Type of Peers Matters: A Study of Peer Effects of Friends, Studymates and Seatmates on Academic Performance*

Tszkin Julian Chan[†] Chungsang Tom Lam[‡]

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Abstract

This paper studies the peer effects of friends, studymates, and seatmates on academic performance. We obtain the information of social networks, personality traits, and cognitive ability measures from a unique data set based on a survey we conducted in three schools in Hong Kong. We estimate a social interaction model which accounts for endogenous network formation and correlation between multiple networks. Our results show that the cognitive ability of studymates and the conscientiousness of friends positively affect a students mathematics exam score, while the conscientiousness of studymates and the cognitive ability of friends do not produce such an effect. We find that students with elder siblings are less affected by the cognitive ability of studymates, but the effect of conscientious friends is not related to that. By contrast, we find no such effect from seatmates. These results are consistent with the idea that studymates influence each other through discussing and teaching where cognitive ability is valued. On the other hand, friends influence each other by creating an atmosphere of studying, so being conscientious is more important. Our study hint at the way different types of peer work and which particular qualities are important for each peer type.

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[†]Boston University, Department of Economics, `cjulian@bu.edu`
[‡]Clemson University, John E. Walker Department of Economics `tomlam@g.clemson.edu`

1 Introduction

Social interaction is studied in economics in a wide range of contexts. From education, health, and crime to finance, marketing, and international trade, social interaction plays an important role in determining key economic decisions and outcomes.[1] Usually each topic is associated with a particular type of network; for example, academic outcomes are affected by classmates, new product adoption is affected by friends, and apartment prices are affected by the neighbourhood quality.

However, most economic outcomes depend on more than one type of peer. A typical student can be affected by both the friends they meet in school and peers they discuss homework with in study groups. It is also possible that students are motivated simply by sitting next to each other in class. Economists have investigated peer effects for a variety of peer types including schoolmates (Evans et al. (1992); Hanushek et al. (2003)), roommates (Sacerdote (2001); Zimmerman (2003); Hoel et al. (2005); McEwan and Soderberg (2006)), classmates (Ammermueller and Pischke (2009); Sojourner (2009)), college coursemates (Parker et al. (2008)), and friends (Cooley (2009); Bramoullé et al. (2009)). While there is an extensive literature on peer effects in schools, research investigating and comparing multiple peer types and alternate channels of peer effects is scant. The availability of friendship and classmate data has facilitated peer effect studies in the existing economic literature. However, there are other peer types that are important to student learning but not included in the analysis. For example, studymates are not necessarily friends, yet studymates could have a more direct effect on achievement. Also, there is no reason to believe that social-based peers such as friends or studymates will have the same effect as proximity-based ones such as seatmates. Facilitated by the availability of new data, our analysis employs three major networks of secondary school students, friends, studymates, aned seatmates.

In this paper, we estimate peer effects from these three types of networks. It is natural for the students to have preferences over being friends or studymates with certain type of person. For example, McPherson et al. (2001) suggest that people in social networks are homophilic. If students choose their peers based on some unobservable characteristics which also affect their academic performance, then the estimation of peer effects would be biased. We correct the peers selection problem by explicitly modeling network formation in the social interaction model.

Evans et al. (1992) hint at the importance of individual choices of peer groups in peer effect estimation. This problem could be solved by random assignment experiment if exogenous random assignment of the peer type of interest is available. This approach is usually more viable for proximity-based peers such as roommates or seatmates. However, for social-based peers such friendship, random assignment is nearly impossible. Even when a randomly assigned proximity-based network is available, one may be tempted to use that as an instrument for an endogenously formed network such as friendship. However, this requires us to assume that seatmates do not produce direct peer effect on each other. This is not a suitable assumption here because we aim to

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estimate and compare peer effects from different types of networks.

The existence of multiple networks introduces another concern in modeling the network formation. Social networks are often overlapping or correlated. For example, if two students are friends, it is more likely for them to study together. If two social networks are not mutually exclusive, peer effects from one type of peer are diluted with the peer effects of another type of peer. The estimates of peer effects of friends combine with peer effects of studymates, since friends can also be studymates. It is a classical problem of confounding factors. To properly estimate the peer effects, it is important to account for the formation and the peer effects of multiple networks in a single model.

