Employment Dynamics in Assignment Markets*

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December 15, 2014

Abstract

We study dynamics of employment and wages in labor markets where assignment plays an important role. We document new facts about inter and intra sector mobility in one example of such a market: the market for PhDs in both academia and the private sector. When the outside option (private sector wage) is higher, the exit rate from academia is higher, and, more strikingly, the rate of job mobility *within* academia is also higher. These dynamics are primarily driven by the relatively inexperienced and are more pronounced when the ratio of PhDs to academic jobs is higher. We develop a dynamic model of assignment and sectoral choice to explain these facts and explore how policies such as wage compression, tenure, and golden parachutes affect the equilibrium distribution of wages, lifecycle job mobility, and the extent of misallocation between workers and jobs.

*All mistakes are our own. Any and all comments welcome. Preliminary - please do not circulate without permission.

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1 Introduction

Many types of markets are well characterized as assignment problems - matches are one-to-one and there is significant heterogeneity in worker, firm, and thus match quality. Some well-known examples of these markets include the market for CEOs, academics, athletes, and entertainers. In assignment markets, in contrast to more conventional labor markets, small differences in quality can lead to large differences in wages and rents. Additionally, these markets are often under intense scrutiny for perceived inequities due to typically very skewed wage distributions.

There is therefore great interest in understanding the impact of policies such as limits to CEO pay, wage compression, or impediments to mobility such as academic tenure and golden parachutes. These policies impact the formation of matches and the distribution of wages in these markets, so understanding their implications is of utmost importance.

In this paper we document several facts about the dynamics of job transitions and sectoral choice in one example of an assignment market: the market for PhDs in academia and the private sector. We categorize PhD recipients as participating either in academia or in the private sector and study the relationship between wages and transitions across jobs and sectors. First, we show that higher wages in the private sector are associated with higher exit rates from academia to the private sector. Second, we show that higher wages in the private sector are associated with higher rates of movement between jobs *within* academia, job mobility we call churn. Documenting this positive relationship between wages in the outside sector and churn within a sector is a previously undocumented fact as far as we are aware. In theory this relationship can go either way, outside wages can either increase or decrease internal churn.

We also show that the magnitude of the effect of private sector wages on both exit from academia and churn within academia depends on the tightness of the current labor market. When conditions are relatively favorable to PhDs in finding a job within academia, as measured by the relative employment of academia to total PhDs within a field, then changes in the outside wage option have a substantially larger effect on both exit and churn.

We then explore a dynamic model of assignment and sectoral choice to explain these stylized facts of job mobility within and across sectors. The model allows us to evaluate counterfactuals such as limits to CEO pay, wage compression, tenure and severance packages. We discipline the model using the stylized facts we documented in the data.

In the model there are two types of agents: workers and jobs. There is one-to-one matching between workers and jobs. Workers and jobs form potential matches with randomly drawn match-specific surplus. Workers have outside options that they may choose instead of choosing a job within academia.[1] There is thus an assignment problem. Pairwise stability of matches[2] therefore pins down the matches, although payoffs depend on bargaining power and how the match surplus is split.

Over time, the random distribution of match-specific productivities and workers' outside options change. We model this with the following process: match-specific productivities are independent and drawn from an exogenous distribution. A match-specific productivity is perfectly persistent except when hit by a Poisson shock, after which it is redrawn from the distribution. Thus across two points in time some match-specific productivities are redrawn and some remain unchanged. Outside options may change but are correlated across periods.

Matches may dissolve for a number of reasons. Match-specific productivities may be redrawn: the worker may get a new better outside offer, either the worker or firm gets a new match-specific productivity with another partner that provides more surplus, or a potential trading partner's match with someone else may dissolve.

Workers can move in and out of academia (exit) and across jobs within academia (churn). When one match dissolves, it may trigger a chain of reallocations within academia.

In general, better outside options may lead to more or less churn within academia. Why might better outside options lead to churn? One possibility is that good outside options are also outside options that change frequently. Churn requires workers matched in academia during the first period to leave academia before the second period. This is more likely to happen when outside options change or get better over time than if outside options are persistent.

The market for Ph.D's in academia and the private sector is particularly well-suited to

[1] Jobs may have outside options too, although this is not necessary if there are more workers than jobs.

[2] or stability with respect to a deviation by any coalition

answer our questions about these types of markets for a variety of reasons. It is clear who market participants are: people with PhDs in particular fields. Although there is some gray area, compartmentalizing the market into two sectors (academia and the private sector) is plausible. Taking together, these features mean that it is relatively straightforward to define and measure both the outside option of academia and the current amount of labor market tightness. Additionally, the clear differences between the academic labor market and the private sector labor market (for PhDs) gives us confidence that there is something distinct about job transitions from academia to the private sector and job transitions from one academic job to another.

Finally, there are advantages in studying the academic labor market in terms of data availability. We are able to use a large panel dataset of PhD recipients and follow their career lifecycle and job transition history. Additionally, there is substantial heterogeneity in both the outside option and the tightness of the labor market over field and time. This is one distinct advantage of academia over, say, the market for CEOs; essentially we have numerous labor markets operating in parallel and are therefore more confident in the patterns we find as representative of a general functioning of these markets.

The paper proceeds as follows. Section 2 describes a set of stylized facts of job and sectoral mobility for academics. Section 3 presents the model of job and sectoral mobility. Section 4 concludes.

2 Facts about sectoral mobility for academics

In this section we establish a set of stylized facts about wages and job transitions for people with PhDs. To do this we combine two datasets: the Survey of Doctoral Recipients (SDR) and the National Survey of College Graduates (NSCG). The SDR is a biennial panel survey of individuals with doctorates in a science, engineering, or health field from a U.S. academic institution. Respondents are drawn from the sample of doctorates responding to the annual cross-sectional Survey of Earned Doctorates who are then followed over time. The National Survey of College Graduates (NSCG) is a panel survey that samples from all students who graduate with a Bachelor's. Since our interest is in individuals with PhDs we further subsample to all individuals who have a PhD by age 30, which is about 4% of the total NSCG sample. Unlike the SDR, this sample includes PhDs in non-STEM fields.

