0.0260 [0.028]	0.0116 [0.025]	0.0229 [0.028]	-0.0283 [0.020]	-0.0131 [0.023]
R^2	0.130	0.061	0.122	0.068	0.191	0.033
State Level Controls	Yes	Yes	Yes	Yes	Yes	Yes
Demographic Dynamics	Yes	Yes	Yes	Yes	Yes	Yes
Expansion Dummy	Yes	Yes	Yes	Yes	Yes	Yes
Observations	919	919	919	919	919	919

Standard errors in brackets

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 3: The effect of Polarization on Post Recession Low-Skill Employment

	(1)	(2)	(3)	(4)	(5)	(6)
	State	Males	Females	1st cohort	2nd cohort	3rd cohort
Panel A: LS Employment in Abstract Jobs						
Polarization ^{GR} * $\delta_t^{s,Recovery}$	0.0156 [0.018]	-0.00967 [0.020]	0.0234 [0.027]	0.0366** [0.016]	0.0208 [0.034]	-0.0379 [0.050]
Polarization ^{GR} * $\delta_t^{s,Expansion}$	-0.0797 [0.064]	-0.0225 [0.050]	-0.0827 [0.084]	0.00972 [0.038]	-0.141 [0.096]	-0.176*** [0.062]
R^2	0.074	0.058	0.064	0.083	0.045	0.070
Panel B: LS Employment in Routine Jobs						
Polarization ^{GR} * $\delta_t^{s,Recovery}$	0.00815 [0.044]	0.150** [0.068]	-0.0858 [0.052]	0.0865* [0.051]	-0.00407 [0.076]	-0.177 [0.116]
Polarization ^{GR} * $\delta_t^{s,Expansion}$	-0.0333 [0.064]	-0.205* [0.120]	0.0585 [0.115]	-0.0131 [0.103]	-0.0261 [0.076]	0.0800 [0.175]
R^2	0.126	0.070	0.080	0.048	0.063	0.087
Panel C: LS Employment in Manual Jobs						
Polarization ^{GR} * $\delta_t^{s,Recovery}$	-0.0214 [0.048]	-0.161** [0.063]	0.0643 [0.056]	-0.128*** [0.047]	-0.00937 [0.076]	0.197* [0.100]
Polarization ^{GR} * $\delta_t^{s,Expansion}$	0.0935 [0.077]	0.176 [0.117]	0.00996 [0.115]	-0.0353 [0.093]	0.135 [0.115]	0.0617 [0.210]
R^2	0.158	0.174	0.089	0.136	0.068	0.090
State Level Controls	Yes	Yes	Yes	Yes	Yes	Yes
Demographic Dynamics	Yes	Yes	Yes	Yes	Yes	Yes
Expansion Dummy	Yes	Yes	Yes	Yes	Yes	Yes
Observations	919	919	919	919	919	919

Standard errors in brackets

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 2 shows results for HS. As reported in Panel A, local labor markets that polarized more during the recession were not able to absorb new HS workers into Abstract jobs afterwards. In fact, during the recovery, we observe a fall of HS employment share in Abstract occupations. Such a fall is particularly strong for women and younger cohorts. This result is in line with Beaudry et al. (2013), where they document a great reversal in HS demand for cognitive jobs in the aftermath of the 2001 recession, with HS workers moving down the job ladder. The same happens here: the only possible occupation in which the share of HS employment could grow during the recovery was a Routine one (Panel B), with women more prone to downgrade than men. However, such a reversal of demand of HS workers in Abstract jobs seems to be finished during the next expansion. As Panel C suggests, Manual jobs could not significantly absorb HS workers after the recession.

Finally, Table 3 shows results for LS workers. Although some unpredictable case for some specific demographic groups, mainly due to the high volatility of LS employment and the small size of the LS sample, the shift in the occupational structure does not benefit this skill group at all.

These results shed light on the reallocation process of human capital, but bring new insights. In fact, differently from McLaughlin and Bils (2001) where it is shown that efficient employment allocation is procyclical -larger mismatch occurs in downturns, but efficiency is restored with employment moving to more productive sectors as the economy recovers- here this is true for HS workers only. In fact, they use the ladder to downgrade in recession only, but go back to their most productive and rewarding job once the economy goes back to its expansion path. This is not true for MS workers: since Routine jobs do not recover because of polarization and the crisis, their mismatch into Manual jobs is persistent during the new expansion phase.

4.2 Polarization, Unemployment and Non Participation

Here I estimate again the same model, now with group j unemployment rate as dependent variables. Controlling for population dynamics and state-level production, we observe that polarization affected mostly MS workers, in particular during expansion periods (Table 4, Panel B). HS and LS employment are not affected by past polarization, at least at state level (Panel A and C).

When considering Non Participation (Table 5), the effects are larger. In particular, States that polarized more by destroying Routine occupations heavily pushed MS workers (in particular MS males) out of the labor force, followed by LS ones.

Table 4: The effect of Polarization on Post Recession Unemployment

	(1) State	(2) Males	(3) Females	(4) 1st cohort	(5) 2nd cohort	(6) 3rd cohort
Panel A: HS Unemployment						
Polarization ^{GR} * $\delta_t^{s,Recovery}$	-0.0102 [0.007]	-0.0101** [0.005]	-0.000176 [0.003]	-0.0000698 [0.004]	-0.00842* [0.004]	-0.00171 [0.002]
Polarization ^{GR} * $\delta_t^{s,Expansion}$	0.0170 [0.017]	0.00531 [0.012]	0.0117 [0.008]	0.00410 [0.007]	0.00762 [0.010]	0.00557 [0.009]
R^2	0.142	0.109	0.098	0.055	0.117	0.042
Panel B: MS Unemployment						
Polarization ^{GR} * $\delta_t^{s,Recovery}$	0.00948 [0.009]	-0.00430 [0.005]	0.0138** [0.007]	0.000642 [0.005]	0.00352 [0.004]	0.00530** [0.002]
Polarization ^{GR} * $\delta_t^{s,Expansion}$	0.0463*** [0.016]	0.0225** [0.009]	0.0238** [0.010]	0.0171 [0.013]	0.0208*** [0.007]	0.00891** [0.004]
R^2	0.080	0.072	0.085	0.031	0.103	0.128
Panel C: LS Unemployment						
Polarization ^{GR} * $\delta_t^{s,Recovery}$	0.0315 [0.032]	0.00847 [0.019]	0.0231 [0.020]	0.00140 [0.029]	0.0295*** [0.010]	0.000757 [0.007]
Polarization ^{GR} * $\delta_t^{s,Expansion}$	0.0597 [0.060]	-0.0457 [0.041]	0.105*** [0.030]	0.0482 [0.042]	0.0150 [0.035]	-0.00286 [0.009]
R^2	0.137	0.123	0.114	0.108	0.104	0.036
State Level Controls	Yes	Yes	Yes	Yes	Yes	Yes
Demographic Dynamics	Yes	Yes	Yes	Yes	Yes	Yes
Expansion Dummy	Yes	Yes	Yes	Yes	Yes	Yes
Observations	919	919	919	919	919	919

Standard errors in brackets

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

5 On the Role of Education around the Great Recession

In this section I loose the classification of skill groups commonly used in the literature when mapping education to job tasks. In practice, I introduce a broader spectrum of educational attainments and study which degree matters most and when for an unemployed individual to be hired. This will help to (i) confirm who was mostly hurt by the recession

Table 5: The effect of Polarization on Post Recession Non Participation

	(1) State	(2) Males	(3) Females	(4) 1st cohort	(5) 2nd cohort	(6) 3rd cohort
Panel A: HS Non Participation						
Polarization ^{GR} * $\delta_t^{s,Recovery}$	0.0152 [0.011]	-0.00710 [0.007]	0.0223** [0.009]	0.00999 [0.007]	0.000595 [0.009]	0.00457 [0.008]
Polarization ^{GR} * $\delta_t^{s,Expansion}$	0.0118 [0.025]	-0.00362 [0.018]	0.0155 [0.021]	0.0173 [0.012]	0.0100 [0.017]	-0.0154 [0.021]
R^2	0.348	0.144	0.274	0.113	0.008	0.330
Panel B: MS Non Participation						
Polarization ^{GR} * $\delta_t^{s,Recovery}$	0.0553*** [0.010]	0.0373*** [0.007]	0.0180*** [0.006]	0.0178*** [0.006]	0.00199 [0.004]	0.0355*** [0.007]
Polarization ^{GR} * $\delta_t^{s,Expansion}$	0.0396** [0.015]	0.0152 [0.017]	0.0244 [0.019]	0.0299* [0.015]	-0.00555 [0.007]	0.0153 [0.016]
R^2	0.657	0.533	0.438	0.376	0.064	0.488
Panel C: LS Non Participation						
Polarization ^{GR} * $\delta_t^{s,Recovery}$	0.0428*** [0.013]	0.0485*** [0.017]	-0.00574 [0.020]	0.0685*** [0.010]	0.000920 [0.011]	-0.0266* [0.013]
Polarization ^{GR} * $\delta_t^{s,Expansion}$	0.0804** [0.037]	0.0737** [0.028]	0.00671 [0.037]	0.0182 [0.033]	0.0259 [0.029]	0.0362* [0.021]
R^2	0.407	0.171	0.158	0.466	0.036	0.071
State Level Controls	Yes	Yes	Yes	Yes	Yes	Yes
Demographic Dynamics	Yes	Yes	Yes	Yes	Yes	Yes
Expansion Dummy	Yes	Yes	Yes	Yes	Yes	Yes
Observations	919	919	919	919	919	919

Standard errors in brackets

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

and who was most likely to be hired during the recovery, (ii) if there was up-skilling within each occupation and (iii) it will shed light on the role of education over the cycle.