We develop an econometrics model to estimate the peer effects with multiple endogenous social networks by explicitly modeling the peer selection using a Bayesian approach. Social networks can be correlated and the correlation is captured by a random effect in the linkage of students between different social networks. Different from the literature on network formation (Christakis et al., 2010; Mele, 2013) and social interaction models with endogenous network formation (Hsieh and Lee, 2013b; Goldsmith-Pinkham and Imbens, 2013), we do not include the number of common peers as an explanatory variable in the formation of networks. Including the number of common peers could induce a bias in the estimation as the number of common peers might be endogenous.[2] The idea is the same as not including endogenous variables as regressors in the first stage of two-stage least squares estimation.

Detailed information on students including their social networks is ssential to this study. To this end, we designed and conducted a network-centric survey "Secondary Education Survey in Hong Kong(SESHK)" in 2011. This survey included cognitive ability tests, personality tests, and assessment scores provided by schools. It contains information on various types of networks including friends, studymates, and seatmates. This newly constructed dataset allows us to estimate the peer effects on academic performance through various channels.

The estimation strategy of this paper follows arguments similar to Brock and Durlauf (2001) and Blume et al. (2010). The expectation of the error in the outcome equation, conditional on endogenously formed social networks create a solution for the non-identification problem studied by Manski (1993). In addition, the intransitivity of social networks provides additional sources of identification to the peer effect (Lee et al., 2010; Bramoullé et al., 2009).

Our results show that smart studymates and conscientious friends positively affect a students mathematics exam score, while conscientious studymates and smart friends do not produce such an effect.[3] Apart from being another piece of evidence to show that peer effects are significant among

[2]For example, whether a student joined the mathematics club is an omitted variable in the outcome equation. If two students are both in the mathematics club, they are more likely to be friends and have more common friends. Hence, the number of common friends explains whether two students are friends because they have more common friends if they are both in the math club.

[3]Smartness are more precisely high cognitive ability, as measure by the progressive matrix test score in the data. Conscientiousness are measured by the Big Five Inventory (John et al. (1991)). It also provide measures for other

students, they also suggest that different type of peers affect students in different ways. It is well known in the literature that higher quality peers are beneficial. What we add via these findings is that different qualities are required for different types of peer.

Understanding the differences between peer types is important to evaluate the value of having certain types of peer or to assign peers because we need to know what it means to be a good peer. Further interpreting our results, they hint at the way different types of peer work. The result that smart studymates and conscientious friends positively affect a student's mathematics score is consistent with the idea that studymates affect their peers by directly discussing and teaching mathematics problems where being smart is important, while friends influence each others by creating a social environment with positive attitude towards studying, so personality traits like being conscientious is valued.

Our interpretations about how different types of peer work are further supported by other evidence. For instance, students are less affected by smart studymates if they have elder siblings, but this is not related to the effect of conscientious friends. These are the expected directions of impact if the effect of studymates works through teaching each other. Students with elder siblings who play similar role will benefit less from this. Also, the effect of conscientious friends is magnified if a student has more friends. This is consistent with the idea that conscientious friends produce peer effect through creating a social environment and the social influence is stronger in a larger peer group.

Our results provide evidence for other aspects of peer effects in schools. We show that the benefits a student can get from their peers not only depends on their peers' characteristics, but also on their own characteristics. A student benefits more from smart studymates if she is smarter. Furthermore, a student benefits more from conscientious friends if she herself is more conscientious. Moreover, certain peer characteristics are found to be complementary. The effects of a conscientious friend is enhanced if that friend is also smart. Simliarly, the effect of a smart studymate is larger if that studymate is also agreeable. Contrast to studymates and friends, seatmates do not produce significant peer effect on Mathematics score and this further indicates that different peer types' effects are very different.

Our estimation are not restricted to mathematics scores. We also investigate the peer effects on English exam score and behavioral outcome measured by the students' conduct grade. While smart studymates are not important in determining whether a student is well-behaved, conscientious friends still have a significant positive effect. For English exam score, conscientiousness play even more important role. While conscientious friend positively affect a student's English exam score like Mathematics exam score, conscientious studymates has positive effect too. Moreover, English exam scores and conduct grades are correlated between studymates even after we control for a series of characteristics of the students. Such correlation is not found for Mathematics score.

personality traits, namely openness, agreeableness, extraversion, and neuroticism. The details of these measures are shown in Section 2.3.