Combining these surveys is relatively straightforward as they were administered by the same organization and thus the questions are largely the same and they were mostly administered in the same years.[3] Both surveys are similarly unbalanced in that eacih wave adds recent PhD graduates as well loses individuals who pass away. We restrict our attention to males aged between 30 and 65 who were surveyed between 1993 and 2010.

Together, we have 50,696 unique respondents and 159,560 individual-year pair observations. The average respondent earned their PhD in 1980, with the earliest earning his in 1950 and the latest in 2010. The sample is roughly three-quarters white and one-fifth asian. The average male PhD recipient does not work in academia, with just over 55% reporting work in either government or the private sector. Not surprisingly, the average wage in the private sector is substantially larger than in academia, with academic PhDs earning roughly $79,000 while private sector PhDs earn roughly $101,000.

Fact 1. *Higher wages in the private sector are associated with increased job transitions from academia to the private sector (exit) as well as increased job transitions within academia (churn).*

The first two stylized facts are presented in Tables 1 and 2 which describe the determinants of job-to-job transitions for PhDs for both exit and churn. Estimates in Table 1 show what predicts an individual who was previously employed in academia leaving their job for the private sector. Exiting academia is positively related to current wages. This result is consistent with the large literature on job search and mobility as individuals who experience job-to-job transitions (instead of job to unemployment and then to job transitions) typically see an increase in their wage. This also partly reflects the higher wages paid in the private sector on average. Interestingly, exit from academia is negatively correlated with their previous wage in an academic job. This is likely due to the fact that people leaving their jobs are more likely to have been a more match for either that job or academia and so have lower average wages.[4]

[3] The NSCG was not administered in 2001 and data from the SDR in 2010 is not presently available.

[4] An interesting question that we do not address at the moment is the possibility of heterogeneity in skill

Looking at Column 2 of Table 1 we see that the outside option, as measured by the log median wage in the private sector, is an important determinant for leaving academia for the private sector.[5] As the value of the outside option increases, individuals are more likely to leave their current job within academia and take advantage of this outside option. Column 3 shows that this effect is robust to the inclusion of a set of demographic and field fixed effects as controls. Columns 4 and 5 of Table 1 decompose this effect by separately estimating it for junior (ages 30-45) and senior (ages 50-65) academics.

Table 2 establishes a similar set of facts for people who move between jobs entirely within academia. Interestingly, Column 1 suggests that people who move between jobs do not experience a net wage improvement as their current wage is not predictive of the move although they do appear to have been negatively selected as having a low previous wage increases the probability of changing jobs. One plausible explanation for this is the relative compression of wages within academia. Another alternative is that the average wage effect is zero but that is obscuring significant heterogeneity in that a large fraction of individuals see substantial wage gains as they move to better opportunities while another see substantial losses as they are denied tenure and move down the academic prestige ladder.

Looking at Column 2 of Table 2 shows that there is a strong relationship between the outside option and the rate of churn within academia. This effect is robust to the inclusion of a set of demographic and field fixed effects (Column 3) and appears to be stronger for junior academics than senior academics (Columns 4 and 5). Figure 1 shows the strength of this positive relationship between the outside option and job churn within academia. For every field and year pair in our data we calculate the outside option of the median wage in the private sector as well as the rate of within academia churn. Figure 1 shows the relationship between these for every decile of the median wage in the private sector. We see that there is a strongly positive association between the outside option and internal churn.

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across these sectors and whether the leavers have a comparative advantage in the private sector a la a Roy model.

[5] Although this section uses the median wage of workers within a PhD field in the private sector as the outside option the facts presented are robust to many other definitions of the outside wage. These alternative definitions include: using the mean wage of the private sector, using the mean wage of recent switchers from academia to private sector, constructing a matching wage based on a large set of worker observables, matching on a large set of worker observables plus applying a parametric selection correction to the mean wage to account for unobserved heterogeneity, and many other slight modifications of the above. Similar patterns are found for all of them. Details available upon request.

Fact 2. *The impact of outside option on both exit churn is heterogeneous based on labor market tightness.*

The next set of stylized shows that the previously established relationship between changes in the outside wage and churn mobility within academia fluctuates depending on the state of the internal academia labor market. When the academic labor market is relatively tight the impact of the outside option

Defining labor market tightness is a challenge with our data because we only have data on employed individuals, not on vacancies. As a proxy for the tightness of the labor market for a particular field and year we use the ratio of the number of PhDs employed in academia to the total number of PhDs in that field/year. If the employment rate of PhDs within academia is particularly high then we interpret that labor market as being looser than a field/year where the employment rate of PhDs within academia is much lower.

We define θ to be this employment rate for a field f in year y:

$$\theta = \frac{A_{f,y}}{A_{f,y} + P_{f,y}}$$

where A is the number of employed PhDs in academia and P is the number of employed PhDs in the private sector.

Tables 3 and 4 present estimates of how the outside option affects exit and churn, interacted with θ. Looking first at Table 3 we see that when we interact the median wage in the private sector with θ the the effect is lessened. When academic labor markets are tighter changes in the outside option have a significantly lower effect on the probability of leaving academia for the private sector. Columns 4 and 5 suggest that this effect is strongest for senior academics.

In Table 4 we see that labor market tightness also effects the relationship between the outside option and churn within academia. Looking at Column 3 we see there is a positive but marginally statistically insignificant ($p = 0.11$) relationship on this interaction. When we break down the effect by junior and senior academics, however, there is clearly a much stronger relationship between the median wage in the private sector and academic churn when labor market conditions are tight for junior academics.