To do so, now I use CPS survey data to look at flows from unemployment. In particular, similarly to Moscarini and Thomsson (2006), I consider a sample of 25 to 55 years old unemployed individuals interviewed in two consecutive months and whose unemployment spell is below or equal to 4 weeks. Each individual unemployed in the first month can flow to one of the three jobs⁸, remain unemployed or flow to non participation in the next month. Therefore, a discrete choice model (i.e. a multinomial logit) can account for the odds of each individual to flow out of unemployment as a function of individual characteristics and business cycle phases. With the baseline choice normalized to remain unemployed, the unconditional probability for individual j to flow (F) from unemployment (U), Abstract (A), Routine (R), Manual (M) or non participation (NLF) between t and $t+1$ can be written as:

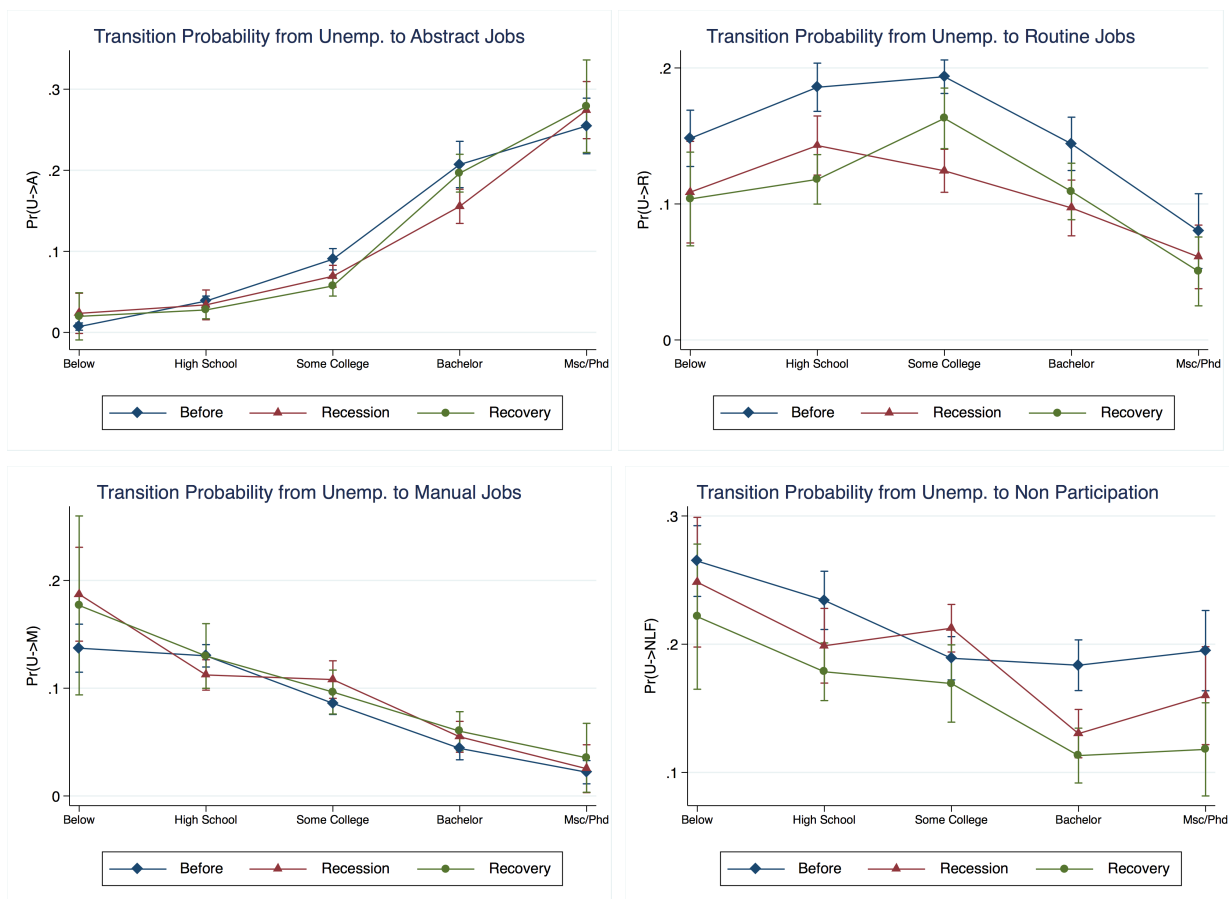
$$Pr(F_{j,t+1}^k | U_{j,t}) = \frac{\exp(\Gamma'_{j,F^k} X_{j,t})}{1 + \sum_k \exp(\Gamma'_{j,F^k} X_{j,t})}$$

where $X_{i,t}$ is a vector of regressors and Γ s are the parameters. The vector X contains individual characteristics: educational attainments, a polynomial of potential experience, marital status, family size, number of children, sex, race, a state dummy and a economic phase dummy accounting for recession and recovery periods. Such a time dummy is interacted with all regressors so to generate the marginal probabilities reported in Figure 6. The direction of the flow is indexed with $k = \{A, R, M, U, NLF\}$.

Consider flows from U to A. By the shape of the probability curve, it is clear that Abstract jobs are typical of highly educated persons. However, the shape changes over different cyclical phases: it is concave in pre-recession periods, it becomes convex during the turmoil and returns concave afterwards. In particular, when the recession hit, a Master/PhD Degree gives 10% more chances to get an Abstract job than a Bachelor Degree and almost 20% more with respect to those with some college education⁹. But if the former catch up during the recovery, the latter do not. In other words, Abstract occupations are experiencing up-skilling, particularly severe during the recession and attenuated afterwards.

⁸I consider flows to employment only for those individuals who report to be in a full time job and not self-employed in the following month. Recalls are excluded. See Appendix B for frequencies.

⁹College dropouts and vocational graduates.



Notice: margins are computed on a sample of 25-to-55 years old individuals with 4 weeks of unemployment spell (maximum). Recalls and self-employed are excluded, as flows from and to manufacture and construction. CPS individual weights used.

Figure 6: Margins

Now consider flows from U to R, i.e. the jobs hit harder by the recession and polarization. The inverted u-shaped curve suggests that middle skill workers (represented by the “High School” and “Some College” category) match more frequently the labor demand in this occupation. When the recession hits, the probability curve shifts down. Only the most skilled ones (i.e. those with Some College) are able to partially recover afterwards. On the other hand, the least educated of the middle skill group (i.e. High School graduates) do not catch up, now facing a probability almost 10% lower than pre-recession periods. Hence, there is a shift in the skill distribution towards higher degrees in post-recession periods so that also Routine jobs are experiencing up-skilling. In addition, the fact that the curve does not shift up again to pre recession levels confirm the disappearance of Routine jobs.

Regarding flows from U to M, the marginal probability is decreasing in education indicating that these jobs are typical of low skilled workers (here represented by the “Below” category). Even though these jobs do not require any particular skill, still there is a

slight increase in marginal probabilities for higher education during recession and recovery. Such increase becomes larger as long as individuals with a longer unemployment spell are included in the sample, thus suggesting that Manual jobs could represent an ultimate outside option for more educated agents in case they cannot find a better job. Notice also that here there is no cyclicity in transition rates to M.

Finally, the probability to flow from U to non participation spikes for middle skilled workers during recession thus confirming previous results: not able to upgrade or to go back to previous occupation or not willing to downgrade, they prefer to stop participating.

To sum up, educational attainments matter most during bad times, in particular to access middle and top jobs. This sheds light on the mismatch process and the importance of skills and skill requirements over the cycle. Moreover, it suggests why a large mass of MS population -not enough educated to get at an Abstract job but too much educated for a Manual one- prefers to move out of the labor force. This confirms what found in Modestino et al. (2015). By using very sophisticate firm-level data on vacancies and vacancy requirements (for different occupations), it shows that there is up-skilling across occupations, mainly due to higher skill requirements. This is because employers opportunistically raise education requirements within occupations in response to increases in the supply of job seekers, i.e. in response to higher unemployment.

6 Wages and Mismatch Penalty

As shown in the pioneering studies by Schultz, Becker and Mincer, education boosts earnings. Yet there are other several channels that can explain returns on education and wage differentials. Here I study how the education-to-occupation match affects wages and how a good (bad) match leads to a wage gain (penalty). The idea is that human capital is productive only if matched to a specific job, i.e. only if associated to a specific technology that allows the worker to deliver a specific task. For example, a Wall Street trader can deliver a cognitive performance (buying and selling stocks) only if his human capital (say a Master in Finance) is associated to an appropriate technology (say a platform of trading). If so, the match productivity affects his earnings. Of course, an imperfect match can be formed too, thus leading to a lower or higher productivity depending whether the worker moves up or down the job ladder. In this case there is a wage penalty or gain. As it is clear, the link between human capital and technology and how much they complement is fundamental in wage determination (see Goldin and Katz (2007) and Krusell et al. (2000)).

To study wages I consider a subsample of the data used in the previous section¹⁰: (25-55) years old individuals who flowed from unemployment to a full-time occupation and that are neither re-entrant (recalls) nor self-employed, whose unemployment spell was below or equal to 4 weeks, and who reported their hourly wage.