The paper outline is as follow: Section 2 describes of the data we collected and analyzed. We discuss our empirical strategy in Section 3 and empirical results in Sections 4 and 5. In Section 6, we perform robustness checks and obtain insights from alternative specifications. Section 7 concludes.

2 Data

Secondary Education Survey in Hong Kong (SESHK) is a survey project[4] in operation since 2011. Our aim is to construct a dataset from this survey about cognitive ability, personality traits, and social network structure for junior secondary school students in Hong Kong. The project involved a face to face administered questionnaire and the gathering of supplemental information directly from the participating schools.

We conducted the survey from March through June 2011. The academic year in Hong Kong starts in September and ends in July, placing the survery in the second semester before the final exam. Data from three secondary schools are included i n the dataset with 873 out of 898 students participating in the survey. The s ample includes grade seven students from all three schools, and grade eight and nine students from one school. Each grade has five classes with an average class size of 35 students.

The empirical analysis in this paper requires three key elements: social network information, exam scores, and student characteristics. We discuss them in detail subsections.

2.1 Social Network

In the survey, students were asked to write down three lists of up to ten peers[5] from among their schoolmates within the same grade. The first list is their friends. The second list is a list of schoolmates who they discuss with when they have schoolwork problems. Finally, a list of schoolmates who sit next to them in class in the first semester. It is possible for seat arrangements to change within semester and the students are asked to write down all schoolmates who sat next to them within the semester. These information allow us to construct three distinctive social networks of friends, studymates, and seatmates.

Since every student was asked to write down their networks, there are two pieces of information, one from each side, about the relationship between every pairs if student. Students are connected as friends only if both sides indicated they were friends. More precisely, the friend social network consists of a node for each student and edges between nodes according the following principle: edge (i, j) is in the graph if and only if student i named student j as a friendand student j named student

[4] The full text of the questionnaire and other information about the survey can be found at `http://`

[5] Some students insisted on writing more than ten answers and squeezed them into the ten boxes provided. Therefore, there are a very small amount of students with more than ten peers, but the extra peers make no substantial difference in the estimation.

i as a friend in our survey. In other words, we only focus on "reciprocal friends" in this paper.[6] We similarly construct the studymate and the seatmate network by this reciprocal peer rule. Table 1 and 2 show the descriptive statistics for the network data. Figure 1 to 3 visualize some examples of friends and studymates networks.

Table 1: Descriptive Statistics of the Degree of Nodes for Different Social Networks

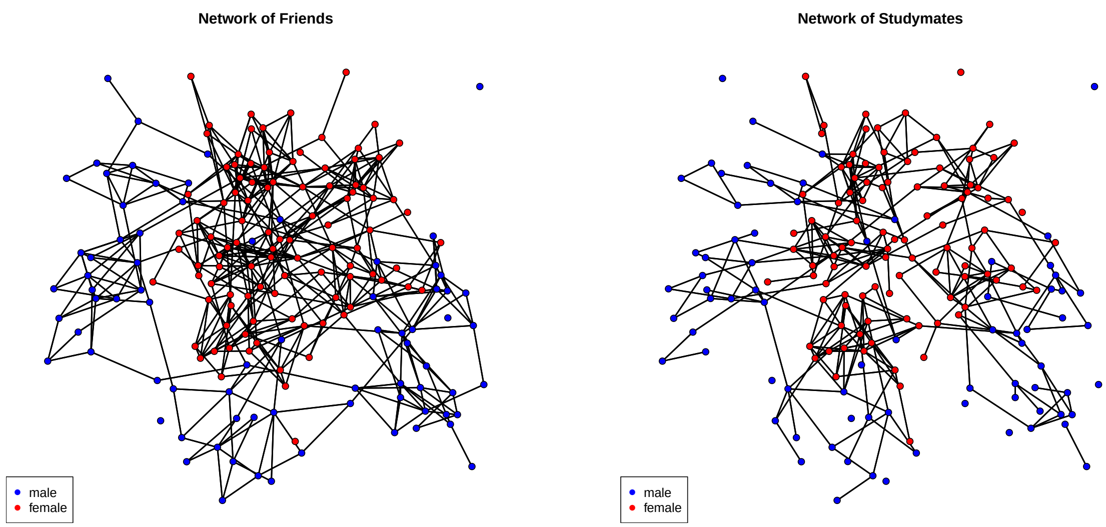
	Mean	SD	Minimum	Maximum#
Friends	4.73	2.20	0	10
Studymates	2.76	1.99	0	11
Seatmates	2.59	2.15	0	11

