Health, Retirement and Consumption

Preliminary and Incomplete

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Abstract

The paper analyzes consumption decisions of retired workers, using Danish register data. A major puzzle, which motivates much of the analysis below, is that that wealth actually increases for a large fraction (roughly half) of the people in our data. One would expect that wealth accumulated before retirement would be used to augment consumption in later life, with the implication that wealth should decline over time. The risk of large out-of-pocket medical expenditures is negligible in Denmark, so although explanations associated with such expenditures might explain similar patterns in U.S. data, these explanations are not plausible for Denmark (and therefore also questionable for the U.S.) Our analysis instead attempts to explain wealth paths using a model that emphasizes health-related fluctuations in the marginal utility of consumption.

1 Introduction

The basic economic forces affecting retirement decisions are still not well understood, although this has been an active area of research recently, and substantial progress has been made.[1] The main question is why many people choose to work full-time for about 40 years, and not work at all for the remaining 15 years or so, rather than taking more time off when they are younger, and working more when they are older. As life expectancy increases, people allocate the added years of life either to working or to retirement, and how this allocation is made has a big effect on the financing of social security systems. A major issue is the extent to which retirement is tied to deteriorating health. This is important because increases in life expectancy may be associated with relatively poor health in later years.

Research on the relationship between health and retirement has been greatly hampered by the lack of suitable data. Recently Christensen and Kallestrup-Lamb (2012) have presented a detailed description of health diagnoses and retirement decisions, using Danish register data. This paper

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[1]See French (2005), French and Jones (2011) and Fan et al. (2015), for example.

analyzes these data useing a dynamic programming model, focusing on the relationship between health shocks and consumption after retirement. Developing an understanding what happens after retirement is an essential first step toward a complete analysis of retirement decisions. The point of the model is to go beyond descriptions of patterns seen in the data, and develop a theory of the decisions that generated these patterns, so that it becomes possible to predict decisions that would be made in other circumstances. Future retirement decisions will be made in an environment that involves substantial changes in life expectancy, and in the health of older workers, and in the generosity of public pension schemes. Merely extrapolating from past data on health and retirement is unlikely to provide accurate predictions of future decisions, and such predictions are essential for informed public policy regarding social security and health insurance systems.

2 Literature

Here, we briefly mention a number of issues and studies that are relevant for various different parts of our work. First off, we study consumption and hence saving by the elderly, and De Nardi et al. (2010) have argued that this is largely driven by risk of large medical expenses late in life; this is especially true for richer people. Here, it should be kept in mind that most expenditures are insured by the government in the Danish welfare state. Nevertheless, utility of consumption may depend on health, and we study this possibility. Secondly, it is well known that there is a relationship between wealth and mortality. Attanasio and Emmerson (2003) study the question of which way the causation runs. It may well be that people in bad health have trouble accumulating wealth. The paper conditions on health in one wave of the British Retirement Survey (1988-89) and asks whether wealth explains differences in health and mortality in the second wave (1994). At present, we are setting this possibility aside in our analysis.

Finkelstein et al. (2013) look at responses to survey questions such as "Much of the time during the past week I was happy. (Would you say yes or no?)" They find that

> "Across a wide range of alternative specifications, we find robust evidence that a deterioration in health is associated with a statistically significant decline in the marginal utility of consumption."

Other relevant papers include French (2005) and French and Jones (2011), French (2005) estimates a life cycle model of labour supply, retirement, and savings behaviour in which future health status and wages are uncertain. The paper uses the method of simulated moments to match life cycle profiles estimated using data from the Panel Study of Income Dynamics (PSID) to life cycle profiles generated by a dynamic programming model. When augmented to include uncertainty over future wages and health status, the model fits the life cycle profile of assets rather well. The model can predict how labour supply and retirement patterns of individuals might change in response to changes in the Social Security rules, with a finding that reducing Social Security benefits by 20% would cause workers to delay exit from the labour force by only three months.

Li Gan (2015) analyze the bequest motive for people with children. The marginal utility of bequests depends on the number of children, but not on the size of the bequest. This is a quasi-linear specification, and it implies that if there is a positive bequest, the marginal utility of consumption is fixed, so the consumption trajectory is independent of initial wealth. There is an initial wealth level, and a constant level of annuity income, and borrowing against future annuity income is not feasible. The focus of the paper is on how subjective mortality beliefs affect consumption in later life. The people in the Oldest Old sample were born between 1890 and 1923 (Asset and Health Dynamics among Oldest Old (AHEAD)). They were all older than 70 when the survey started. Only singles are analyzed. The first wave was in 1993 (but the asset data from that wave are believed to be unreliable). The second wave was in 1995, the third in 1998, and the fourth was in 2000. Wealth levels in two waves of the survey are used to estimate parameters: risk aversion coefficient, marginal utility of bequests, and discount factor. The wealth in the second wave is taken as the initial wealth, and the parameters are chosen to fit the wealth levels in the third wave; wave 4 is used for out of sample predictions.