Following a simple Mincerian approach and recent developments in the literature (Lemieux (2014), Fortin et al. (2012) and Firpo et al. (2011)), I propose the following wage equation:

$$\begin{aligned} \log(w_j) = & \alpha_j + \beta_1 \mathcal{M}(j, k) + \beta_2 Experience_j + \beta_3 Experience_j^2 + \gamma_1 [Experience_j * \mathcal{M}(j, k)] \\ & + \gamma_2 [Experience_j^2 * \mathcal{M}(j, k)] + \eta_1 X_j + \eta_2 Sector_j + \eta_3 LastJob_j + \delta + \varepsilon_j \end{aligned}$$

where α captures the unobservable individual ability; $\mathcal{M}(j, k)$ is matching vector collecting dummies that take value 1 if -for any j and k - worker j is matched to occupation k so that β_1 is a vector collecting education-to-job productivity coefficients; $Experience$ and $Experience^2$ accounts for worker's potential experience (i.e. age - years of education). Since experience can play a role in the attenuation of wage penalties in case of mismatch down (or up) the job ladder, I interact it with the matching vector. Finally, X controls for demographic characteristics (marital status, race, family size, number of children); $Sector$ is a dummy for the sector where the worker has been hired, $LastJob$ is a dummy indicating agent j last reported job; δ is an economic phase time dummy; ε is the error term. Table 6 shows estimates: column (1) and (2) is for the entire sample (without and with interactions), columns (3) to (6) repeat for the male and female subsamples; in all models the baseline case is a low-skill individual matched to a Manual job during the recession phase. All results must be read in light of the facts shown in Section 4 and 5: (i) more frequent mismatches happen from the top to the bottom of the job ladder, and (ii) there is up-skilling within Routine and Abstract occupations during recession and recovery.

Consider column (1) and HS workers first. Moving from an Abstract to a Routine job implies a significant productivity penalty of 0.3 points, thus making HS workers the most damaged group for a downgrade to the next qualifying job. On the other hand, bringing their knowledge to a Manual occupation do not give any benefit: the HS worker accepts the lowest wage on the job ladder, i.e. the baseline wage. MS workers have a significant

¹⁰Individual from CPS Outgoing Rotation Group.

Table 6: Wages and education-to-occupation (mis)match

	(1)	(2)	(3)	(4)	(5)	(6)
	All	All	Males	Males	Females	Females
$\mathcal{M}(ls, R)$	0.129*** [0.047]	0.0465 [0.102]	0.164 [0.101]	0.159 [0.166]	0.0209 [0.049]	-0.00572 [0.138]
$\mathcal{M}(ls, A)$	0.0841 [0.082]	0.00559 [0.097]	0.276** [0.137]	0.0631 [0.161]	-0.0692 [0.081]	1.123 [0.854]
$\mathcal{M}(ms, M)$	0.0497 [0.040]	-0.0531 [0.107]	0.138 [0.096]	0.0114 [0.171]	0.0183 [0.041]	-0.0915 [0.133]
$\mathcal{M}(ms, R)$	0.243*** [0.040]	0.193** [0.088]	0.241** [0.099]	0.393** [0.162]	0.236*** [0.040]	0.00978 [0.094]
$\mathcal{M}(ms, A)$	0.379*** [0.054]	0.0218 [0.153]	0.441*** [0.125]	0.714** [0.360]	0.350*** [0.058]	-0.164 [0.160]
$\mathcal{M}(hs, M)$	-0.00847 [0.071]	-0.129 [0.193]	-0.131 [0.139]	-0.0667 [0.267]	0.0632 [0.078]	0.0409 [0.234]
$\mathcal{M}(hs, R)$	0.379*** [0.054]	0.187** [0.093]	0.526*** [0.116]	0.394** [0.167]	0.282*** [0.053]	0.0540 [0.104]
$\mathcal{M}(hs, A)$	0.612*** [0.063]	0.371*** [0.124]	0.833*** [0.138]	0.720*** [0.263]	0.544*** [0.068]	0.239* [0.131]
<i>Experience</i>	0.0147*** [0.004]	-0.00426 [0.008]	0.00667 [0.006]	-0.00143 [0.018]	0.0167*** [0.005]	-0.00689 [0.008]
<i>Experience</i> ²	-0.000286*** [0.000]	0.000175 [0.000]	-0.0000671 [0.000]	0.000322 [0.000]	-0.000330*** [0.000]	0.000160 [0.000]
$\mathcal{M}(j, k) * \textit{Experience}$	No	Yes	No	Yes	No	Yes
$\mathcal{M}(j, k) * \textit{Experience}^2$	No	Yes	No	Yes	No	Yes
Demographic Controls	Yes	Yes	Yes	Yes	Yes	Yes
Economic Phase	Yes	Yes	Yes	Yes	Yes	Yes
Sector	Yes	Yes	Yes	Yes	Yes	Yes
Last Job	Yes	Yes	Yes	Yes	Yes	Yes
Observations	2765	2765	1066	1066	1699	1699
<i>R</i> ²	0.216	0.222	0.204	0.219	0.265	0.278

Standard errors in brackets

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

productive match when working in a Routine occupation, but being matched to an Abstract job leads to a higher wage, even though smaller than wages of HS peers in the same job. In other words, HS workers benefit of a skill premium in the Abstract market that cannot be gained by MS workers through experience. This is in line with what found so far: demand of MS workers in Abstract jobs falls, with wages and quantities moving in the same direction. On the other hand, when they move down the job ladder they accept the same wage of a low skill worker. Finally, LS workers have a benefit only if upgraded to a Routine job. Notice also that there is a skill premium in the Routine market, while there is an occupation premium when climbing up the job ladder. Experience affects wages in a concave fashion.

Now consider model (2) where interactions with *Experience* and *Experience*² are added so that the vector β_1 now captures the match productivity coefficients for (potential) new entrant¹¹. The same conclusions hold for high skill workers: both Routine and Abstract market are willing to reward HS workers in accomplishing the same tasks of their more unskilled peers, although not experienced. This gives evidence to the fact that the skill demand in Routine jobs is actually changing. Finally, MS and LS workers can upgrade to a higher wage in a more qualifying job only through experience. In fact, their productivity match is significant only if their qualifications are in line with their job.

As shown in columns (3) to (6), even though females do better during recession periods, there is a gender gap within Routine and Abstract jobs for both middle and high skill workers. In fact, if not a Manual worker or if not a Routine and experienced MS worker, men benefit of larger and significant matches than women. Experience matters the most for women, but it does not allow to close the wage gap. Unexperienced men performs better in the Routine and Abstract sector.

7 A Model of Endogenous Skill Requirements and Up-Skilling

To sum up, the empirical analysis conducted so far led to the following conclusions: (i) polarization and the crisis accelerate the process of destruction of Routine Jobs; (ii) there is evidence of up-skilling -a rise in skill and educational requirements- in Routine and (more pronounced) in Abstract jobs during the recession; (iii) large mismatches occur down the job ladder, with more HS workers getting Routine jobs and MS workers getting

¹¹See Appendix C, Table 7 and 8 for all interactions and controls.

Manual ones during the recovery; (iv) being mismatched has a cost and the quality of the match has an impact on wage; (v) as a reaction to the lack of job opportunities, many MS workers left the labor force for non participation.

In this section, I build a theoretical model of labor search and matching that captures the first four patterns observable in the data. To make things easier, I develop a simple model with only two categories of workers and two jobs. The set up is based on Albrecht and Vroman (2002) and it reconciles results of models of skill-biased technical change, direct technical change and job polarization as in Acemoglu (2002), Acemoglu and Autor (2011) and Jaimovich and Siu (2012). The labor market frictions are modeled under the framework of Pissarides (1985) and Mortensen and Pissarides (1994).

7.1 Set Up

Assume there is an unitary population exogenously divided in two types of agents: a share p of MS individuals, and a share $1 - p$ of HS ones. Moreover, assume that each agent i in the population is endowed of an individual skill level $y_i \stackrel{i.i.d}{\sim} dG(y)$, with $dG(\cdot)$ being a continuous distribution function on the $[0, 1]$ support. Therefore, for an exogenously given threshold \bar{y} , HS agents are characterized by a skill level $y_i \geq \bar{y}$, while MS agents are characterized by a skill level $y_i < \bar{y}$. In this way, agents are continuously ranked from the worst to the best not only over the entire population (respectively $y_i = 0$ and $y_i = 1$), but also across categories (respectively $y_i = 0$ and $y_i = \lim_{y \rightarrow \bar{y}} y_i$ for MS, and $y_i = \bar{y}$ and $y_i = 1$ for HS).

In this world, there are two jobs. Abstract jobs requires to be HS, while Routine jobs require to be at least MS. In other words, both HS and MS agents can fill Routine vacancies such that the labor market is not perfectly segmented by educational attainment. When working in a Routine job, a MS worker is subject to an endogenous skill cut-off: if his skill level falls below a certain value $\varepsilon_1 \in [0, \bar{y})$ he will be fired. Similarly, when working in a Abstract job, a HS worker is subject to an endogenous skill cut-off: if his individual skill level falls below a certain value $\varepsilon_2 \in [\bar{y}, 1]$ he will be fired. However, for a HS agent working in a Routine job, the minimum skill necessary to keep the job is not binding because -by definition- his skill level will be always above \bar{y} and therefore above any possible cut-off ε_1 . Figure 7 summarizes the features of the skills distributions and cut-offs.