3 Model

3.1 Static Model

There is a continuum of workers with mass W and a continuum of jobs of with measure J. Utility is transferable. There is a matching process in which workers are matched to jobs, creating a set of potential matches $\mathcal{M}$ with mass $M(W, J)$. Each potential match is characterized by a particular worker, a particular job, and an match-specific productivity, z, drawn from a distribution with CDF $H(z)$. For an individual worker or job, the identity of the partner of the match is uniform, and the productivity of a potential match is independent of the identity of the partner and of other matches. The probability that a worker matches to n jobs is p_n^W and the probability that a job matches to n workers is p_n^J. Thus

$$W \sum_{n=0}^{\infty} p_n^W n = M(W, J) = J \sum_{n=0}^{\infty} p_n^J n$$

Later, we will assume that the number of matches that arrive to any worker or job follow Poisson distributions.

In addition, workers and jobs also have outside options u_i^o and v_i^o that are randomly from the distributions with respective CDFs $F_{u^o}(u^o)$ and $F_{v^o}(v^o)$ and are independent of the individual matches.

3.1.1 Equilibrium

Given the set of potential matches, $\mathcal{M}$, workers and firm must select which partner to actually match with. We will study the set of pairwise stable matches.

The set of pairwise is a well studied object. The set of pairwise stable equilibria corresponds to the core of the economy. There is a unique (up to sets of measure zero) set of matches consistent with pairwise stability. However, the set of payoffs an individual may receive is an interval: for a matched pair, any split of surplus is consistent with pairwise stability. For each individual, there is a maximum and minimum payoff consistent with pairwise stability. For worker i call these $\bar{u}_i$ and $\underline{u}_i$, and for a job j call these $\bar{v}_j$ and $\underline{v}_j$. If worker i is matched with j in a match that produces output z, then $\bar{u}_i + \underline{v}_j = \underline{u}_i + \bar{v}_j = z$. The numbers.$\underline{u}_i$ and $\underline{v}_j$ should be thought of as worker i's and job j's best alternative if the match dissolved. For simplicity we will assume that workers have all of the bargaining power so that $u_i = \bar{u}_i$ and $v_j = \underline{v}_j$. This assumption is relaxed in Appendix **??**.[6]

We now define several distributions that will be useful in characterizing the equilibrium. Let $\bar{F}_u(u)$ and $\bar{F}_v(v)$ be the fraction of workers and jobs with *maximal* payoffs no greater than u and v respectively.

For a job, consider a single potential trading partner; let $\tilde{F}_u(u)$ be that trading partner's best other option. Similarly, for a worker, consider a single potential job she has matched with; let $\tilde{F}_v(v)$ be that job's best other option.

Define $G_u(u) = \int_{-\infty}^{\infty} H(u+v)d\tilde{F}_v(v)$ and $G_v(v) = \int_{-\infty}^{\infty} H(u+v)d\tilde{F}_u(u)$. To interpret G_u, consider a single potential match. If the worker is in that match in equilibrium, there would be a best possible payoff $\bar{u}$. $G_u(u)$ is the fraction of matches for which the worker's maximum payoff from that match would be no better than u. To understand the formula, the potential match is associated with a partner with a best alternative v and a match specific productivity z. The probability that the match would deliver a maximum payoff no greater than u is the same as the probability that the match specific productivity is no greater than $u+v$. To get G_u, we simply integrate over the possible realization of the partner's best alternative and the realization of the match-specific productivity.

[6] Appendix **??** parameterizes workers' bargaining power as β, so that

$$
\begin{aligned}
w_i &= \underline{u}_i + \beta\left[z - \underline{u}_i - \underline{v}_j\right] = (1-\beta)\,\underline{u}_i + \beta\bar{u}_i \\
z - w_i &= \underline{v}_j + (1-\beta)\left[z - \underline{u}_i - \underline{v}_i\right] = \beta\underline{v}_j + (1-\beta)\,\bar{v}_i
\end{aligned}
$$

Under the functional forms of Section 3.1.2, this lowers the average equilibrium wage by $\frac{1-\beta}{\lambda}$.

The distribution of payoffs satisfy

$$\begin{aligned}
\bar{F}_u(u) &= F_{u^o}(u)\sum_{n=0}^{\infty} p_n^W G_u(u)^n \\
\bar{F}_v(v) &= F_{v^o}(v)\sum_{n=0}^{\infty} p_n^J G_v(v)^n \\
\tilde{F}_u(u) &= F_{u^o}(u)\sum_{n=1}^{\infty} \frac{p_n^W}{\sum_{\tilde{n}=1}^{\infty} p_{\tilde{n}}^W} G_u(u)^{n-1} \\
\tilde{F}_v(v) &= F_{v^o}(v)\sum_{n=1}^{\infty} \frac{p_n^J}{\sum_{\tilde{n}=1}^{\infty} p_{\tilde{n}}^J} G_v(v)^{n-1}
\end{aligned}$$

$\bar{F}_u(u)$ is the probability that, across all potential matches, a worker's maximum payoff is no better than u. p_n^W is the probability that the worker has n potential matches, and $G_u(u)^n$ is the probability that none of those n matches provide a maximum payoff better than u.

Consider a single match. $\tilde{F}_v(v)$ is the probability the job's maximal payoff from its best alternative is no better v. $\frac{p_n^J}{\sum_{\tilde{n}=1}^{\infty} p_{\tilde{n}}^J}$ is the probability that the job has $n-1$ alternative matches (this is the probability that job has n matches in total conditioning on having at least one) and $G_v(v)^{n-1}$ is the probability that none of those alternatives provide a maximal payoff better than v.

A nice feature is that $\tilde{F}_u(u)$ and $\tilde{F}_v(v)$ form a recursive system. This happens because of the assumption that a continuum of players on each side of the market.

3.1.2 Functional Form Assumptions

Define $A_W \equiv \frac{M(W,J)}{W}$ and $A_J \equiv \frac{M(W,J)}{J}$ to be the average number of matches per worker and per firm respectively. To draw out the implications of the model, it will be useful to make several functional form assumptions.

Assumption 1. *The arrival of matches for each side is poisson:* $p_n^W = \frac{e^{-A_W} A_W^n}{n!}$ *and* $p_n^J = \frac{e^{-A_J} A_J^n}{n!}$.

Assumption 2. *The distribution of match-specific productivities is exponential:*

$$H(z) = \max\left\{0, 1 - e^{-\lambda(z-z_0)}\right\}$$

We next make a third assumption, which corresponds to a limiting economy.