The model generates three types of consumption paths. For high initial wealth levels, the consumption path just equates the (discounted) marginal utility of consumption in each period with the marginal utility of bequests. Then consumption is insensitive to wealth – higher wealth is just left as a bequest. For lower wealth levels, it can be optimal to leave no bequest (if he maximal age is reached). And for even lower wealth levels it is optimal to run wealth down to zero, and consume just the annuity income afterward. The main results exclude housing wealth. It isn't clear how this makes sense – is it just that the value of the house becomes part of the bequest, for those people who are planning to leave a positive bequest anyway? There is a footnote describing alternative estimates using total wealth: the parameter estimates are similar. Mean regressions indicate strong bequest motives, but this is because a few rich families leave large bequests, and this can't be explained without a strong bequest motive. But median regressions indicate that the request motive is neligible.

Our paper is most closely related to the work of Laitner et al. (2014), which focuses mainly on the implications of large health expenditures late in life. The aim is to explain why the average wealth of a cohort might rise after retirement, rather than to explain the decisions of individuals. The paper also offers explanations of the lack of demand for annuities, as well as the "relative scarcity of bequests."

The argument about increasing wealth seems to involve retirement with assets below some target level. Why would a healthy person choose to retire in this situation? Wealth accumulation after retirement is explained by showing that there is an optimal level of wealth in the good health stage, and if initial wealth after retirement is below this, it is optimal to consume less than the amount of the annuity, so as to build up savings for the bad health stage, when the marginal utility of expenditures will be higher. This begs the question of why the individual would choose to retire with insufficient wealth. It might be that productivity fell to the point where continuing to work wouldn't help. But this can't be typical – if it were, people would work more when they were younger. In other words, a proper explanation of wealth accumulation after retirement really requires a sensible theory of retirement.

The first-best solution involves purchasing an annuity at retirement, and consuming the proceeds until death. There is just one health transition, at rate λ, from good to bad health. After this, death comes at rate Λ. On the optimal path, expenditure rises and consumption drops when the transition occurs. In the low health phase, "bequeathable" wealth B is run down to zero in finite time. Consumption thereafter is just the annuity flow.

Estimates of the effect of health on retirement behavior are plagued by upward justification bias stemming from self-reporting of health conditions in surveys. Christensen and Kallestrup-Lamb (2012) show how the justification bias in the estimated impact of health shocks on retirement is mitigated by using objective health measures. They consider merged register data on individual objective medical diagnosis codes and early retirement behavior for a large, representative Danish sample of older workers. Their study follows individuals year by year from the age of 50, with precisely measured period by period changes in objectively measured individual health, labor market status, and other relevant financial and socio-economic variables on. They estimate a single risk parametric retirement duration model lumping all labor market exit routes, and extensions involving unobserved heterogeneity, nonparametric baseline (for a semiparametric model structure), separation by gender, and a competing risks specification. In the latter case, besides the disability state, involving specific medical eligibility criteria, and voluntary early retirement, the competing risk approach also distinguishes two complete multi-period routes to retirement, namely, unemployment followed by early retirement (due to an exemption in the rules, this unemployment spell may have commenced at age 51, and continued until the early retirement eligibility age of 60), and unemployment followed by other programs (such as disability, civil service pension, or welfare), thus allowing an investigation of whether the former combined unemployment-early retirement route appears more similar than the latter to early retirement itself, and hence more voluntary.

In the present paper, we study these data with the purpose of estimating and assessing an optimizing intertemporal model.

3 Data

We use the unique Danish register data from Statistics Denmark linked with detailed health registers from Statens Serum Institut for the period 1980 through 2012. The registers contain annual individual level information on the entire Danish population on age, gender, marital status, income, and wealth which can be linked to health care registers which contain daily data on individual hospitalization, use of primary health care such as GP and privately practising medical specialists, drug purchases, etc. The data are based on administrative registers and contain no survey element.

We extract data for the entire population of men who are exactly 68 years old, single and alive in 1980.[2] We follow them until death or 2012.[3] The financial indicators, all deflated to 2000 levels, are own income and household net wealth based on tax data. Moreover, we are able to identify children of these men and their individual demographic and socio-economic characteristics such as age, gender, income, and wealth.

The health data are drawn from the Danish National Registry for Patients and includes information about admissions, actual diagnoses, treatments, and discharges for all patients either through admission ($1980-2012$) or outpatient treatment ($1994-2012$). It contains daily information on both somatic and psychiatric treatment in hospitals. Within each year we have multiple observations for a given patient since the possibility of several admissions exists (approximately one third of the patients experience more than one admission within a given year). Furthermore, in relation to an admission, the patient is diagnosed with a main condition and possibly several additional conditions. The different diagnoses are organized in relation to WHO's international classification of diseases (ICD). From 1980 through 1993, ICD-8 is used, and from 1994 through 2012 ICD-10. This information is summarized in 12 dummy variables, each indicating whether a person has been diagnosed with a disease in the associated category within the year. The categories we consider are: (1) Malignant cancer (leukemia, melanoma, and other malignant cancers); (2) Benign cancer (various types of tumors); (3) Endocrine, nutritional, and metabolic diseases (e.g., diabetes, obesity, etc.); (4) Mental and behavioral disorders (dementia, delirium, schizophrenia, stress-related disorders, etc.); (5) Diseases of the nervous system and sensory organs (Alzheimer's, Parkinson's, epilepsy, sclerosis, migraine, apnoea, cataract, hearing loss, etc.); (6) Diseases of the circulatory system (ischemic and other heart diseases, angina pectoris, acute rheumatic fever, high blood pressure, hypertension, stroke, etc.); (7) Diseases of the respiratory system (influenza, pneumonia, bronchitis, asthma, and other lung diseases); (8) Diseases of the digestive system (gastric ulcer, hernia, diseases of the liver and gallbladder, etc.); (9) Diseases of the genitourinary system (kidney stone, renal failure, other diseases of the urinary system and genital organs); (10) Diseases of the musculoskeletal system and connective tissue (arthritis, osteoarthritis, Lyme disease, herniated disc, lumbago, osteoporosis, sclerosis, rheumatism, gout); (11) Injury, poisoning, and other consequences of external causes (bone fractures, dislocations, etc.); (12) Other diseases. Both main and additional diagnoses are included. Furthermore, the number of days of treatment, number of diagnoses, and number of admissions within a given year are included.