Both Routine and Abstract employers use the following matching function:

$$m(u, v) = m\left(1, \frac{v}{u}\right)u = m(\theta)u$$

where u is the unemployment rate for the entire population, v is the total number of

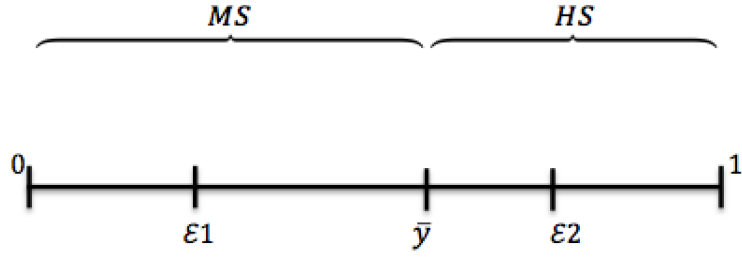


Figure 7: Individual Skills Distribution

vacancies posted by both firms together, $\theta = \frac{v}{u}$ is market tightness. The function $m(\cdot)$ is such that $m'(\theta) > 0$ and $\lim_{\theta \rightarrow 0} m(\theta) = 0$. In the universe of vacancies v , a fraction ϕ is opened for Routine occupations while a fraction $1 - \phi$ for Abstract ones. Finally, assume that there is a fraction γ of MS unemployed workers and a fraction $1 - \gamma$ of HS unemployed workers. Hence, I can define

$$\frac{m(\theta)}{\theta}$$

as the rate at which vacancies meet unemployed workers. Whereas, we define

$$\frac{\gamma m(\theta)}{\theta}$$

and

$$\frac{(1 - \gamma)m(\theta)}{\theta}$$

as the effective arrival rates respectively for MS and HS workers. When hired, individual skills are matched with a job specific technology z^R or z^A , respectively if working in Routine or Abstract occupations. These technologies move according to the following diffusion processes:

$$\xi = \begin{cases} dz^R = -\rho z^R dt + \sigma dB \\ dz^A = \bar{z}^A - \rho z^A dt + \sigma dB \end{cases}$$

with dB being a standard Brownian motion. Notice that -although the two processes have same persistence and volatility- technology in Abstract jobs is moving towards a long run (higher) level \bar{z}^A . In this way, I introduce Skill Biased Technical Change (SBTC) and allow also for random shocks to hit productivity in both occupations.

In light of this, now I can write the value functions characterizing the demand and supply of labor in the economy for each type of worker and employer.

7.1.1 Value Functions

Time is continuous. Conditional on being HS ($y \geq \bar{y}$), the value of being employed in an Abstract job is:

$$rN^{hs}(y, z^A) = w^{hs}(y, z^A) + \delta \mathbb{E}_{\xi'|\xi} \left[\int_{\varepsilon_2}^1 N^{hs}(s, z^A) dG(s|hs) + U^{hs} \int_{\bar{y}}^{\varepsilon_2} dG(s|hs) - N^{hs}(y, z^A) \right] \quad (1)$$

while, conditional on being MS ($y < \bar{y}$), the value of being employed in a Routine job is:

$$rN^{ms}(y, z^R) = w^{ms}(y, z^R) + \delta \mathbb{E}_{\xi'|\xi} \left[\int_{\varepsilon_1}^{\bar{y}} N^{ms}(s, z^R) dG(s|ms) + U^{ms} \int_0^{\varepsilon_1} dG(s|ms) - N^{ms}(y, z^R) \right] \quad (2)$$

where δ is the separation rate and $w^{hs}(y, z^A)$ and $w^{ms}(y, z^R)$ are the wages paid to HS and MS workers in the two occupations and are functions of their individual skill. Notice that, for both agent types, the value of employment depends on the value of continuation while the value of unemployment enters weighted by the probability of not satisfying the minimum skill level.

Differently, conditional on being HS ($y \geq \bar{y}$), the value of being employed in a Routine job is simply

$$rN^{hs}(y, z^R) = w^{hs}(y, z^R) + \delta \mathbb{E}_{\xi'|\xi} \left[U^{hs} - N^{hs}(y, z^R) \right] \quad (3)$$

because these agents can never fall under the minimum requirement ε_1 .

The value of production for a HS agent into an Abstract job is:

$$rJ^{hs}(y, z^A) = z^A y - w^{hs}(y, z^A) + \delta \mathbb{E}_{\xi'|\xi} \left[\int_{\varepsilon_2}^1 J^{hs}(s, z^A) dG(s|hs) - J^{hs}(y, z^A) \right] \quad (4)$$

while the value of production for a MS agent into a Routine job is:

$$rJ^{ms}(y, z^R) = z^R y - w^{ms}(y, z^R) + \delta \mathbb{E}_{\xi'|\xi} \left[\int_{\varepsilon_1}^{\bar{y}} J^{ms}(s, z^R) dG(s|ms) - J^{ms}(y, z^R) \right] \quad (5)$$

where z^A and z^R (with $z^A > z^R$) is the technology available in Abstract and Routine productions. Notice that the value of production depends again on continuation for both type of agents. Differently, the value of production for a HS worker in a Routine job is independent on the continuation value imposed by ε_1 . So it reduces to:

$$rJ^{hs}(y, z^R) = z^R y - w^{hs}(y, z^R) - \delta \mathbb{E}_{\xi'|\xi} \left[J^{hs}(y, z^R) \right]. \quad (6)$$

The value of unemployment depends on the type of worker. In fact, for HS workers we have:

$$rU^{hs} = b + m(\theta)\mathbb{E}_{\xi'|\xi}\left\{\phi[N^{hs}(y, z'^R) - U^{hs}] + (1 - \phi)[N^{hs}(y, z'^A) - U^{hs}]\right\} \quad (7)$$

i.e. the value of leisure for HS workers depends on unemployment benefit b and the weighted probability of being matched in a Routine occupation or being mismatched in an Abstract occupation. Differently, the value of unemployment for MS workers is just:

$$rU^{ms} = b + m(\theta)\phi\mathbb{E}_{\xi'|\xi}\left[N^{ms}(y, z'^R) - U^{ms}\right] \quad (8)$$

Finally, Abstract and Routine employers face different values of posting a vacancy:

$$rV^A = -c + \frac{m(\theta)}{\theta}(1 - \gamma)\mathbb{E}_{\xi'|\xi}\left[J^{hs}(y, z'^A) - V^A\right] \quad (9)$$

$$rV^R = -c + \frac{m(\theta)}{\theta}\mathbb{E}_{\xi'|\xi}\left\{\gamma[J^{ms}(y, z'^R) - V^R] + (1 - \gamma)[J^{hs}(y, z'^R) - V^R]\right\} \quad (10)$$

where c is the costs of posting a vacancy.

7.1.2 Equilibrium Conditions

Given this set up, now I proceed by characterizing the economy at its deterministic steady state. Such equilibrium it will define not only the (mis)match of each category of workers within each occupation but also which subgroups will be fired because not satisfying skills/productivity requirement within each job. In this sense, this model captures both endogenous skill requirement (as in Albrecht and Vroman (2002)), but also up-skilling as a form of job destruction of the least productive/skilled workers within each job (Mortensen and Pissarides (1994)).

Nash Bargaining and Wages For every agent with individual skill level y belonging to category $i = \{ms, hs\}$ and employable in job $k = \{R, A\}$, the sharing rule is

$$N^i(y, z^k) - U^i = \beta[J^i(y, z^k) + N^i(y, z^k) - V^k - U^i].$$

Using the definition of value function above combined with the sharing rule, wages are so defined¹²:

$$w^{hs}(y, z^A) = \beta z^A y + (1 - \beta)[rU^{hs}] \quad (11)$$

$$w^{ms}(y, z^R) = \beta z^R y + (1 - \beta)[rU^{ms}] \quad (12)$$

¹²See Appendix D.1, D.2 and D.3 for details.

$$w^{hs}(y, z^R) = \beta z^R y + (1 - \beta)[rU^{hs}]. \quad (13)$$

As it is clear, the wage depends on individual skill level and the value of unemployment, but -since HS workers can access to more markets, it easy to show that $rU^{hs} > rU^{ms}$. As a consequence, even though wages are posted at a common productivity level, HS workers will earn always a higher wage than their MS counterpart when working in a Routine job¹³.

Job Creation and Job Destruction Consider the Abstract market first. Using the equilibrium condition $V^A = 0$ and the fact that employers post jobs at the highest skill level available among HS workers ($y = 1$), we define the following job creation condition¹⁴:

$$c = \frac{m(\theta)(1 - \gamma)(1 - \beta)}{\theta} \left[\frac{z^A - rU^{hs} + \frac{\delta z^A}{r + \delta} \int_{\varepsilon_2}^1 (s - \varepsilon_2) dG(s|hs)}{r + \delta} \right] \quad (14)$$

while job destruction is

$$z^A \varepsilon_2 = w^{hs}(\varepsilon_2, z^A) + \frac{\delta(1 - \beta)z^A}{r + \delta} \int_{\varepsilon_2}^1 (s - \varepsilon_2) dG(s|hs) \quad (15)$$

Now, consider the Routine market. Using the equilibrium condition $V^R = 0$ and the fact that employers post jobs at the highest skill level available among MS workers ($y = \bar{y}$), we define the following job creation condition¹⁵:

$$c = \frac{m(\theta)(1 - \beta)}{\theta} \left\{ \gamma \left[\frac{z^R \bar{y} - rU^{ms} + \frac{\delta z^R}{r + \delta} \int_{\varepsilon_1}^{\bar{y}} (s - \varepsilon_1) dG(s|ms)}{r + \delta} \right] + (1 - \gamma) \left[\frac{z^R \bar{y} - rU^{hs}}{r + \delta} \right] \right\} \quad (16)$$

while job destruction is

$$z^R \varepsilon_1 = w^{ms}(\varepsilon_1, z^R) + \frac{\delta(1 - \beta)z^R}{r + \delta} \int_{\varepsilon_1}^{\bar{y}} (s - \varepsilon_1) dG(s|ms) \quad (17)$$

Flows from and to Employment Finally, in equilibrium flows from employment to unemployment and vice versa must equate for both types of agents. This implies that,

¹³The explicit form of rU^{hs} and rU^{ms} as functions of parameters and endogenous variables only are shown in Appendix D.4 and D.5, equation (20) and (21).

¹⁴See Appendix D.4 for details.