Assumption 3. *For any z, the arrival of matches with match-specific productivity greater than z is $me^{-\lambda z}$*

To understand this as a limiting economy, define $m(W,J) \equiv M(W,J)e^{-\lambda z_0}$. We will look at the limit of a sequence of economies as $z_0 \to -\infty$ (holding m fixed). This means that as $z_0 \to -\infty$, the number of matches grows large $M \to \infty$ but also that the distribution of match-specific productivities deteriorates. The limiting economy is such that the arrival of matches with productivity z is $me^{-\lambda z}$.

Next define $a_W \equiv \frac{m(W,J)}{W}$ and $a_J \equiv \frac{m(W,J)}{J}$.

Claim 1. *Under assumptions 1-3,*

$$\begin{aligned} F_{\bar{u}}(u) &= \tilde{F}_u(u) = F_{u^o}(u)e^{-\phi^i_W e^{-\lambda u}} \\ F_{\bar{v}}(v) &= \tilde{F}_v(v) = F_{v^o}(v)e^{-\phi^i_J e^{-\lambda v}} \end{aligned}$$

where

$$\begin{aligned} \phi^i_W &= a_W \int_{-\infty}^{\infty} e^{-\lambda v} d\tilde{F}_v(v) \\ \phi^i_J &= a_J \int_{-\infty}^{\infty} e^{-\lambda u} d\tilde{F}_u(u) \end{aligned}$$

With additional structure on the outside options, we can further characterize the these distributions.

Assumption 4. *The distributions of outside options are $F_{u^o}(u) = e^{-\phi^o_W e^{-\lambda u}}$ and $F_{v^o}(v) = e^{-\phi^o_J e^{-\lambda v}}$*

Under this additional assumption, we can more sharply characterize the distributions.

Corollary 1. *Under assumptions 1-4,*

$$
\begin{aligned}
F_{\bar{u}}(u) &= \tilde{F}_u(u) = e^{-\phi_W e^{-\lambda u}} \\
F_{\bar{v}}(v) &= \tilde{F}_v(v) = e^{-\phi_J e^{-\lambda v}}
\end{aligned}
$$

where

$$
\begin{aligned}
\phi_W &= \phi_W^o + \phi_W^i \\
\phi_J &= \phi_J^o + \phi_J^i \\
\phi_W^i &= \frac{a_W}{\phi_J} \\
\phi_J^i &= \frac{a_J}{\phi_W}
\end{aligned}
$$

Claim 2. *Under Assumptions 1-3,*

1. *The fraction of workers using inside options is* $\frac{\phi_W^i}{\phi_W}$
2. *The average wage among workers using their outside options is* $\frac{1}{\lambda}\left[\ln \phi_u + \gamma\right]$
3. *The average wage among workers using their inside options is* $\frac{1}{\lambda}\left[\ln \phi_u + \gamma\right]$

where γ *is the Euler-Mascheroni constant.*

Note that if jobs have no outside options[7], then

$$
\phi_W = \frac{1}{1 - \frac{a_W}{a_J}} \phi_W^o = \frac{1}{1 - \frac{J}{W}} \phi_W^o
$$

Thus the tightness of the market acts as a multiplier of the the value of the outside option. With fewer workers, each worker is likely to have a better alternative, which raises the value of alternatives even more, etc..

[7] This requires that $W > J$ or the wages of workers would be infinite.

3.2 Dynamics

Conceptually, the dynamic model is a straightforward extension of the static model. Over time, matches arrive and decay, and outside options may change. We will make the assumption dissolving a match is costless, and that each instant the economy reorganizes to a pairwise stable allocation. Thus the cross-section of the dynamic model will have the same properties as the static model.

However, we are interested in churn, which is requires at least two periods to analyze. Tracking individual workers and jobs is difficult, because the state of an individual consists of their potential matches, their potential trading partners' potential matches, etc. Instead, it will be useful to characterize the joint distribution of payoffs across two points in time.[8]

We are interested in following workers over time. Suppose there are two points in time, t_1 and t_2. Suppose there are M_1 matches that form before t_1 but end between t_1 and t_2, M_{12} matches that are formed before t_1 and survive beyond t_2, and M_2 matches that are formed between t_1 and t_2 that survive beyond t_2.

One example is if there is a constant mass M and matches are formed and decay at rate δ. Then $M_{12} = e^{-\delta(t_2-t_1)}M$ and $M_1 = M_2 = \left[1 - e^{-\delta(t_2-t_1)}\right] M$.

Workers and jobs have random (and potentially changing) outside options. Let $F_{u^\circ}(u_1, u_2)$ be the fraction of workers with outside options no better than u_1 and u_2 at times t_1 and t_2 respectively. Similarly, let $F_{v^\circ}(v_1, v_2)$ be the fraction of jobs with outside options no better than v_1 and v_2 at times t_1 and t_2 respectively.

We now derive formulas for the joint distribution of payoffs across times t_1 and t_2. This is more cumbersome than in the static model, although the limiting economy the formulas will simplify considerably.

Among workers, we consider four distributions. $\bar{F}_u(u_1, u_2)$ is the fraction of workers with maximal payoffs no better than u_1 and u_2 at times t_1 and t_2. Among matches that exist at t_1 but not t_2, let $\tilde{F}_{u_1}(u_1)$ be the fraction for which the worker's maximal alternative t_1 payoff is no greater than u_1. Similarly, among matches that exist at t_2 but not t_1, let $\tilde{F}_{u_2}(u_2)$ be the fraction for which the worker's maximal alternative t_2 payoff is no greater than u_2.

[8] It is straightforward (though cumbersome) to generalize this method to more than two points in time.

Finally, among matches that exist at both times, let $\tilde{F}_{u_{12}}(u_1, u_2)$ be the fraction for which the worker's maximal payoffs are no greater than u_1 and u_2. We can similarly define $\bar{F}_v(v_1, v2)$, $\tilde{F}_{v_1}(v_1)$, $\tilde{F}_{v_2}(v_2)$, and $F_{v_{12}}(v_1, v_2)$ for jobs.