The National Health Insurance Service Registry contains daily registrations from 1990 through 2012 on services provided by GPs, specialist doctors, dentists, physiotherapists, chiropractors, and psychologists. The register includes information on date of visit and type of service. Moreover, we have access to the Register of Medicinal Product Statistics that contains monthly data from 1994 through 2012 on medicine sold by pharmacies. Information on medicine sales from hospital

[2] We consider only single men (as a starting point) in order to avoid having to model joint household decisions. Here single means neither married nor cohabiting.

[3] Of $18,050$ men alive and single in 1980, 177 were still alive in 2012 (at age 100). Thus (for these single men who survived to age 68) the chance of living to 100 was a little less than one in a hundred.

pharmacies and use in hospitals is included since 1997. Prescription drugs sold in the primary health sector are registered at the individual level.

There is a general subsidy on medicine. This subsidy is calculated based on the cheapest generic alternative. Subsidies for adults are calculated as follows: for annual spending of less than \$165 (at 2014 prices) there is no subsidy; beyone this point there is a 50% subsidy up to \$270; then 70% from \$270 to \$584, and 85% above \$584. Moreover it is possible to apply for a 100% subsidy for chronicially ill individuals once spending exceeds \$584.

3.1 Some Descriptive Statistics

Income after retirement is dominated by annuity flows from the public pension system. Figure 1 shows annual income in Danish Kroner (6 Kroner being roughly equivalent to one U.S. dollar). Clearly, the dispersion in income is quite low.

Figure 1: Income after Retirement

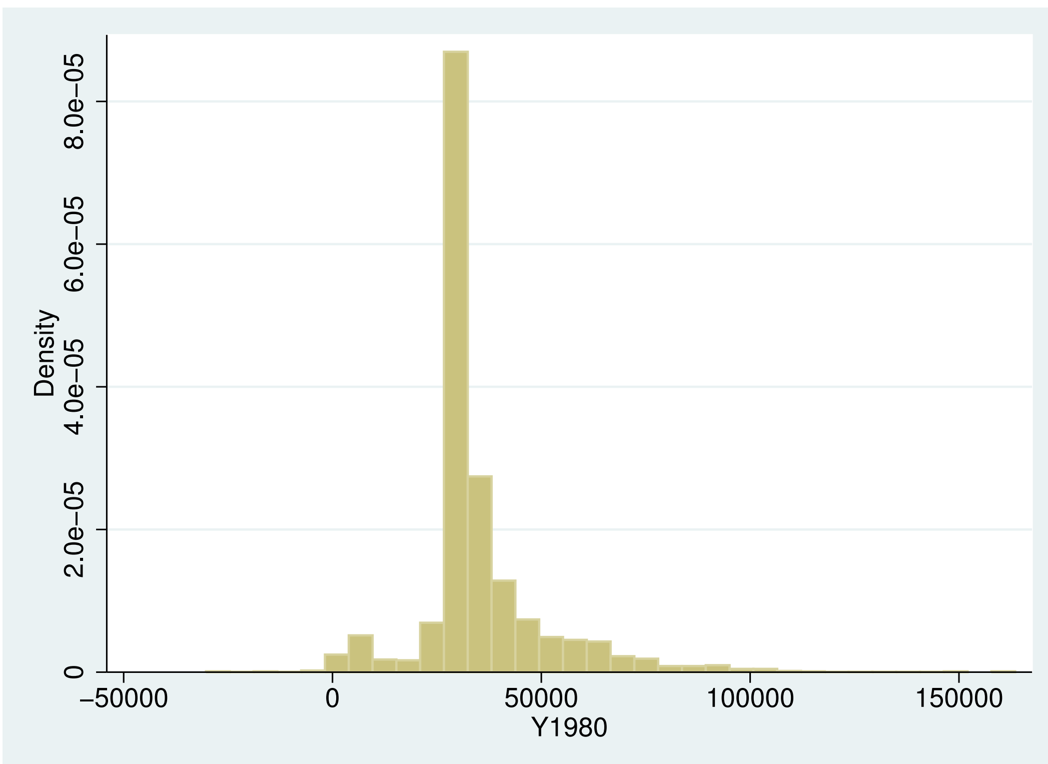
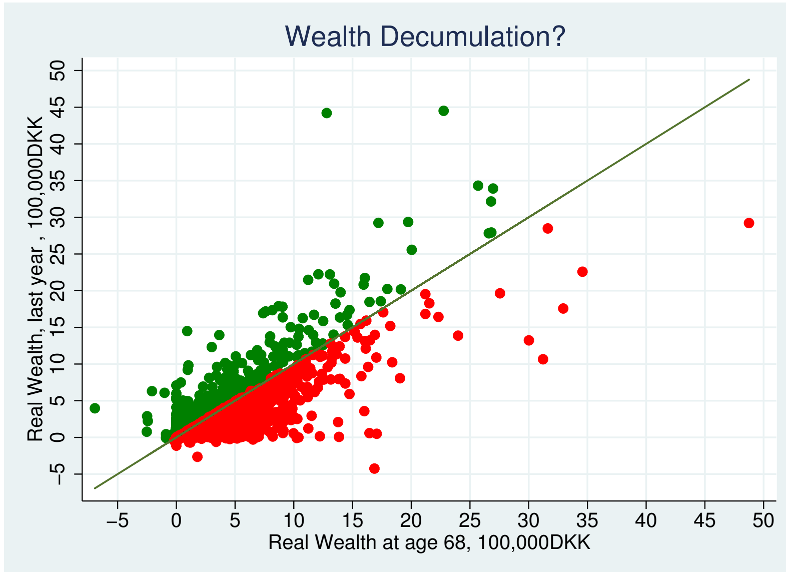