¹⁵See Appendix D.5 for details.

conditional on $y \geq \bar{y}$, the equilibrium condition for HS employment is

$$\delta[(1-p) - (1-\gamma)u] \int_{\bar{y}}^{\varepsilon_2} dG(s|hs) = m(\theta)(1-\gamma)\phi u + m(\theta)(1-\gamma)(1-\phi)u \quad (18)$$

while, conditional $y < \bar{y}$, the equilibrium condition for MS employment

$$\delta[p - \gamma u]G(\varepsilon_1|ms) = m(\theta)\gamma\phi u \quad (19)$$

7.2 Cross-Skill Steady State Equilibrium

In this model, there exist two alternative steady state equilibria, both depending heavily on parameterization. The first is a *cross-skill matching* equilibrium, under which it is beneficial for HS workers to match with Routine vacancy. The second is an *ex-post segmentation* equilibrium, under which Abstract jobs are so numerous and productive that HS workers never accept Routine vacancies. Since data suggest an existence of an equilibrium of the first type, here I treat only the *cross-skill matching* case¹⁶.

Definition 1 *A cross-skill matching equilibrium is a vector $\{\theta^*, u^*, \phi^*, \gamma^*, \varepsilon_1^*, \varepsilon_2^*\}$ satisfying job creation and destruction in each market, free entry condition in each market and conditions on flows for both type of agents, i.e. it solves simultaneously equations (14), (15), (16), (17), (18) and (19).*

Lemma 1 *A cross-skill matching equilibrium exists if Routine employers find profitable to hire HS workers, and HS workers find profitable to accept Routine jobs. This requires*

$$S(hs, z^R) = N^{hs}(\bar{y}, z^R) + J^{hs}(\bar{y}, z^R) - V^R - U^{hs} \geq 0$$

i.e. the surplus from such a match must be positive. From equation (16), this condition reduces simply to

$$z^R \bar{y} - rU^{hs} \geq 0$$

Finally, it is necessary to rule out the *corner solution* for which only Routine vacancies are posted ($\phi = 1$ and $V^R \geq V^A$). This requires a restriction on parameters which ensures that -for any individual productivity level- an interior solution ($\phi < 1$) exists. This condition is:

¹⁶For *ex-post segmentation* equilibria, see Albrecht and Vroman (2002), and Blazquez and Jansen (2003).

$$z^R \bar{y} + X - b < (1 - p) \left[z^A + Y - b + \frac{m(\hat{\theta})\beta(z^A - z^R \bar{y} + Y)}{r + \delta} \right]$$

with $X = \frac{\delta z^R}{r + \delta} \int_0^{\bar{y}} (s - \varepsilon_1) dG(s|ms)$ and $Y = \frac{\delta z^A}{r + \delta} \int_y^1 (s - \varepsilon_2) dG(s|hs)$ and $\hat{\theta}$ uniquely satisfies the *equal value* condition $V^R = V^A = 0$ ¹⁷.

7.3 Model's Predictions

The process governing the diffusion of technology z^A implies that productivity in Abstract jobs is increasing and converging from below¹⁸ to a long-run level \bar{z}^A , so that the entire economy is transitioning from one initial steady state to another along its equilibrium path. As z^A increases during the transition, more Abstract vacancies are posted with respect to Routine ones in order to exploit the increase in productivity. Therefore, this model generates job polarization under SBTC. Moreover, as long as the Abstract sector expands, more HS workers are conveyed from unemployment and the Routine sector into these new jobs. A higher demand for HS workers traduces into a fall in the skill/productivity minimum requirement to access Abstract jobs.

The situation is different for the other skill group. Since MS workers cannot upgrade to Abstract jobs, polarization leads to an increase of MS unemployment. Although fewer Routine vacancies are posted and less MS workers are demanded, also in this market the minimum skill/productivity requirement falls. This is because the value of being a Routine worker decreases so much during polarization that Routine employers can keep these jobs alive (i.e they respect the “no corner solution” condition stated above) only by decreasing requirements and by keeping wages low, i.e. giving access to anyone willing to forgo unemployment benefits for a very sluggish job. To sum up, the SBTC process moves the economy from a cross-skill equilibrium towards a separating one: the switching market is closing as productivity increases in Abstract jobs, employment polarization occurs, requirements are falling in both markets (but for different reasons).

These facts are not only compatible with the evidence provided in previous sections, but also with the idea that workers move towards jobs where they have a comparative advantage. Figure 8 (blue line) summarizes the dynamics of the economy transitioning to the long-run equilibrium under SBTC¹⁹.

¹⁷The *equal value* condition is obtained by equating (14) to (16) under $\phi = 1$. This reduces simply to $c = \frac{m(\theta(1-\beta))}{\theta} [z^R \bar{y} + X - b]$.

¹⁸True if $z_0^A < \frac{\bar{z}^A}{1-\rho}$, with z_0^A being the initial steady-state level of technology in Abstract jobs.

¹⁹The parameters used in the simulation are from Albrecht and Vroman (2002): $r = 0.05$, $p = 2/3$, $b = 0.1$, $\delta = 0.2$, $\beta = 0.5$, $z^A = z_0^A = 1.2$, $z^R = z_0^R = 1$, $m(x) = 2x^2$. To keep things easy, I assume

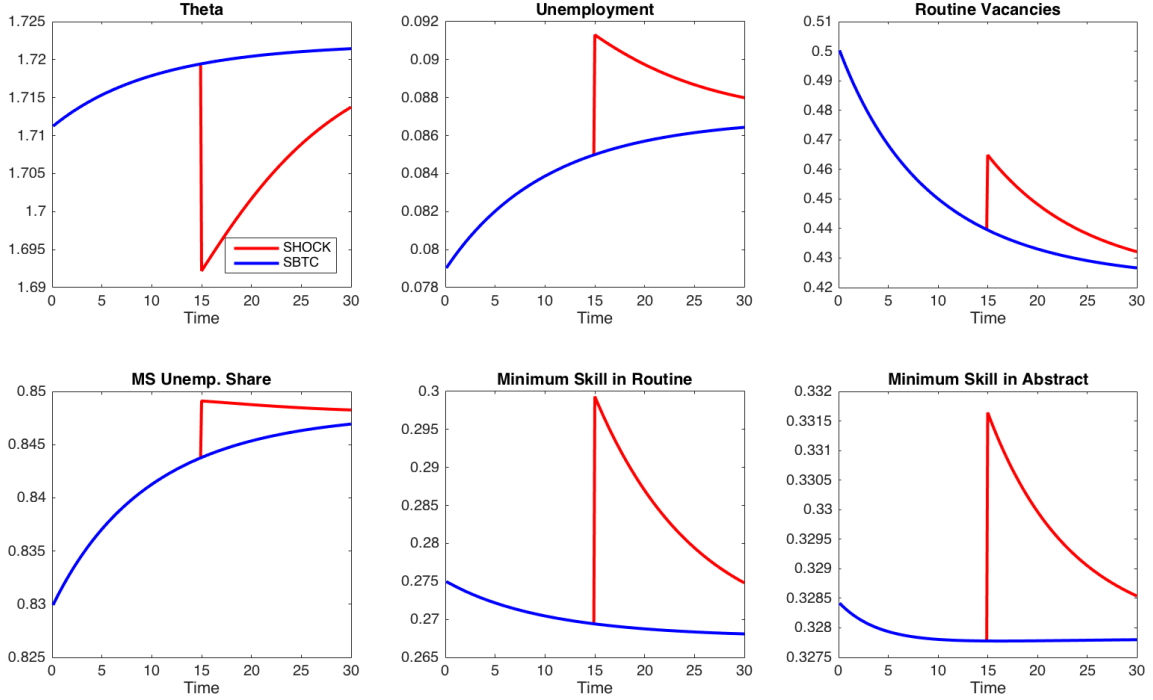


Figure 8: Model's Predictions under SBTC

Now assume that, during the transition, the economy is hit (in period $t = 15$) by an aggregate shock strong enough to destroy a large mass of jobs across sectors (Figure 8, red line)²⁰. What does the model predict? In this scenario, when the shock hits unemployment rises, with more MS workers losing their jobs relative to HS ones. At the same time, given the larger pool of unemployment, skill requirements rises in both markets because employer can select a more productive labor force to fill the new vacancies posted after the shock. In other words, in both markets we observe up-skilling and labor hoarding, with the best agents of each group matched first.

It is important to notice that, when the shock hits, more Routine vacancies are posted. Why? And for whom? The existence of a switching market, here represented by Routine jobs for HS agents, allows HS workers to move from the Abstract sector to the Routine one. This is because now minimum skill requirements in Abstract jobs have risen enough to make it profitable for HS workers to get Routine jobs, i.e. for HS agents the value of a Routine job is increasing relative to the value of an Abstract job. At the same time, Routine employers can exploit the larger productivity of this group to post some productive vacancy. Therefore, such a mutually beneficial match drags the least skilled of the HS group down the job ladder. At the same time the share of MS unemployment increases.

$dG(.) = U_{[0,1]}$ and $\bar{y} = 2/3$. These values still grant existence of a *cross-skill matching* equilibrium under my set-up. The long-run steady state is driven by a higher level of z^A : $\bar{z}^A = 1.5$. I assume $\rho = 0.95$
²⁰ $\sigma = 0.08$ for both technology diffusion processes.

To sum up, under a parameterization ensuring the existence of a *cross-skill matching* equilibrium, this model describes the main dynamics we observe in the data: (i) job polarization, (ii) larger mismatch down the job ladder, (iii) up-skilling across occupations in bad times and (iv) movements from the top to the bottom of the ladder along with wage dispersion. The dynamics that characterizes movements of HS workers down the ladder can be easily replicated for MS workers by including a market for Manual occupations.