To characterize these distributions, it will be useful to consider payoffs that come from single potential matches. We will derive formulas for the workers, the derivation for jobs is identical.

Among matches that exist in t_1 but not t_2, let $G_{u_1}(u_1)$ be the fraction that would deliver a minimum payoff no greater than u_1 to the worker. Similarly, $G_{u_2}(u_2)$ is the fraction of matches that exist at t_2 but not t_1 that would deliver payoff no greater than u_2. Finally, among matches that exist at both times, $G_{u_{12}}(u_1, u_2)$ is the fraction that that would deliver minimal payoffs no greater than u_1 and u_2. These satisfy

$$\begin{aligned}
G_{u_1}(u_1) &= \int_{-\infty}^{\infty} [1 - \tilde{F}_{v_1}(z - u_1)] dH(z) \\
G_{u_2}(u_2) &= \int_{-\infty}^{\infty} [1 - \tilde{F}_{v_2}(z - u_2)] dH(z) \\
G_{u_{12}}(u_1, u_2) &= \int_{-\infty}^{\infty} \left[1 - \tilde{F}_{v_{12}}(z - u_1, \infty) - \tilde{F}_{v_{12}}(\infty, z - u_2) + \tilde{F}_{v_{12}}(z - u_1, z - u_1)\right] dH(z)
\end{aligned}$$

The formulas for G_{u_1} and G_{u_2} are direct analogues of the static case. The formula for $G_{u_{12}}$ is derived by noting that $G_{u_{12}}(u_1, u_2) = \int_{-\infty}^{\infty} \Pr\left(z - u_2 < V_2, z - u_2 \leq V_2\right) dH(z)$.

Given all of her matches, the joint distribution for the best possible payoff for a worker is

$$\bar{F}_u(u_1, u_2) = F_{u^o}(u_1, u_2) F_{u^i}(u_1, u_2)$$

where

$$F_{u^i}(u_1, u_2) = \left(\sum_{n=0}^{\infty} p_n^{W_1} G_{u_1}(u_1)^n\right) \left(\sum_{n=0}^{\infty} p_n^{W_{12}} G_{u_{12}}(u_1, u_2)^n\right) \left(\sum_{n=0}^{\infty} p_n^{W_2} G_{u_2}(u_2)^n\right)$$

To derive formulas for $\tilde{F}_{u_1}$, $\tilde{F}_{u_2}$, and $\tilde{F}_{u_{12}}$, we simply note that we must condition on the worker having at least one of the specified match.

$$
\begin{aligned}
\tilde{F}_{u_1}(u_1) &= F_{u^o}(u_1,\infty)\left(\frac{\sum_{n=1}^{\infty} p_n^{W_1} G_{u_1}(u_1)^{n-1}}{\sum_{n=1}^{\infty} p_n^{W_1}}\right)\left(\sum_{n=1}^{\infty} p_n^{W_{12}} G_{u_{12}}(u_1,\infty)^n\right)\\
\tilde{F}_{u_2}(u_2) &= F_{u^o}(\infty,u_2)\left(\sum_{n=0}^{\infty} p_n^{W_{12}} G_{u_{12}}(\infty,u_2)^n\right)\left(\frac{\sum_{n=1}^{\infty} p_n^{W_2} G_{u_2}(u_2)^{n-1}}{\sum_{n=1}^{\infty} p_n^{W_2}}\right)\\
\tilde{F}_{u_{12}}(u_1,u_2) &= F_{u^o}(u_1,u_2)\left(\sum_{n=1}^{\infty} p_n^{W_1} G_{u_1}(u_1)^n\right)\left(\frac{\sum_{n=1}^{\infty} p_n^{W_{12}} G_{u_{12}}(u_1,u_2)^{n-1}}{\sum_{n=1}^{\infty} p_n^{W_{12}}}\right)\left(\sum_{n=0}^{\infty} p_n^{W_2} G_{u_2}(u_2)^n\right)
\end{aligned}
$$

The equations for $\tilde{F}_{u_1}$, $\tilde{F}_{u_2}$, and $\tilde{F}_{u_{12}}$ along with the analogous equations for jobs is a system of functional equations that fully characterize the equilibrium.

3.2.1 Payoffs and Flows

The fraction of workers with payoffs no greater than u_1 and u_2 in the first and second periods respectively is

$$\bar{F}_u(u_1,u_2) = F_{u^o}(u_1,u_2)F_{u^i}(u_1,u_2)$$

Since $1 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_u(du_1,du_2)$, the product rule gives the following identity:[9]

$$
\begin{aligned}
1 &= \underbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_{u^o}(u_1,u_2)F_{u^i}(du_1,du_2)}_{\text{stay in}} + \underbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_{u^o}(u_1,du_2)F_{u^i}(du_1,u_2)}_{\text{switch out}}\\
&\quad + \underbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_{u^o}(du_1,du_2)F_{u^i}(u_1,u_2)}_{\text{stay out}} + \underbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_{u^o}(du_1,u_2)F_{u^i}(u_1,du_2)}_{\text{switch in}}
\end{aligned}
$$

Thus the measure of workers with payoffs u_1 and u_2 can be divided into four categories. The first are those that spend both periods within academia, and we give them the label "stay in." Their inside options are u_1 and u_2 and their outside options are no greater than u_1 nd u_2. Second are those the are in academia the first period but switch to the outside option for the second period. These are individuals whose first period inside option is u_1 and second period outside option is u_2, and these are respectively better than their first period outside

[9] We use this notation because F_u may not be twice differentiable.

options and second period inside options. The third category are those that begin using their outside option but enter academia in the second period. The fourth category are those that use their outside options in both periods.