Figure 2 and Table 1 present a major puzzle which motivates much of the analysis below. One would expect that wealth accumulated before retirement would be used to augment consumption in later life, with the implication that wealth should decline over time. Figure 2 compares wealth in the year before death with initial wealth (at age 68), showing that wealth actually increases for a large fraction (roughly half) of the men in our data. Zero wealth at age 68 is very common in these data, as shown in Figures 3 and 4. And many of those who begin with zero wealth accumulate

Figure 2: Wealth Paths



substantial amounts before they die, by saving part of their annuity income. It is important to emphasize here that the risk of large medical expenditures is negligible in Denmark.

One possible interpretation of these data is that there are changes in bequest incentives due to unexpected events occurring after retirement (since anticipated bequests should presumably be covered by wealth accumulated before retirement). But on average, such changes should be zero. Another relevant consideration is that the wealth increases might be associated with increase in property values. But Table indicates that wealth increases were actually most frequent for those without property.

Table 1: **Wealth Changes**

Age 68 to year before death			
Single men, survived at least 5 years from age 68			
	Decrease	Increase	
	999	1153	2152
Property			
Yes	481	253	734
No	518	900	1418

In the analysis below, we attempt to explain wealth paths using a model that emphasizes health-related fluctuations in the marginal utility of consumption. Table 2 summarizes the incidence of major health events late in life, and Table 3 summarizes the relationship between health events and wealth changes.

Figure 3: Post-Retirement Wealth Distribution

Figure 4: Zero-Wealth Spike

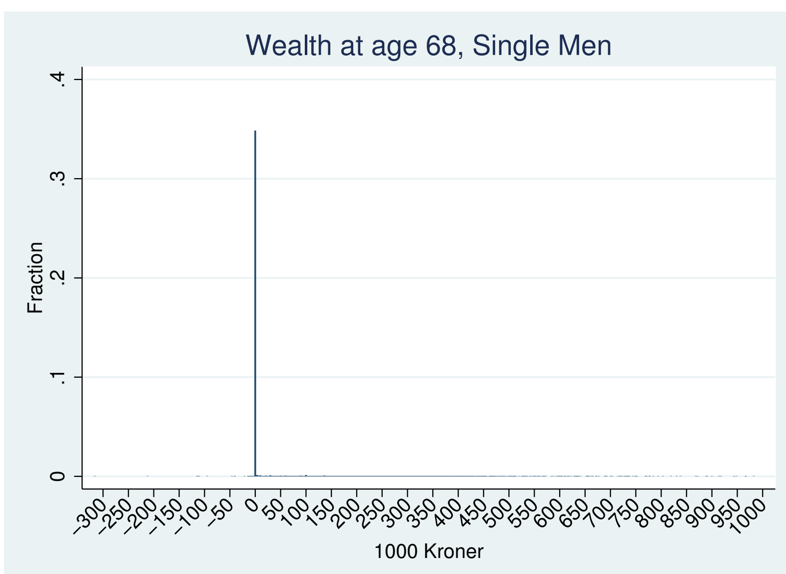


Figure 5: Wealth Paths

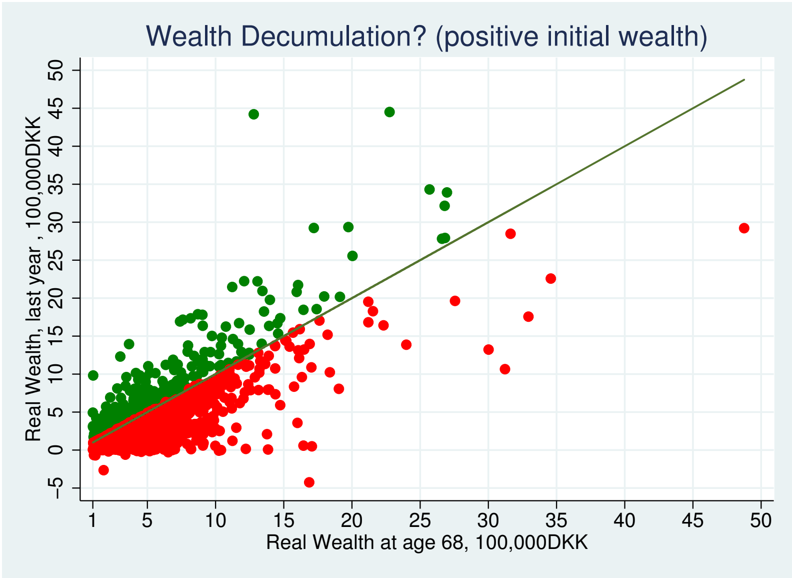


Table 2: Health Events (**Diagnosis Frequencies, Final Two Years**)