These results are in line with recent developments of the theory on capital reallocations, job mobility, and mismatch. In particular, Birchenall (2008) shows how changes in demand and technology can lead to larger mismatches and inefficient allocation of capital and labor in segmented markets; Visschers and Carrillo-Tudel (2013) and Alvarez and Shimer (2011) shows how negative shocks affect search and reallocation unemployment on different jobs, so to explain the high volatility of unemployment fluctuations over the cycle and the long lasting unemployment, unemployment rest and inactivity during downturns. Krolkowski (2014) shows how reallocations on the job ladders and match quality matter to explain wage dispersion and earning losses. Finally, Cortes (2014) shows that, in a polarizing world, workers from the middle move towards Abstract jobs if they have a comparative advantage in that task.

8 Conclusions

This paper provides evidence that the Great Recession and job polarization have violently reshaped the structure of the labor market and influenced the reallocation of human capital in recent years. Three important dynamics come from the data. First, polarization and the recession mostly harmed MS agents by destroying Routine occupations across sectors. Second, after the downturn, MS workers could recover only through a downgrade to Manual jobs, HS workers through a downgrade to Routine jobs, while LS ones were dismissed everywhere. This is because of a rise of minimum skill requirements in both the Abstract and Routine sector that did not allow HS and MS workers to be perfectly matched, thus pushing HS and MS searchers down job ladder. Precisely, during recession periods, only the best individuals of each skill group were most likely to be perfectly matched; the others were misallocated to easier tasks. Third, larger skill-to-job mismatch led to larger wage loss, that could not be attenuated by worker experience. Nonetheless, when mismatched into Routine occupations, high skills are significantly more rewarded than middle skills thus suggesting a shift in the demand for higher skills in these activities. This is not true for any mismatch in Manual jobs: Manual jobs pay the minimum wage on the market, independently on education. Moreover, when the economy goes back on its expansion path, only HS agents reallocate efficiently to Abstract jobs while MS

workers are forced to a persistent downgrade in a polarizing world. These facts shed light on the mismatch process and the importance of skills over the cycle and suggests why MS unemployment was long term.

From the theoretical perspective, I show how a simple model with heterogeneous agents can capture polarization, skill mismatch, and a rise of minimum skill requirements in a fairly easy way. As the model predicts, the existence of switching markets allows heterogeneous agents to move down the ladder. When an aggregate negative shock hits the economy, these markets can even expand. The intuition is that, when the labor market is so sluggish, switching markets represent a better option to unemployment for HS searchers, and an opportunity for employers to hire more skilled and productive workers at a lower wage into a task-easier occupation. When the shock is washed away, matching efficiency is restored with HS workers going back to Abstract occupations, while MS workers are condemned to long term unemployment in a polarizing world.

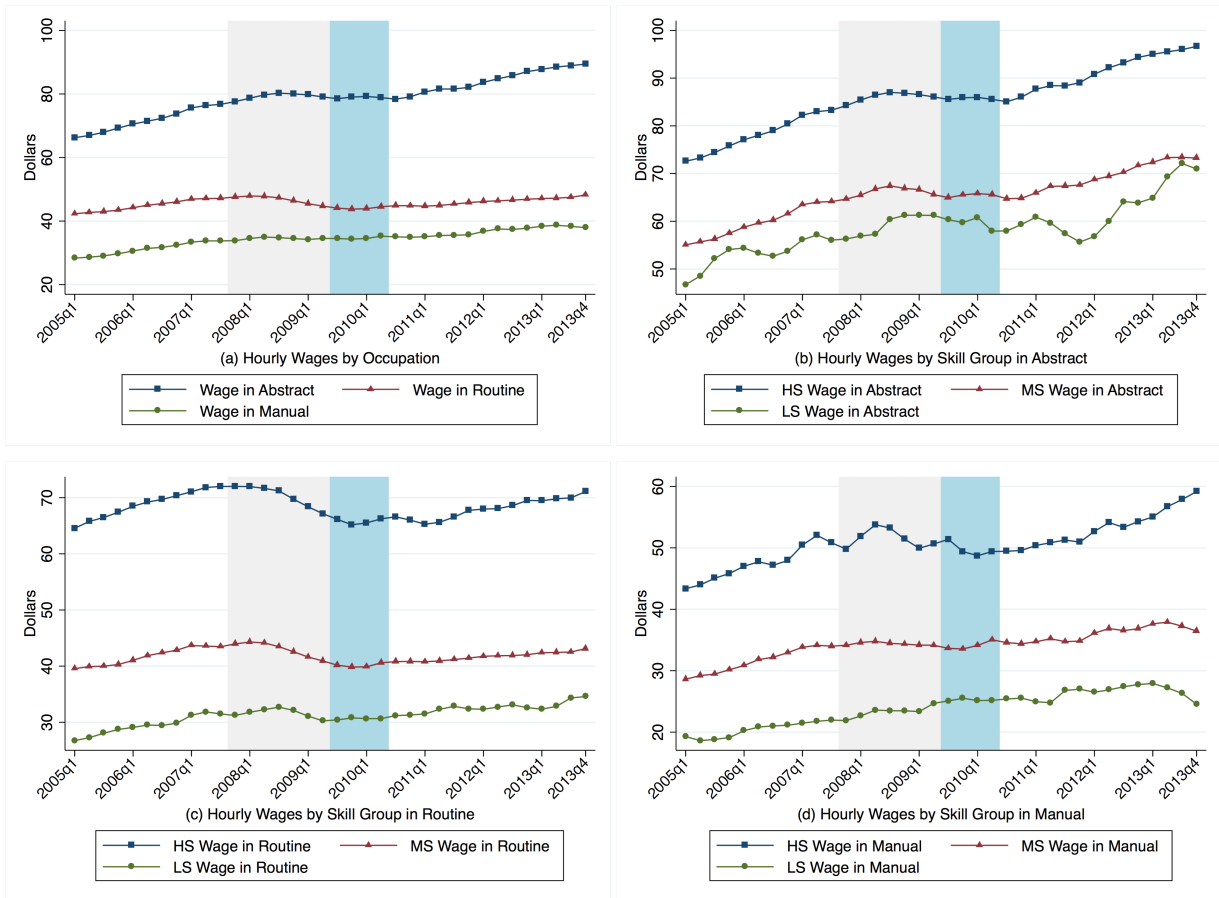
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A Hourly Wages (Levels)



Notice: average values by occupation; manufacture and construction excluded; reference period 2005Q1.

Figure 9: Wage Dynamics

B Summary Statistics for Discrete Choice Model

	Flow to					
Schooling	Abstract	Routine	Manual	Unemp.	Non LF	Total
Below	25	446	327	1165	631	2594
High School	173	1264	659	2929	1259	6284
Some College	455	1159	547	2888	1137	6186
Bachelor	674	479	179	1653	541	3526
Master/Phd	347	94	31	592	195	1259
Total	1674	3442	1743	9227	3763	19846

C Wage Penalty: Controls

Table 7: Wages and education-to-occupation (mis)match (interactions)

	(1)	(2)	(3)	(4)	(5)	(6)
	All	All	Males	Males	Females	Females
$\mathcal{M}(ls, R)*Experience$		0.0170 [0.012]		0.00693 [0.022]		0.0118 [0.015]
$\mathcal{M}(ls, A)*Experience$		0.0392** [0.019]		0.0521** [0.025]		-0.0844 [0.076]
$\mathcal{M}(ms, M)*Experience$		0.0144 [0.012]		0.0308 [0.023]		0.00789 [0.015]
$\mathcal{M}(ms, R)*Experience$		0.0105 [0.010]		-0.00602 [0.020]		0.0262** [0.012]
$\mathcal{M}(ms, A)*Experience$		0.0475** [0.019]		-0.0518 [0.052]		0.0636*** [0.019]
$\mathcal{M}(hs, M)*Experience$		0.0135 [0.030]		-0.0115 [0.048]		-0.00440 [0.038]
$\mathcal{M}(hs, R)*Experience$		0.0259* [0.014]		0.0297 [0.028]		0.0198 [0.014]
$\mathcal{M}(hs, A)*Experience$		0.0262 [0.019]		0.0343 [0.044]		0.0252 [0.020]
Observations	2765	2765	1066	1066	1699	1699
R^2	0.216	0.222	0.204	0.219	0.265	0.278

Standard errors in brackets

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 8: Wages and education-to-occupation (mis)match (interactions)

	(1)	(2)	(3)	(4)	(5)	(6)
	All	All	Males	Males	Females	Females
$\mathcal{M}(ls, R)*Experience^2$		-0.000479*		-0.000304		-0.000384
		[0.000]		[0.001]		[0.000]
$\mathcal{M}(ls, A)*Experience^2$		-0.00127**		-0.00164**		0.00150
		[0.001]		[0.001]		[0.002]
$\mathcal{M}(ms, M)*Experience^2$		-0.000327		-0.00107*		-0.0000599
		[0.000]		[0.001]		[0.000]
$\mathcal{M}(ms, R)*Experience^2$		-0.000286		-0.000107		-0.000573*
		[0.000]		[0.001]		[0.000]
$\mathcal{M}(ms, A)*Experience^2$		-0.00117**		0.00170		-0.00153***
		[0.000]		[0.002]		[0.000]
$\mathcal{M}(hs, M)*Experience^2$		-0.000224		0.000368		0.000300
		[0.001]		[0.002]		[0.001]
$\mathcal{M}(hs, R)*Experience^2$		-0.000583		-0.000906		-0.000269
		[0.000]		[0.001]		[0.000]
$\mathcal{M}(hs, A)*Experience^2$		-0.000431		-0.00127		-0.000231
		[0.001]		[0.001]		[0.001]
Observations	2765	2765	1066	1066	1699	1699
R^2	0.216	0.222	0.204	0.219	0.265	0.278