This decomposition useful in that it facilitates solving for many equilibrium objects. For example, among workers in academia in the first period, the fraction that switch out before the second is

$$\frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_{u^i}(du_1,u_2)F_{u^o}(u_1,du_2)}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[F_{u^i}(du_1,du_2)F_{u^o}(u_1,u_2)+F_{u^i}(du_1,u_2)F_{u^o}(u_1,du_2)]}$$

It can also be used to compute conditional moments of payoffs. For example, the average change in payoff among those that switch out of academia is

$$\frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(u_2-u_1)F_{u^i}(du_1,u_2)F_{u^o}(u_1,du_2)}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_{u^i}(du_1,u_2)F_{u^o}(u_1,du_2)}$$

Finally we derive an expression for churn. Among those who begin in academia, we want to calculate the share of workers that switch jobs.

Consider all matches that last for both periods. If there are n such matches, the probability that a single one delivers maximal payoffs u_1 and u_2 and the others deliver payoffs no greater than u_1 and u_2 is

$$nG_{u_{12}}(u_1,u_2)^{n-1}G_{u_{12}}(du_1,du_2)$$

Integrating over possible realizations of n, the unconditional probability that a single match delivers maximal payoffs u_1 and u_2 and all others deliver payoffs that are no better is

$$\sum_{n=1}^{\infty} p_n^{W_{12}} nG_{u_{12}}(u_1,u_2)^{n-1}G_{u_{12}}(du_1,du_2)$$

Multiplying this by the probability that none of the matches that exist only at t_1 deliver payoff better than u_1, none of the matches that exist only at t_2 deliver payoff better than u_2, and the outside option does not deliver payoffs better than u_1 or u_2 gives

$$\sum_{n=1}^{\infty} p_n^{W_{12}} nG_{u_{12}}(u_1,u_2)^{n-1}G_{u_{12}}(du_1,du_2)\frac{F_{u^i}(u_1,u_2)}{\sum_{n=0}^{\infty} p_n^{W_{12}} G_{u_{12}}(u_1,u_2)^{n}}F_{u^o}(u_1,u_2)$$

Integrating over u_1 and u_2 and dividing by the fraction in academia gives an expression for churn

$$\frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\frac{\sum_{n=1}^{\infty}p_n^{W12}nG_{u_{12}}(u_1,u_2)^{n-1}}{\sum_{n=0}^{\infty}p_n^{W12}G_{u_{12}}(u_1,u_2)^n}G_{u_{12}}(du_1,du_2)F_{u^i}(u_1,u_2)F_{u^o}(u_1,u_2)\right]}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[F_{u^i}(du_1,du_2)F_{u^o}(u_1,u_2)+F_{u^i}(du_1,u_2)F_{u^o}(u_1,du_2)\right]}$$

3.2.2 Functional Form Assumptions

As in the static environment it will be useful to make some parametric assumptions.

Assumption 5. *The arrival of matches for each side is poisson:* $p_n^{W_1} = \frac{e^{-A_{W1}}A_{W1}^n}{n!}$ *and* $p_n^{J_1} = \frac{e^{-A_{J1}}A_{J1}^n}{n!}$ *etc...*

It will also be useful to give some structure to the distributions of outside options.

Assumption 6. *The joint CDFs of outside options for workers and jobs can be written as*

$$\begin{aligned}
F_{u^o}(u_1,u_2) &= \exp\left\{-e^{-\lambda u_1}\Phi_W^o\left(e^{-\lambda(u_2-u_1)}\right)\right\}\\
F_{v^o}(v_1,v_2) &= \exp\left\{-e^{-\lambda v_1}\Phi_J^o\left(e^{-\lambda(v_2-v_1)}\right)\right\}
\end{aligned}$$

Assumption 6 is analogous to Assumption 4. We can provide examples of ways of parameterizing Φ_W^o and Φ_J^o so that the marginal distribution of outside options at a single date follows a Gumbel distribution, but the distributions are correlated across the two dates.

Claim 3. *Under Assumptions 2, 3, 5 and 6, there is are functions* F_u *and* F_v *such that*

$$\begin{aligned}
\bar{F}_u(u_1,u_2) &= \tilde{F}_{u_{12}}(u_1,u_2) = F_u(u_1,u_2)\\
\tilde{F}_{u_1}(u_1) &= F_u(u_1,\infty)\\
\tilde{F}_{u_2}(u_2) &= F_u(\infty,u_2)\\
\bar{F}_v(v_1,v_2) &= \tilde{F}_{v_{12}}(v_1,v_2) = F_v(v_1,v_2)\\
\tilde{F}_{v_1}(v_1) &= F_v(v_1,\infty)\\
\tilde{F}_{v_2}(v_2) &= F_v(\infty,v_2)
\end{aligned}$$

These functions F_u and F_v are defined by

$$\begin{aligned} F_u(u_1, u_2) &= e^{-e^{-u_1}\Phi_W\left(e^{-\lambda(u_2-u_1)}\right)} \\ F_v\left(v_1, v_2\right) &= e^{-e^{-v_1}\Phi_J\left(e^{-\lambda(v_2-v_1)}\right)} \end{aligned}$$

where the functions $\Phi_W(\cdot))$ and $\Phi_J(\cdot)$ satisfy

$$\begin{aligned} \Phi_W\left(s\right) &= \Phi_W^o\left(s\right) + \frac{a_{W1} + a_{W12}}{\Phi_J\left(0\right)} + s\frac{a_{W2} + a_{W12}}{\lim_{t\to\infty}\Phi_J\left(t\right)/t} - \frac{a_{W12}}{\Phi_J\left(s^{-1}\right)} \\ \Phi_J\left(s\right) &= \Phi_J^o\left(s\right) + \frac{a_{J1} + a_{J12}}{\Phi_W\left(0\right)} + s\frac{a_{J2} + a_{J12}}{\lim_{t\to\infty}\Phi_W\left(t\right)/t} - \frac{a_{J12}}{\Phi_W\left(s^{-1}\right)} \end{aligned}$$

Appendix 1 (to be added) provides closed form expressions for $\Phi_W(s)$ and $\Phi_J(s)$. Note that this generalizes claim 1. The marginal distribution of payoffs for workers at t_1 is

$$\exp\left\{\Phi_W(0)e^{-\lambda u}\right\}$$

and the marginal distribution of payoffs for workers at t_2 is

$$\exp\left\{\left[\lim_{s\to\infty}\frac{\Phi_W(s)}{s}\right]e^{-\lambda u}\right\}$$

3.2.3 Numerical example

While the model is stylized, we can provide some simple comparative statics. Figure 2 shows the impact of increasing the level of the outside option on internal churn. Here, outside options across the two periods are almost uncorrelated. There is a clear pattern that when the outside option is higher, churn within academia is also higher.