Examination, no diagnosis	1.3%
Infectious diseases	1.4%
Malignant cancer	6.4%
Benign cancer	1.5%
Endocrine etc	4.7%
Blood diseases	1.7%
Mental, behavioral	2.7%
Nervous system	3.7%
Circulatory system	14.4%
Respiratory system	8.2%
Digestive system	5.7%
Genitourinary system	5.4%
Pregnancy, birth etc	0.3%
Skin diseases	1.0%
Musculoskeletal diseases	2.7%
Symptoms deficient for examination	7.1%
Injury, poisoning, etc	6.3%

Table 3: Health and Wealth

Wealth Changes			
Age 68 to year before death			
	Decrease	Increase	
	999	1,153	2,152
Health Event			
Yes	821	943	1,764
	46.5%	53.5%	
No	178	210	388
	45.9%	54.1%	

4 Consumption After Retirement

4.1 Bequests and Consumption with Complete Markets

When death is stochastic, the amount left as a bequest is random. But if markets are complete, the bequest can be converted to a sure thing. For example, a life insurance policy can be designed to achieve this. The complete markets case provides a benchmark, where all contingencies have been covered in advance. If wealth shocks have been insured, there should be no observed changes in wealth, aside from drawing down assets to pay for planned consumption. This is an extreme case – it is hard to believe that changes in the value of houses are fully insured, for example (although this could be achieved by simply selling the house and signing a long-term lease). But it is a good starting point.

The problem then is just how much to set aside as a bequest, and how much to spend on consumption over the remaining lifetime. For a given wealth level, it makes sense that an older person would choose a larger bequest. The choice is between a consumption rate for the remaining years of life and an amount to be passed on to heirs. If the same person is observed later, and the bequest has not actually been transferred, it might seem that the level of consumption should be increased. But this is misleading, because measured wealth includes the amount set aside as a bequest. It might be more reasonable to say that the consumption rate actually reveals the planned bequest amount. The problem is more complicated when health affects the marginal utility of consumption; this will be considered later.

Consider an individual who has already retired, and who chooses a consumption plan to maximize the present value of utility over the remaining lifetime, subject to a budget constraint governed by an initial wealth level W_0.

The utility maximization problem is

$$\max_c \int_0^\infty e^{-(\rho+\delta)t} u\left(c\left(t\right)\right) dt$$

subject to

$$\int_0^{\infty} \left(c\left(t\right)-y\left(t\right)\right) e^{-(r+\delta)t} dt = W_0$$

where ρ is the rate of time preference, δ is the death rate, and r is the market interest rate.

Note that the budget constraint here reflects the complete markets assumption. This can be illustrated using a simple two-period example. Consider the price of a contingent claim to a dollar to be paid next period only if this person is alive. This claim is worthless if the person dies, and it is worth e^{-r} now otherwise, and the survival probability is $e^{-\delta}$ so if the market is risk-neutral (meaning that the uncertainty about this person's survival has no aggregate component), then the expected value of the claim is just $e^{-(r+\delta)}$. If the individual maximizes the expected discounted value of a strictly concave utility function over two periods, and if contingent claims to future consumption are priced at $e^{-(r+\delta)}$, then the problem is

$$\max_{c_1,c_2} u\left(c_1\right) + e^{-(\rho+\delta)} u\left(c_2\right)$$

subject to the budget constraint

$$c_1 + e^{-(r+\delta)} c_2 = W$$

Equating marginal utility per dollar gives

$$u'\left(c_1\right) = \frac{e^{-(\rho+\delta)}}{e^{-(r+\delta)}} u'\left(c_2\right)$$

so consumption is constant if $\rho = r$, and otherwise consumption is increasing or decreasing according to whether the consumer is more or less patient than the market. And in any case (with complete markets) the survival probability affects only the level of consumption, not whether it rises or falls.

Now consider the incomplete markets case, where life annuities are not available at fair prices, although it is possible to borrow and lend at the interest rate r. The problem then is that money set aside for future consumption will be lost if the individual doesn't survive. The basic Euler equation calculation is as follows. The budget constraint implies

$$\dot{W} = rW + y - c$$

Thus the utility maximization problem can be written as

$$\int_0^{\infty} e^{-(\rho+\delta)t} u\left(y\left(t\right) + rW - \dot{W}\right) dt = \int_0^{\infty} F\left(W(t), \dot{W}, t\right) dt$$

The Euler equation is

$$F_1\left(W(t), \dot{W}, t\right) - \frac{d}{dt}\left(F_2\left(W(t), \dot{W}, t\right)\right) = 0$$

where F_1 is the derivative of F with respect to W, and F_2 is the derivative of F with respect to

$\dot{W}$. This gives

$$re^{-(\rho+\delta)t}u'(c(t)) + \frac{d}{dt}e^{-(\rho+\delta)t}u'(c(t)) = 0$$

Let

$$x(t) = e^{-(\rho+\delta)t}u'(c(t))$$

Then

$$rx(t) + \dot{x}(t) = 0$$

so

$$\log(x(t)) = \log(x(0)) - rt$$

and

$$x(t) = e^{-rt}x(0)$$

which means

$$e^{-(\rho+\delta)t}u'(c(t)) = e^{-rt}u'(c(0))$$

Thus the law of motion for the marginal utility of consumption is

$$u'(c(t)) = e^{(\rho+\delta-r)t}u'(c(0))$$

The marginal utility of consumption increases unless the interest rate exceeds the sum of the time preference rate and the death rate. If the marginal utility of consumption is rising, then consumption must be falling (assuming that the utility function is concave). In standard formulations the death rate is set to zero, with death occuring at some known date. But for someone who is already over 70, the death rate is too large to be ignored.