Standard errors in brackets

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

D Model's Appendix

D.1 Wages for HS workers in Abstract jobs

To obtain the wage equation for HS individuals in A, multiply equation (4) by β and equation (1) by $1 - \beta$ and subtract one from the other, so to get:

$$\begin{aligned}
\beta r J^{hs}(y, z^A) - (1 - \beta) r N^{hs}(y, z^A) &= \beta \left\{ z^A y - w^{hs}(y, z^A) + \delta \int_{\varepsilon_2}^1 J^{hs}(s, z^A) dG(s|hs) - \delta J^{hs}(y, z^A) \right\} \\
-(1 - \beta) \left\{ w^{hs}(y, z^A) + \delta \int_{\varepsilon_2}^1 N^{hs}(s, z^A) dG(s|hs) + \delta U^{hs} \int_{\bar{y}}^{\varepsilon_2} dG(s|hs) - \delta N^{hs}(y, z^A) \right\} \\
&= \beta z^A y - \delta \left\{ \beta J^{hs}(y, z^A) - (1 - \beta) r N^{hs}(y, z^A) \right\} \\
&+ \delta \left\{ \int_{\varepsilon_2}^1 [\beta J^{hs}(s, z^A) - (1 - \beta) r N^{hs}(s, z^A)] dG(s|hs) - (1 - \beta) U^{hs} \int_{\bar{y}}^{\varepsilon_2} dG(s|hs) \right\} - w^{hs}(y, z^A)
\end{aligned}$$

This reduces to

$$\begin{aligned}
(r + \delta) [\beta J^{hs}(y, z^A) - (1 - \beta) N^{hs}(y, z^A)] &= \beta z^A y \\
+ \delta \left\{ \int_{\varepsilon_2}^1 [\beta J^{hs}(s, z^A) - (1 - \beta) r N^{hs}(s, z^A)] dG(s|hs) - (1 - \beta) U^{hs} \int_{\bar{y}}^{\varepsilon_2} dG(s|hs) \right\} &- w^{hs}(y, z^A)
\end{aligned}$$

Now, exploit the fact that in equilibrium $V^A = 0$ so that the sharing rule can be written as $[\beta J^{hs}(y, z^A) - (1 - \beta) N^{hs}(y, z^A)] = -(1 - \beta) U^{hs}$. Using this trick into the previous result gives:

$$\begin{aligned}
(r + \delta) [-(1 - \beta) U^{hs}] &= \\
\beta z^A y + \delta \left\{ \int_{\varepsilon_2}^1 [-(1 - \beta) U^{hs}] dG(s|hs) - (1 - \beta) U^{hs} \int_{\bar{y}}^{\varepsilon_2} dG(s|hs) \right\} &- w^{hs}(y, z^A) \\
= \beta z^A y + \delta \left\{ \int_{\varepsilon_2}^1 [-(1 - \beta) U^{hs}] dG(s|hs) - (1 - \beta) U^{hs} \left(1 - \int_{\varepsilon_2}^1 dG(s|hs)\right) \right\} &- w^{hs}(y, z^A)
\end{aligned}$$

Finally, this leads to the result:

$$w^{hs}(y, z^A) = \beta z^A y + (1 - \beta) r U^{hs}$$

D.2 Wages for MS workers in Routine jobs

To obtain the wage equation for MS individuals in R, multiply equation (5) by β and equation (2) by $1 - \beta$ and subtract one from the other, so to get:

$$\begin{aligned}
\beta r J^{ms}(y, z^R) - (1 - \beta)r N^{ms}(y, z^R) &= \beta \left\{ z^R y - w^{ms}(y, z^R) + \delta \int_{\varepsilon_1}^{\bar{y}} J^{ms}(s, z^R) dG(s|ms) - \delta J^{ms}(y, z^R) \right. \\
&\quad \left. - (1 - \beta) \left\{ w^{ms}(y, z^R) + \delta \int_{\varepsilon_1}^{\bar{y}} N^{ms}(s, z^R) dG(s|ms) + \delta U^{ms} \int_0^{\varepsilon_1} dG(s|ms) - \delta N^{ms}(y, z^R) \right\} \right\} \\
&= \beta z^R y - \delta \left\{ \beta J^{ms}(y, z^R) - (1 - \beta)r N^{ms}(y, z^R) \right\} \\
&\quad + \delta \left\{ \int_{\varepsilon_1}^{\bar{y}} [\beta J^{ms}(s, z^R) - (1 - \beta)r N^{ms}(s, z^R)] dG(s|ms) - (1 - \beta)U^{ms} \int_0^{\varepsilon_1} dG(s|ms) \right\} - w^{ms}(y, z^R)
\end{aligned}$$

This reduces to

$$\begin{aligned}
(r + \delta)[\beta J^{ms}(y, z^R) - (1 - \beta)r N^{ms}(y, z^R)] &= \beta z^R y \\
+ \delta \left\{ \int_{\varepsilon_1}^{\bar{y}} [\beta J^{ms}(s, z^R) - (1 - \beta)r N^{ms}(s, z^R)] dG(s|ms) - (1 - \beta)U^{ms} \int_0^{\varepsilon_1} dG(s) \right\} &- w^{ms}(y, z^R)
\end{aligned}$$

Now, exploit the fact that in equilibrium $V^R = 0$ so that the sharing rule can be written as $[\beta J^{ms}(y, z^R) - (1 - \beta)r N^{ms}(y, z^R)] = -(1 - \beta)U^{ms}$. Using this trick into the previous result gives:

$$\begin{aligned}
(r + \delta)[-(1 - \beta)U^{ms}] &= \\
\beta z^R y + \delta \left\{ \int_{\varepsilon_1}^{\bar{y}} [-(1 - \beta)U^{ms}] dG(s|ms) - (1 - \beta)U^{ms} \int_0^{\varepsilon_1} dG(s|ms) \right\} &- w^{ms}(y, z^R) \\
= \beta z^R y + \delta \left\{ \int_{\varepsilon_1}^{\bar{y}} [-(1 - \beta)U^{ms}] dG(s|ms) - (1 - \beta)U^{ms} \left(1 - \int_{\varepsilon_1}^{\bar{y}} dG(s|ms)\right) \right\} &- w^{ms}(y, z^R)
\end{aligned}$$

Finally, this leads to the result:

$$w^{ms}(y, z^R) = \beta z^R y + (1 - \beta)r U^{ms}$$

D.3 Wages for HS workers in Routine jobs

To obtain the wage equation for HS individuals in R, multiply equation (4) by β and equation (1) by $1 - \beta$ and subtract one from the other, so to get:

$$\begin{aligned} \beta r J^{hs}(y, z^R) - (1 - \beta) r N^{hs}(y, z^R) &= \beta \left\{ z^R y - w^{hs}(y, z^R) - \delta J^{hs}(y, z^R) \right\} \\ - (1 - \beta) \left\{ w^{hs}(y, z^R) + \delta U^{hs} - \delta N^{hs}(y, z^R) \right\} \\ &= \beta z^R y - \delta \left\{ \beta J^{hs}(y, z^R) - (1 - \beta) r N^{hs}(y, z^R) \right\} - \delta (1 - \beta) U^{hs} - w^{hs}(y, z^R) \end{aligned}$$

This reduces to

$$(r + \delta) [\beta J^{hs}(y, z^R) - (1 - \beta) N^{hs}(y, z^R)] = \beta z^R y - \delta (1 - \beta) U^{hs} - w^{hs}(y, z^R)$$

Now, exploit the fact that in equilibrium $V^R = 0$ so that the sharing rule can be written as $[\beta J^{hs}(y, z^R) - (1 - \beta) N^{hs}(y, z^R)] = -(1 - \beta) U^{hs}$. Using this trick into the previous result gives:

$$(r + \delta) [-(1 - \beta) U^{hs}] = \beta z^R y - \delta (1 - \beta) U^{hs} - w^{hs}(y, z^R)$$

Finally, this leads to the result:

$$w^{hs}(y, z^R) = \beta z^R y + (1 - \beta) r U^{hs}$$

D.4 Job Creation and Destruction in the Abstract Market

The value of production for a HS type in an Abstract job (equation (4) in the model) can be written as follows:

$$(r + \delta) J^{hs}(y, z^A) = z^A y - w^{hs}(y, z^A) + \delta \int_{\varepsilon_2}^1 J^{hs}(s, z^A) dG(s|hs)$$

Evaluate the latter at ε_2 and subtract it from the previous one, so to get:

$$\begin{aligned} (r + \delta) J^{hs}(y, z^A) - (r + \delta) J^{hs}(\varepsilon_2, z^A) &= z^A y - w^{hs}(y, z^A) + \delta \int_{\varepsilon_2}^1 J^{hs}(s, z^A) dG(s|hs) \\ - \left\{ z^A \varepsilon_2 - w^{hs}(\varepsilon_2, z^A) + \delta \int_{\varepsilon_2}^1 J^{hs}(s, z^A) dG(s|hs) \right\} \end{aligned}$$