See footnote 5

Table 2: Correlation between Social Networks

	Friends	Studymates	Seatmates
Friends	1.00	0.50	0.14
Studymates	0.50	1.00	0.15
Seatmates	0.14	0.15	1.00

Figure 1: Friends and studymates network by gender (From a participated school, Form 1 (Grade 7) only)



[6] We explore non-reciprocal peers in Section 6.2.

Figure 2: Friends and studymates network by class (From a participated school, Form 1 (Grade 7) only)

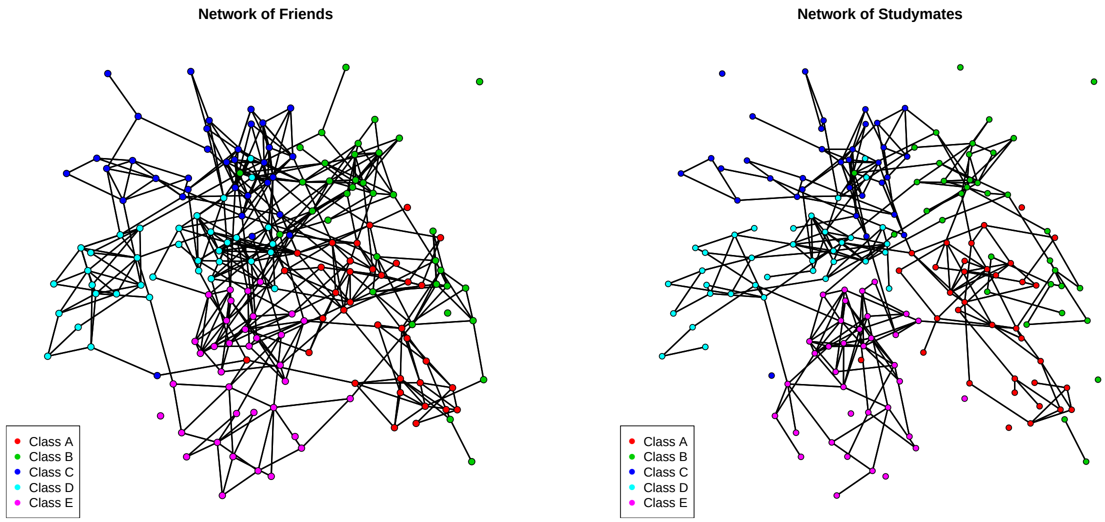
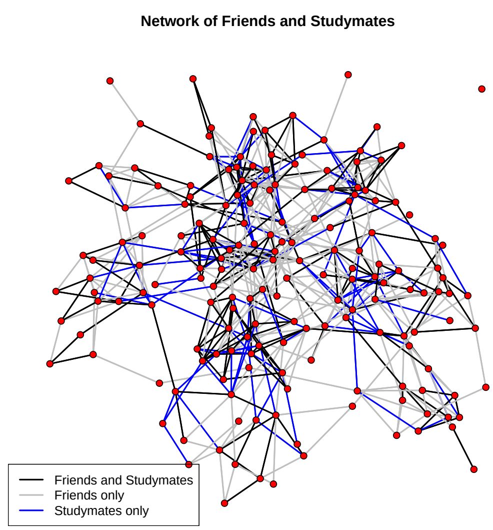


Figure 3: Friends and studymates network combined (From a participated school, Form 1 (Grade 7) only)



We only investigate peers within the same grade in the same school because the survey data do not include information about interschool or intergrade peers.[7] However, since students' social

[7]

networks are not confined to be within class, the data do capture their friends or studymates outside class but within the same grade in their schools. The availability of interclass peers data allow us to estimate how classes affect peer choices. The high participation rate gives us an almost complete within grade social network data for each schools. High participation rate together with the availability of multiple types of peers makes SESHK an ideal data source for our analysis in this paper.