The CRRA utility function has derivatives

$$\begin{aligned} u(c) &= \frac{c^{1-\gamma}-1}{1-\gamma} \\ u'(c) &= c^{-\gamma} \\ u''(c) &= -\gamma c^{-\gamma-1} \end{aligned}$$

For this utility function (with a positive coefficient of relative risk aversion, γ), consumption is given by

$$c(t)^{-\gamma} = e^{(\rho+\delta-r)t}c_0^{-\gamma}$$

so that

$$\log(c(t)) = \log(c_0) - \frac{\rho+\delta-r}{\gamma}t$$

Thus the rate at which consumption declines depends on the curvature of the utility function. If curvature is large, marginal utility declines a lot when consumption falls a little, so the decline in the marginal utility of consumption is achieved with a very gradual decline in consumption.

The Euler equation is a local optimality condition. The level of consumption is governed by c_0, which is determined by the budget constraint. But the rate of change of consumption is determined by the discounting coefficient $(\rho + \delta - r)$, and by the curvature of the utility function. This suggests that it would be useful to distinguish between health events that have a significant effect on life expectancy, and those that do not. If a change in health status affects the marginal utility of consumption without affecting life expectancy, then there will be a jump in the level of consumption, but the rate of change after this jump will remain as it was. On the other hand following a health event that changes life expectancy without affecting the marginal utility of consumption there will be no jump in consumption, but consumption will start declining more quickly if life expectancy has fallen.

The Bequest Choice Assume log utility for simplicity. Then the value of the optimal consumption plan is

$$U(c_0) = \frac{\log(c_0)}{\rho + \delta}$$

If a bequest is made to someone who also has log utility, so as to maximize a weighted sum of utilities, then the fraction of wealth allocated to the bequest is given by

$$B = \frac{\frac{\alpha}{\rho+\delta^*}}{\frac{\alpha}{\rho+\delta^*} + \frac{1}{\rho+\delta}} A$$

where $\frac{1}{\delta^*}$ is the life expectancy of the recipient, and α represents the degree of altruism, and A represents total wealth to be divided. If the hazard rate of death is zero for the recipient, then the bequest is

$$B = \frac{1}{1 + \frac{1}{\alpha}\frac{\rho}{\rho+\delta}} A$$

This suggests that older people would leave a larger fraction of wealth as bequests, to the extent that δ increases with age. But the Euler equation analysis assumed a constant death hazard, so this result is just an approximation. Given complete markets, what is relevant is the remaining life expectancy of the donor relative to the recipient at the point when the bequest decision is made.

4.2 Health and Consumption

The relationship between health and consumption decisions can be illustrated using a simple two-period model with no bequest motive, and with income in the form of a life annuity.

Suppose that flow utility is described by the function $u(c, h)$, where h is an index of health. The individual's decision problem is

$$\max_s u(A + y - s, h) + \beta\sigma(h) E\left(u\left(y + Rs, h'\right) \mid h\right)$$

where A is initial assets, y is income from an annuity, s is saving, σ is the probability of surviving to the next period, R is the rate of return on savings, β is the discount factor, and $E(h' \mid h)$ is expected health status in the next period, conditional on current health. The first-order condition for this problem is

$$R\beta\sigma(h)\,E\left(u_c\left(y+Rs,h'\right) \mid h\right) \leq u_c\left(A+y-s,h\right)$$

with equality if borrowing against future income is feasible, or if $s > 0$. Here u_c is the marginal utility of consumption, and there is positive saving if and only if the expected marginal utility of future consumption would exceed the marginal utility of current consumption if no assets are carried into the next period.

4.2.1 Example 1

Suppose $u(c,h) = h\log(c)$, with $R = \beta = 1$, and suppose that h is determined by a two-state Markov chain with positive persistence, with $h \in \{h_b, h_g\}$ and $h_b < h_g$. Also assume that borrowing against future income is not feasible. Then there are two possibilities, depending on whether the first-order condition is satisfied with equality. If it is, then

$$\frac{\sigma(h)\,\bar{h}}{y+s} = \frac{h}{A+y-s}$$

where $\bar{h}$ denotes the expected future value of h, conditional on the current value. This implies

$$s = \frac{\sigma(h)\,\bar{h}\,(A+y) - hy}{\sigma(h)\,\bar{h} + h}$$

If there are no initial assets and if $h = h_g$ then $\bar{h}_g < h_g$, so saving is zero. On the other hand if $h = h_b$ and

$$\sigma(h_b)\,\bar{h}_b > h_b$$

then saving is positive. Thus the individual saves now if current health is worse than expected future health, adjusted for survival probability. In the opposite case, it would be desirable to borrow against future income, if this were feasible. If A is sufficiently large than the borrowing constraint does not bind. Write the equation for savings as