Making use of the fact that $J^{hs}(\varepsilon_2, z^A) = 0$ and the definition of the wage function $w^{hs}(y, z^A)$ as stated in equation (11), we can reduce the latter into the following explicit functional form:

$$J^{hs}(y, z^A) = \frac{(1 - \beta)z^A(y - \varepsilon_2)}{r + \delta}$$

Now, before showing how to derive the job destruction condition, it is necessary to define an explicit function for the value of unemployment rU^{hs} . To do so, make use of the sharing rule and the explicit functional form of $J^{hs}(y, z^A)$ into the integral part of equation (1) so to get:

$$\begin{aligned} (r + \delta)N^{hs}(y, z^A) &= w^{hs}(y, z^A) + \delta \int_{\varepsilon_2}^1 N^{hs}(s, z^A)dG(s|hs) + \delta U^{hs} \int_{\bar{y}}^{\varepsilon_2} dG(s|hs) \\ &= w^{hs}(y, z^A) + \delta \left\{ \int_{\varepsilon_2}^1 \left[\frac{\beta z^A(y - \varepsilon_2)}{r + \delta} + U^{hs} \right] dG(s|hs) + \delta U^{hs} \int_{\bar{y}}^{\varepsilon_2} dG(s|hs) \right\} \\ &= w^{hs}(y, z^A) + \delta \int_{\varepsilon_2}^1 \left[\frac{\beta z^A(y - \varepsilon_2)}{r + \delta} \right] dG(s|hs) + \delta U^{hs} \end{aligned}$$

Hence, the value of employment for a HS worker into an Abstract job is:

$$N^{hs}(y, z^A) = \frac{w^{hs}(y, z^A) + \delta \int_{\varepsilon_2}^1 \left[\frac{\beta z^A(y - \varepsilon_2)}{r + \delta} \right] dG(s|hs) + \delta U^{hs}}{r + \delta}$$

From equation (3), it is easy to get the value of employment for an HS worker into a Routine job:

$$N^{hs}(y, z^R) = \frac{w^{hs}(y, z^A) + \delta U^{hs}}{r + \delta}$$

Since Abstract job are created at $y = 1$ while Routine jobs at $y = \bar{y}$, equation (7) can be written as:

$$rU^{hs} = b + m(\theta) \left\{ \phi [N^{hs}(\bar{y}, z^R) - U^{hs}] + (1 - \phi) [N^{hs}(1, z^A) - U^{hs}] \right\}.$$

Now, by using the definition of employment values as expressed above combined with wage functions (11) and (13), we finally plug $N^{hs}(\bar{y}, z^R)$ and $N^{hs}(1, z^A)$ into rU^{hs} . With

some algebra, the value of HS unemployment is:

$$rU^{hs} = \frac{b(r + \delta) + \beta m(\theta) \{ \phi z^R \bar{y} + (1 - \phi) [z^A + \frac{\delta z^A}{r + \delta} \int_{\varepsilon_2}^1 [y - \varepsilon_2] dG(s|hs)] \}}{r + \delta + m(\theta)\beta} \quad (20)$$

As it is clear, the value of unemployment depends on the average return between being employed in a Routine job and being employed in an Abstract one. The main difference with respect to Albrecht and Vroman (2002) is the integral component in the equation: the agent internalizes the chance that moving from unemployment to an Abstract job exposes him to the threat of being fired in the next period if not above a certain skill level ε_2 .

Finally, I can express the wage for HS workers as a functions of endogenous variables and parameters only. Moreover, we can now express equation (4) from the model in a fully explicit form. By simply using the explicit version of $J^{hs}(y, z^A)$ in the integral part of (4) and $w^{hs}(y, z^A)$ with the explicit form of rU^{hs} , we get:

$$(r + \delta)J^{hs}(y, z^A) = z^A y - w^{hs}(y, z^A) + \frac{\delta(1 - \beta)z^A}{r + \delta} \int_{\varepsilon_2}^1 (s - \varepsilon_2) dG(s|hs)$$

Evaluation of the latter at $y = \varepsilon_2$ leads to the job destruction curve in the Abstract market:

$$0 = z^A \varepsilon_2 - w^{hs}(\varepsilon_2, z^A) + \frac{\delta(1 - \beta)z^A}{r + \delta} \int_{\varepsilon_2}^1 (s - \varepsilon_2) dG(s|hs)$$

For job creation, use the explicit expression of $J^{hs}(y, z^A)$ into the value of an Abstract vacancy (equation(9) in the model). Since in equilibrium $V^A = 0$ and wages are posted at $y = 1$, we finally obtain the job creation condition for Abstract jobs:

$$c = \frac{m(\theta)(1 - \gamma)(1 - \beta)}{\theta} \left[\frac{z^A - rU^{hs} + \frac{\delta z^A}{r + \delta} \int_{\varepsilon_2}^1 (s - \varepsilon_2) dG(s|hs)}{r + \delta} \right]$$

D.5 Job Creation and Destruction in the Routine Market

The value of production for a MS type in an Routine job (equation (5) in the model) can be written as follows:

$$(r + \delta)J^{ms}(y, r) = z^R y - w^{ms}(y, z^R) + \delta \int_{\varepsilon_1}^{\bar{y}} J^{ms}(s, z^R) dG(s|ms)$$

Now evaluate the latter at ε_1 and subtract it from the previous one so to get:

$$(r + \delta)J^{ms}(y, z^R) - (r + \delta)J^{ms}(\varepsilon_1, z^R) = z^R y - w^{ms}(y, z^R) + \delta \int_{\varepsilon_2}^1 J^{ms}(s, z^R) dG(s|ms) - \left\{ z^R \varepsilon_1 - w^{ms}(\varepsilon_1, z^R) + \delta \int_{\varepsilon_1}^{\bar{y}} J^{ms}(s, z^R) dG(s|ms) \right\}$$

Making use of the fact that $J^{ms}(\varepsilon_2, z^R) = 0$ and the definition of the wage function $w^{ms}(y, z^R)$ as stated in equation (12), we can reduce the latter into the following:

$$J^{ms}(y, z^R) = \frac{(1 - \beta)z^R(y - \varepsilon_1)}{r + \delta}$$

Now, before showing how to derive the job destruction condition, it is necessary to define an explicit function for the value of unemployment rU^{ms} . To do so, make use of the sharing rule and the explicit functional form of $J^{ms}(y, z^R)$ into the integral part of equation (2) so to get:

$$\begin{aligned} (r + \delta)N^{ms}(y, z^R) &= w^{ms}(y, z^R) + \delta \int_{\varepsilon_1}^{\bar{y}} N^{ms}(s, z^R) dG(s|ms) + \delta U^{ms} \int_0^{\varepsilon_1} dG(s|ms) \\ &= w^{ms}(y, z^R) + \delta \left\{ \int_{\varepsilon_1}^{\bar{y}} \left[\frac{\beta z^R (y - \varepsilon_1)}{r + \delta} + U^{ms} \right] dG(s|ms) + \delta U^{ms} \int_0^{\varepsilon_1} dG(s|ms) \right\} \\ &= w^{ms}(y, z^R) + \delta \int_{\varepsilon_1}^{\bar{y}} \left[\frac{\beta z^R (y - \varepsilon_1)}{r + \delta} \right] dG(s|ms) + \delta U^{ms} \end{aligned}$$

Hence, the value of employment for a MS worker into an Abstract job is:

$$N^{ms}(y, z^R) = \frac{w^{ms}(y, z^R) + \delta \int_{\varepsilon_1}^{\bar{y}} \left[\frac{\beta z^R (y - \varepsilon_1)}{r + \delta} \right] dG(s|ms) + \delta U^{ms}}{r + \delta}$$

Since Routine jobs are created at $y = \bar{y}$, equation (8) can be written as:

$$rU^{ms} = b + m(\theta)\phi[N^{ms}(\bar{y}, z^R) - U^{ms}]$$

Now, by using the definition of employment value as expressed above combined with wage functions (12), we finally plug $N^{ms}(\bar{y}, z^R)$ into rU^{ms} . With some algebra, the value of HS unemployment is:

$$rU^{ms} = \frac{b(r + \delta) + \beta m(\theta)\phi[z^R\bar{y} + \frac{\delta z^R}{r + \delta} \int_{\varepsilon_1}^{\bar{y}} [y - \varepsilon_1] dG(s|ms)]}{r + \delta + \phi m(\theta)\beta} \quad (21)$$

As it is clear, the value of unemployment depends on the return from being employed in a Routine job. The main difference with respect to Albrecht and Vroman (2002) is the integral component in the equation: the agent internalizes the chance that moving from unemployment to a Routine job exposes him to the threat of being fired in the next period if not above a certain skill level ε_1 .

Finally, I can express the wage for MS workers as a functions of endogenous variables and parameters only. Moreover, we can now express equation (5) from the model in a fully explicit form. By simply using the explicit version of $J^{ms}(y, z^R)$ in the integral part of (5) and $w^{ms}(y, z^R)$ with the explicit form of rU^{ms} , we get:

$$(r + \delta)J^{ms}(y, z^R) = z^R y - w^{ms}(y, z^R) + \frac{\delta(1 - \beta)z^R}{r + \delta} \int_{\varepsilon_1}^{\bar{y}} (s - \varepsilon_1) dG(s|ms)$$

Evaluation of the latter at $y = \varepsilon_1$ leads to the job destruction curve in the Routine market:

$$0 = z^R \varepsilon_1 - w^{ms}(\varepsilon_1, z^R) + \frac{\delta(1 - \beta)z^R}{r + \delta} \int_{\varepsilon_1}^{\bar{y}} (s - \varepsilon_2) dG(s|ms)$$

For job creation, use the explicit expression of $J^{ms}(y, z^R)$ into the value of an Routine vacancy (equation (10) in the model). Since in equilibrium $V^R = 0$ and wages are posted at $y = \bar{y}$, we finally obtain the job creation condition for Routine jobs:

$$c = \frac{m(\theta)(1 - \beta)}{\theta} \left\{ \gamma \left[\frac{z^R \bar{y} - rU^{ms} + \frac{\delta z^R}{r + \delta} \int_{\varepsilon_1}^{\bar{y}} (s - \varepsilon_1) dG(s|ms)}{r + \delta} \right] + (1 - \gamma) \left[\frac{z^R \bar{y} - rU^{hs}}{r + \delta} \right] \right\}$$