2.2 Assessment Scores

Assessment scores were provided by schools. Table 3 shows the descriptive statistics of the assessment scores. In addition to the regular subjects such as mathematics and English, each student also get a conduct grade, which is a score given by class teachers to evaluate how well-behaved a student is. Our analysis focus on the mathematics exam score and we supplement it with other the English score and conduct grades. All subjects have similar mean scores even without standardization. [8]

Table 3: Descriptive Statistics of Assessment Scores

	Mean	SD	Minimum	Maximum	Scale
Mathematics	69.233	14.363	25.00	99.00	$[0, 100]$
English	67.331	11.325	20.36	89.79	$[0, 100]$
Conduct Grade	67.027	12.088	23.81	80.95	$[0, 100]$

2.3 Personalities and Skills Measures

The dataset includes a progressive matrix test and the Big Five Inventory. The progressive matrix test is a series of context-free logical deduction tests on space and shapes. They are designed to measure students' cognitive ability.

Big Five Inventory includes a 44 item questionnaire from John et al. (1991) which produces five personality measures: conscientiousness, openness, agreeableness, neuroticism, and extraversion. Conscientiousness captures elements such as self-discipline, carefulness and diligence. Openness captures elements such as imagination and curiosity. Agreeableness captures whether a student is considerate or kind. Neuroticism captures whether a student worries or becomes anxious easily. Extraversion captures whether a student is outgoing or talkative.

Table 4 shows the mean and standard deviation of each personality and cognitive ability measure.[9] Some characteristics such as cognitive ability or conscientiousness are related to studying. Others, such as agreeableness or extraversion, are related to how students making friend. A dataset

[8]Standardization could have been performed by the schools before the assessment scores were provided to the authors.

[9]The descriptive statistics of the personality measures are not far away from other studies. Reference to Srivastava et al. (2003)

with these measures allows us to see whether these characteristics produce effects which conform to conventional wisdom, and whether peer effects are significant though any of these characteristics.

Table 4: Descriptive Statistics of Cognitive Ability Tests and Personality Trait Measures

	Mean	SD	Minimum	Maximum	Scale
Cognitive Ability	8.556	1.911	1	14	$[0, 16]$
Conscientiousness	26.418	5.482	12	45	$[9, 45]$
Agreeableness	27.063	4.007	12	40	$[9, 45]$
Openness	35.891	5.473	18	51	$[10, 50]$
Neuroticism	22.982	5.589	8	38	$[8, 40]$
Extraversion	26.805	5.035	10	39	$[8, 40]$

2.4 Hobbies and Other Information

In addition to the variables mentioned above, SESHK also provides information about students' hobbies and other demographic information. Students were asked to write down their hobbies, which were classified into 120 different types. These hobbies include piano, violin, basketball, and badminton ..... etc. Similar to personality traits, hobby could be related to peer formation and exam scores and hence useful in our analysis. The full list of hobbies is shown in the Appendix IV. Table 5 contains descriptive statistics for hobbies and other demographics.

Table 5: Descriptive Statistics of Hobbies and other variables

	Mean	SD
Number of Hobbies	3.699	1.654
Height (cm)	159.668	8.907
Weight (kg)	47.780	10.571
Male	0.444	0.497

3 Empirical Strategy

In this section, we describe our empirical strategy. Using the data from the Secondary Education Survey in Hong Kong mentioned in Section 2, we construct the following five sets of data for each student i : Individual Characteristics ($X_{self,s,i}$), Peer Characteristics ($X_{peer,s,j}$), Network Formation Characteristics ($Z_{s,r,i,j}$), Academic Performance ($Y_{s,i}$), and Network ($D_{s,r}$). We define $i, j = 1, \ldots, N$ as the indices for students and N is the number of students, $r = 1, \ldots, R$ is an index for networks, and $s = 1, \ldots, S$ is an index uniquely defined for each grade in each school.[10] Each students can only be in one school and in a particular grade (s), but they can be connected in all different networks (r).

For the academic performance ($Y_{s,i}$), we focus on the mathematics exams score as our measure of academic achievement in most sections, then we supplement the results with a language subject (English exam score) and a measure of behavioral outcome (conduct grade).

Individual characteristics ($X_{self,s,i}$) include a series of student characteristics which could affect a student's exam scores or behavior, such as their personality traits and cognitive ability. Gender, family characteristics and types of hobbies involved are also included as controls. We include personality traits and cognitive ability in peer characteristics ($X_{peer,s,j}$) so that we can examine how peers are affected by them.