$$s = \frac{x(A+y) - y}{x+1}$$

where

$$x = \frac{\sigma(h)\,\bar{h}}{h}$$

Thus the amount saved is increasing in x. But

$$\frac{\bar{h}_b}{h_b} = p_b \frac{h_g}{h_b} + 1 - p_b = 1 + p_b \left(\frac{h_g}{h_b} - 1 \right) > 1$$

and

$$\frac{\bar{h}_g}{h_g} = p_g + (1 - p_g) \frac{h_b}{h_g} = 1 - (1 - p_g) \left(1 - \frac{h_b}{h_g} \right) < 1$$

so if the survival probability is independent of current health (or nearly so), then saving is higher in the bad health state. On the other hand the bad health state is associated with a relatively large reduction in survival probability, then saving may be higher in the good health state, even though the marginal utility of income is positively related to health.

4.2.2 Example 2

If the utility function is $U = u(z)$, where $z = hc$, then marginal utility may be decreasing with health. In this case

$$\begin{aligned} \frac{\partial^2 U}{\partial h \partial c} &= \frac{\partial}{\partial h} u'(z) h \\ &= u'(z) + u''(z) z \\ &= u'(z) (1 - r_R(z)) \end{aligned}$$

where $r_R(z)$ is the coefficient of relative risk aversion. If the relative risk aversion coefficient is greater than 1, then the marginal utility of consumption decreases when health improves. For example, in the case of a constant relative risk aversion utility function, $u(z) = \frac{z^{1-\gamma}-1}{1-\gamma}$, with derivatives $u'(z) = z^{-\gamma}$ and $u''(z) = -\gamma z^{-\gamma-1}$, the cross-partial derivative is

$$\frac{\partial^2 U}{\partial h \partial c} = (1 - \gamma) z^{-\gamma}$$

4.3 Saving After Retirement

A surprising feature of the data is that many individuals have substantially higher wealth shortly before death than they had when first observed (at age 68). This means that they choose to save out of their annuity income, even though the annuity continues at a constant level until death. The simplest explanation for this behavior is that the marginal utility of consumption (at a given level of consumption) is expected to be higher in the future, perhaps because of changes in health.

Suppose the marginal utility of consumption is determined by a two-state Markov process. An individual with (at most) n periods remaining chooses consumption by solving the dynamic programming problem

$$\begin{aligned} V_n^L(w) &= \max_{c\leq w+a}\left(\theta u(c)+\gamma\left(\rho V_{n-1}^L(w+a-c)+(1-\rho)V_{n-1}^H(w+a-c)\right)\right) \\ V_n^H(w) &= \max_{c\leq w+a}\left(u(c)+\gamma\left((1-\sigma)V_{n-1}^L(w+a-c)+\sigma V_{n-1}^H(w+a-c)\right)\right) \end{aligned}$$

The notation is as follows. The parameter $\theta < 1$ governs the marginal utility of consumption in the low state, with the normalization $\theta^H = 1$. Annuity income in each period is a, and the wealth level before receiving this income is w. The probability of survival is γ, and the state persistence probabilities are ρ and σ. Consumption, c cannot exceed current resources $w + a$. Discounting is ignored.

One interesting case is when there is a corner solution in the high state, and an interior solution in the low state. Then

$$\begin{aligned} V_n^L(w) &= \max_{c\leq w+a}\left(\theta u(c)+\gamma\left(\rho V_{n-1}^L(w+a-c)+(1-\rho)V_{n-1}^H(w+a-c)\right)\right) \\ V_n^H(w) &= u(w+a)+\gamma\left((1-\sigma)V_{n-1}^L(0)+\sigma V_{n-1}^H(0)\right) \end{aligned}$$

This model is related to the Laitner et al. (2014) model. The difference is that neither state is absorbing. Thus there may be positive saving even late in life. For example if poor health is associated with a higher marginal utility of consumption expenditure, as Laitner et al. (2014) assume, someone currently in good health has an incentive to save, running wealth down to zero when there is an unfavorable transition, and starting to save again later if health improves.

4.4 Transient and Permanent Health Shocks

We extend the model by allowing both transient and permanent health shocks. More specifically, health is modelled as a bivariate process, $h = (T, P)$, where T and P are the transient respectively permanent health states, each taking value high or low (H or L).The permanent health state is absorbing, and if $P = H$, a permanent health shcok induces a transition to $P = L$.

The transient health state is modelled using a health stock variable, s. With t indicating the discrete time periods, the transition equation for the health stock is

$$s_t = \lambda s_{t-1} + v_t,$$

where $\lambda \in (0, 1)$ is the persistence and v_t the innovation to the health stock. Based on this, the transient health state is high when the health stock is at or above a suitable threshold, $T = H$ if $s \geq 1$, and $T = L$ otherwise.

In total, there are thus four possible configurations of $h \in \{HH, HL, LH, LL\}$. When convenient, we index these by $i = 1, \dots, 4$, with $i = 1$ indicating $h = HH$, and so on. Thus, we may write $p = \{p(i, j)\}_{ij}$ for the 4×4 health transition probability matrix, where $p(i, j) = P(h_{t+1} = i|h_t = j)$.