4 Conclusion

In this paper we have documented several facts about a labor market where assignment and outside options play an important role: the market for individuals with PhDs. The market for PhDs within academia is responsive to changes in the private sector wages for PhDs;

increases in the outside option wage increase both exit from academia to the private sector as well as job-to-job transitions *within* academia. Further, the magnitude of this relationship varies by the condition of the labor market within academia. When labor market conditions favor workers, changes in the outside option have a weaker role in explaining exit from academia but a stronger role in explaining internal churn within academia.

To explore the mechanisms that underly these facts we develop a dynamic model of assignment and job choice. The model generates a predictions that fit the stylized facts of the academic labor market. Further, the model will allow us to evaluate counterfactuals such as changes in the distribution of worker or firm quality, changes in the match quality distribution, and the importance of tenure and severance packages.

Table 1: Determinants of exit (academia to private sector transitions)

	(1)	(2)	(3)	(4)	(5)
	1	2	3	4	5
	b/se	b/se	b/se	b/se	b/se
Median w_P		0.0611	0.0621*	0.245**	-0.152**
		(0.0520)	(0.0328)	(0.0798)	(0.0494)
Current log salary	0.0759***	0.0755***	0.0740***	0.108***	0.0167*
	(0.00765)	(0.00761)	(0.00725)	(0.0127)	(0.00932)
Previous log salary	-0.106***	-0.106***	-0.0956***	-0.125***	-0.0394**
	(0.00876)	(0.00873)	(0.00827)	(0.0112)	(0.0107)
N	31817	31817	31817	14349	13161
Demographic controls	No	No	Yes	Yes	Yes
Field of study controls	No	No	Yes	Yes	Yes
Sample	Full	Full	Full	Ages 30-45	Ages 50-65

Data from Survey of Doctorate Recipients (SDR) and National Survey of College Graduates (NSCG), 1993-2010. Both samples are restricted to males aged 30 to 65. We restrict the NSCG sample to individuals with a reported PhD by age 30. Robust standard errors are clustered by PhD field of study.

$*$: significant at 10% level. $**$: significant at 5% level. $***$: significant at 1% level.

Table 2: Determinants of churn (academia to academia transitions)

	(1)	(2)	(3)	(4)	(5)
	1	2	3	4	5
	b/se	b/se	b/se	b/se	b/se
Median w_P		0.124*	0.506***	0.751***	0.261***
		(0.0666)	(0.0827)	(0.114)	(0.0598)
Current log salary	-0.000575	-0.00135	0.000437	0.00936	-0.0136***
	(0.00726)	(0.00718)	(0.00642)	(0.00939)	(0.00311)
Previous log salary	-0.104***	-0.104***	-0.0902***	-0.143***	-0.0366**
	(0.0195)	(0.0192)	(0.0164)	(0.0188)	(0.0123)
N	45528	45528	44629	19065	19500
Demographic controls	No	No	Yes	Yes	Yes
Field of study controls	No	No	Yes	Yes	Yes
Sample	Full	Full	Full	Ages 30-45	Ages 50-65

Data from Survey of Doctorate Recipients (SDR) and National Survey of College Graduates (NSCG), 1993-2010. Both samples are restricted to males aged 30 to 65. We restrict the NSCG sample to individuals with a reported PhD by age 30. Robust standard errors are clustered by PhD field of study.

$*$: significant at 10% level. $**$: significant at 5% level. $***$: significant at 1% level.

Table 3: Heterogeneity of impact by labor market tightness: EXIT

	(1)	(2)	(3)	(4)	(5)
	1	2	3	4	5
	b/se	b/se	b/se	b/se	b/se
Median w_P	0.0611	0.138**	0.0563	0.209	-0.0608
	(0.0520)	(0.0448)	(0.0586)	(0.123)	(0.113)
θ		2.962	4.286	0.0106	10.16**
		(1.724)	(3.010)	(5.629)	(4.135)
θ X Median w_P		-0.265*	-0.418	-0.0205	-0.947**
		(0.149)	(0.273)	(0.516)	(0.370)
Current log salary	0.0755***	0.0749***	0.0744***	0.108***	0.0164
	(0.00761)	(0.00731)	(0.00738)	(0.0127)	(0.00934)
Previous log salary	-0.106***	-0.106***	-0.106***	-0.125***	-0.0386**
	(0.00873)	(0.00860)	(0.00871)	(0.0114)	(0.0108)
N	31817	31817	31817	14349	13161
Demographic controls	No	No	Yes	Yes	Yes
Field of study controls	No	No	Yes	Yes	Yes
Sample	Full	Full	Full	Ages 30-45	Ages 50-65

Data from Survey of Doctorate Recipients (SDR) and National Survey of College Graduates (NSCG), 1993-2010. Both samples are restricted to males aged 30 to 65. We restrict the NSCG sample to individuals with a reported PhD by age 30. Robust standard errors are clustered by PhD field of study.

$*$: significant at 10% level. $**$: significant at 5% level. $***$: significant at 1% level.