Network formation characteristics ($Z_{s,r,i,j}$) are student characteristics whcih could affect how networks are formed. The student characteristics mentioned above are also included in the construction of the network formation characteristics ($Z_{s,r,i,j}$) since many of them affect both acdemic performance as well as network formation. Typical examples of network formation characteristics are agreeableness and extraversion. In addition, we construct variables from multiple students' information, such as the number of hobbies in common between each student pairs or difference in personality traits.[11]

Network ($D_{s,r}$) refers to the network structure. In particular, $D_{s,r}$ is the adjacency matrix of network r. $D_{s,r,i,j} = 1$ if i and j is connected in network r, otherwise it is equal to zero. Each student is connected to R networks and all of them are allowed to cause peer effects which affect that student's academic performance.

3.1 Model

In this paper, we consider a linear-in-means social interaction model which accounts for the formation of multiple endogenously formed social networks. First, we define the score formation equation as follows.

$$Y_{s,i} = X'_{self,s,i}\beta + \sum_{r=1}^{R} \left(\sum_{j} W_{s,r,i,j} X'_{peer,s,j}\theta_r + W_{s,r,i,j} Y_{s,j}\lambda_r + \rho_r e_{s,r,i} \right) + \alpha_s + \varepsilon_{s,i} \tag{1}$$

where the weighted adjacency matrix W is a row normalized adjacency matrix D,[12]

[10] As mentioned in Section 2.1, we focus on the peers within the same grade. Therefore s is an index for each grade in each school.

[11] This is precisely why $Z_{s,r,i,j}$ is indexed by i and j instead of only i.

[12] The elements of the weighted adjacency matrix do not have to be the same across rows. The row sum of the weighted adjacency matrix is assumed to be one. It is an essential assumption to remove school level fixed effect by differencing (Lee et al., 2010) and the method is shown in Appendix III(ii)

$$W_{s,r,i,j} = \frac{D_{s,r,i,j}}{\sum_j D_{s,r,i,j}} \tag{2}$$

The academic performance ($Y_{s,i}$) of student i depends on *i)* student i's own characteristics ($X_{self,s,i}$), *ii)* the weighted average of characteristics of students connected to i in each network r ($\sum_j W_{s,r,i,j} X_{peer,s,j}$), *iii)* the weighted average of the outcome variable of students connected to i in each network r ($\sum_j W_{s,r,i,j} Y_{s,j}$), *iv)* an unobservable error that also affects the formation of network r ($e_{s,r,i}$), *v)* school fixed effects (α),[13] and *vi)* a score formation specific unobservable error (ε_i).

Second, we model the network formation as follows. Students i and j are connected in network r if and only if both of them get positive utility from the connection. In other words, $D_{s,r,i,j} = 0$ if either i or j or both of them get non-positive utility. In other words,

$$D_{s,r,i,j} = 1(U_{s,r,i,j} > 0) \times 1(U_{s,r,j,i} > 0) \tag{3}$$

while the utility to student i or having a connection with student j in network r is:

$$U_{s,r,i,j} \equiv Z_{s,i,j}^{'} \gamma_r + |e_{s,r,i} - e_{s,r,j}| \psi_r + u_{s,r,i,j} \tag{4}$$

The network formation specific unobservable errors ($u_{s,r,i,j}$) are normally distributed with covaiance matrix Ψ.[14]

$$u_{s,i,j} = (u_{s,1,i,j}, \ldots, u_{s,R,i,j})^{'} \text{ and } u_{s,i,j} \sim N(0, \Psi) \tag{5}$$

Similar utility function are also used by Christakis et al. (2010) and Mele (2013). For simplicity, we will omit the subscript s in the rest of the paper.