Because of the permanent health state, the second and fourth columns of p have zeroes in the first and third entries. We assume that all the agent knows about the health process is its current value, say h_t, and the conditional distribution of the future health states h_{t+1}, h_{t+2},... given h_t and Markov transitions based on p. Thus, the detailed level of the health stock is not accounted for, only whether it is above or below the threshold. The health stock s is instead a device for operationalizing the transient health state when taking the model to the data. Specifically, we assume that each occurence of a hospital diagnosis in any of a specified list of illness categories contributes to the transitory health shock,

$$v_t = \sum_{k=1}^{5} \beta_k 1\{k, t\},$$

where $1\{k, t\}$ is the indicator function for at least one diagnosis in category k in year t. Here, different diagnoses have different impact β_k on the health stock innovation v_t. In contrast, any diagnosis of a chronic condition constitutes a permanent health shock and so induces a transition to the absorbing low permanent health state. The five categories of diagnoses generating innovations to the health stock are diseases of the digestive and genitourinary systems, benign cancer, injuries/poisoning, and other conditions. The chronic conditions that we consider are mental and endochrine diseases, malignant cancer, and diseases of the circulatory, respiratory, nervous, and musculoskeletal systems.

4.5 The Likelihood Function

We assume that the individual knows current and future (annuity) income with certainty, subject to being alive. There is uncertainty about future health and time of death. Given current wealth and health, the individual chooses current consumption, and hence next period wealth. In the model specified so far, this determines consumption as a function of wealth and health. The value function satisfies the Bellman equation

$$V_n^h(w) = \max_{c \leq w+a} \left(\theta_h u(c) + \gamma(n, h) \sum_{i=1}^{4} p(i, h) V_{n-1}^{i}(w + a - c) \right).$$

Here, the survival probability is allowed to depend on health and age (but not income or wealth). A separate shift θ_h to the utility function (and hence marginal utility) is allowed for each health condition $h \in \{HH, HL, LH, LL\}$ (we normalize $\theta_{HH} = 1$). Up to parameters of the health stock process, wealth and health are observed and part of our data set. So is consumption. In practice, no (consumption choice) function will satisfy the strict requirement that it fully explains all observed consumption choices in terms of the health and wealth data. To get a useful empirical specification, we consider a random utility version of the model. Note that health is a discrete and wealth a continuous state variable. The control variable in the dynamic programming formulation is consumption, also continuous. In the implementation, we consider a finite grid for wealth and consumption. With each consumption choice in the grid, we associate an additive random current period utility shock. We assume this to be extreme value distributed, independently across consumption choices and time. Current (but not future) utility shocks are observed by the agent before making the consumption choice.

With these specifications, the observed process $\{h_t, w_t, c_t\}_t$ is Markov. Thus, the likelihood function for the individual is

$$\prod_t \gamma(n_{t-1}, h_{t-1}) p(h_t, h_{t-1}) q(c_t \mid h_t, w_t),$$

where the product is over periods where the agent stays alive, and $q(c_t \mid h_t, w_t)$ is the probability of choosing the observed consumption level c_t when faced with the observed state (h_t, w_t). Again, given this state, there is a non-degenerate probability distribution over consumption choices, due to the additional component of the state vector seen by the agent but not in our data, namely, the consumption choice specific random utility shocks. We have

$$q(c_t|h_t, w_t) = \frac{\exp(\xi W_{n_t}^{h_t}(w_t, c_t))}{\sum_k \exp(\xi W_{n_t}^{h_t}(w_t, c^k))},$$

where ξ is the scale parameter of the random utility shock, and $\{c^k\}_k$ are the consumption levels in the grid, observed c_t taking on one of these values (in this approximation). Finally, the choice specific value function components $W_n^h(w, c)$ are readily computed from the system

$$W_n^h(w, c^j) = \theta_h u\left(c^j\right) + \gamma(n, h) \sum_{i=1}^{4} p(i, h) \log\left(\sum_k \exp(\xi W_{n-1}^i(w + a - c^j, c^k))\right).$$

This may be solved by iterating on the contraction mapping defined on the argument on the lect hand side by the expression on the right hand side. It runs fast since there is no optimization on the right hand side. The contraction property follows because the survival probabilities are strictly less than unity. This represents an extension relative to the existing literature, where sure survival has been assumed, and a subjective discount factor (common across all states) has been used instead (e.g., Harold Zurcher). To interpret the choice specific value function, note that the full value function for the agent's decision problem is

$$\tilde{V}_n^h(w) = \max_{c^j \leq w+a} \left(W_n^h(w, c^j) + \varepsilon^j\right),$$

where $\{\varepsilon^j\}_j$ are the random utility shocks, and the (slight) difference between this and the previous value function (ignoring randomness in the utility function) is indicated by writing $\tilde{V}$ in place of V.

This completes the description of the likelihood function for the individual. The full log likelihood is the sum over individuals of the logs of the individual likelihoods. This is maximized numerically with respect to the unknown parameters. Asymptotic standard errors are read off the squareroots of the diagonal elements of the negative inverse Hessian. The parameters are $(\beta_1, \ldots, \beta_5, \lambda, \theta_{HL}, \theta_{LH}, \theta_{LL}, \xi, p)$. We read $\gamma(n, h)$ off life tables (actually, life tables calculated within our data). Thus, there are 18 parameters to be estimated, as there are four zero restrictions (from the absorbing state), four adding-up constraints, and thus eight parameters in the health transition matrix p.

5 Empirical Results

[to be added]

6 Conclusion

[to be added]

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