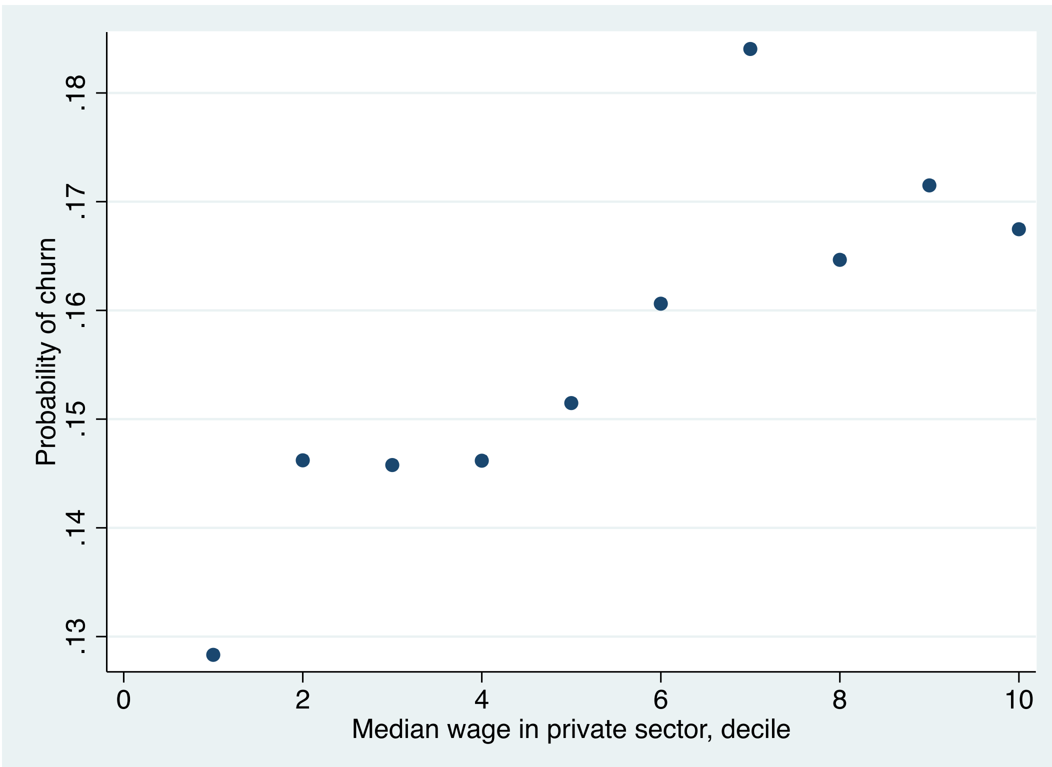
Table 4: Heterogeneity of impact by labor market tightness: CHURN

	(1)	(2)	(3)	(4)	(5)
	1	2	3	4	5
	b/se	b/se	b/se	b/se	b/se
Median w_P	0.124*	0.128**	0.387***	0.453**	0.327**
	(0.0666)	(0.0506)	(0.0770)	(0.114)	(0.109)
θ		-0.0672	-4.163	-14.46**	3.616
		(2.311)	(2.959)	(4.837)	(3.584)
θ X Median w_P		0.00503	0.421	1.357**	-0.307
		(0.203)	(0.269)	(0.447)	(0.319)
Current log salary	-0.00135	-0.00137	-0.00212	0.00943	-0.0135***
	(0.00718)	(0.00728)	(0.00738)	(0.00937)	(0.00308)
Previous log salary	-0.104***	-0.104***	-0.103***	-0.143***	-0.0368**
	(0.0192)	(0.0191)	(0.0190)	(0.0188)	(0.0121)
N	45528	45528	45528	19065	19500
Demographic controls	No	No	Yes	Yes	Yes
Field of study controls	No	No	Yes	Yes	Yes
Sample	Full	Full	Full	Ages 30-45	Ages 50-65

Data from Survey of Doctorate Recipients (SDR) and National Survey of College Graduates (NSCG), 1993-2010. Both samples are restricted to males aged 30 to 65. We restrict the NSCG sample to individuals with a reported PhD by age 30. Robust standard errors are clustered by PhD field of study.

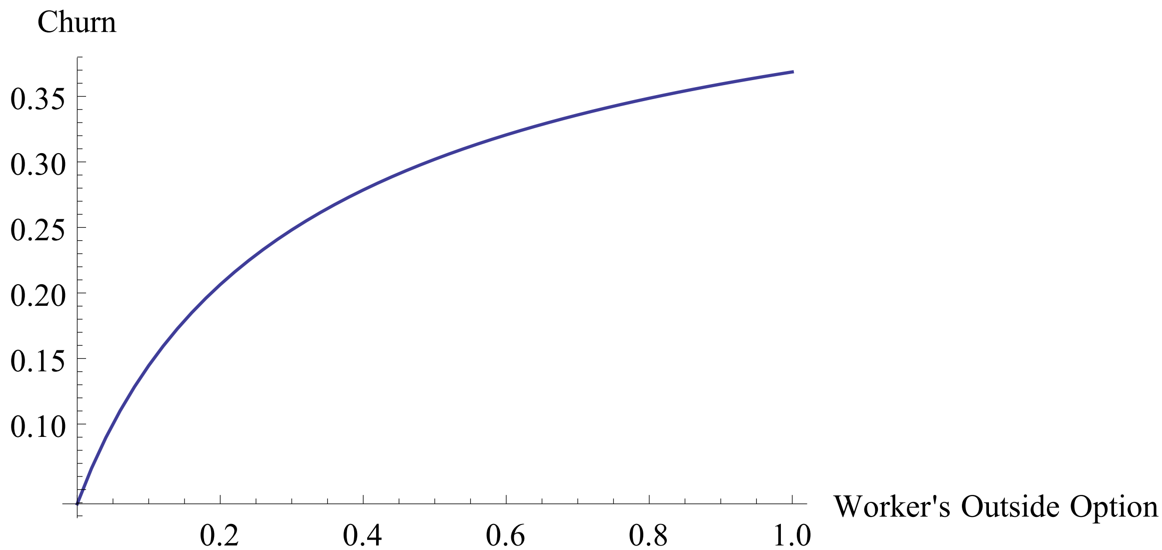
$*$: significant at 10% level. $**$: significant at 5% level. $***$: significant at 1% level.

Figure 1: Outside option and churn



Data from Survey of Doctorate Recipients (SDR) and National Survey of College Graduates (NSCG), 1993-2010. Both samples are restricted to males aged 30 to 65. We restrict the NSCG sample to individuals with a reported PhD by age 30.

Figure 2: Modeled relationship between outside option and churn



A Proofs