The disturbance terms in the score formation equation include ε_i and $e_{r,i}$. The ε_i appear in the score formation equation while the $e_{r,i}$ capture unobservable error that also appears in the utility in network formation. $e_{r,i}$ enters into the utility to i for having a connection with j in network r in the form of $|e_{r,i}, e_{r,j}|\psi_r$. The quantity $|e_{r,i}, e_{r,j}|$ can be interpreted as a similarity measure between i and j. If ψ_r is negative, the preference exhibits homophily (heterophily if $\psi_r > 0$) (Kolaczyk, 2009). It is important to have $|e_{r,i}, e_{r,j}|$ instead just $e_{r,i}$ in the utility because including $|e_{r,i}, e_{r,j}|$ allows students to select peers who are similar (or different) to them. In contrast, e_i only affects the amount of connection student i has with others. The endogeneity problem arises when $\rho_r \neq 0$ and $\psi_r \neq 0$, that is the error in the outcome equation is correlated with the error in network formation equations. This problem can be caused by an omitted variable affecting both the network formation and the acdemic performance.

[13]Referred as correlated effect in Manski (1993)

[14]Ψ is an $R \times R$ covariance matrix with all diagonal elements equal to one.

The error terms in the utility of network formation $u_{r,i,j}$ are correlated over different networks. Thus, $u_{i,j}$ is a random effect component across networks. The assumption that allows for a random effect is motivated by the correlation between the formation of different networks.

With all these components we estimate the own effects (β), contextual effects (θ_r), spillover effect (λ_r), selection effect (ρ_r), network formation parameters (γ_r), and Network Formation Random Effects ($\Psi_{ij}|_{i\neq j}$)[15] in equation (1) to (3).

The contextual effects refer to how a students' scores are affected by the average characteristics of their peers. A positive estimate indicates that average peer characteristics are positively correlated with a student's exam scores. The spillover effect[16] refer to how a students' academic performance are affected by the average academic performance of their peers. A positive estimate indicates that peers are more likely to have similar exam scores, because their exam scores positively affect each other. While contextual effects capture peer effects generated from predetermined observable characteristics of peers, the spillover effect captures peer effects, which do not go through any observable characteristics in the model. The network formation parameters describe how different characteristics affect network formation. The selection effect captures how students are self selected into peer groups and the network formation random effects captures how the formation of networks are related to each others. Finally, the own effects describe how students characteristics affect their own outcome variable.[17]

By including multiple networks we acknowledge that the formation of different networks are correlated. For example, students may want to study with their friends and those who study together can easily become friends easily. The network formation random effects ($\Psi_{ij}|_{i\neq j}$) takes into account this effect in multiple-network models. The covariance matrix Ψ has diagonal elements equal to one and the estimates of the off diagonal elements Ψ_{ij} captures whether the formation of network i and j are correlated or not.

3.2 Identification

The identification of social interaction model can be achieved through an exogenous social network Bramoullé et al. (2009), Lee et al. (2010). In our model, the network matrices are endogenously formed and so the identification strategy instead follows Blume et al. (2010) and Blume et al. (2011). The expectation of Y conditional on X and $D_1, \cdots, D_R$ is

[15]Network Formation Random Effects ($\Psi_{ij}|_{i\neq j}$) is the parameter for the distribution function of $u_{i,j}$

[16]It is also known as the endogenous effect in Manski (1993)

[17]Since the non-dummy characteristics and subject scores are standardized, we can interpret the contextual effect and own effect paramters in the unit of standard deviation.

$$
\begin{aligned}
E\left(Y|X,D_1,\ldots,D_R\right) &= E\left[\left(\sum_i^{\infty}\left(\sum_{r=1}^R W_r\lambda_r\right)^i\right)\left(X\beta+\sum_{r=1}^R W_rX\theta_r+e\right)\middle|X,D_1,\ldots,D_R\right] \\
&\quad +E\left[\sum_i^{\infty}\left(\sum_{r=1}^R W_r\lambda_r\right)^i e\middle|X,D_1,\ldots,D_R\right] \qquad (6)
\end{aligned}
$$

The non-zero correlation between e_i and $u_{r,i,j}$ implies $\mathbb{E}(e|X,D_1,\ldots,D_R)\neq 0$, precisely an endogeneity problem. On the other hand, $\mathbb{E}(e|X,D_1,\ldots,D_R)$ provides a new opportunity to identify the model. With the distributional assumption of e_i and $u_{i,j}$, $E(e|X,D_1,\ldots,D_R)$ is not perfectly collinear with the first part of the equation 6. Therefore, the last term of the equation can facilitate identification even without an instrumental variable. This strategy requires the model to be correctly specified. In addition, it is possible that $E(e|X,D_1,\ldots,D_R)$ is highly correlated with other control variables, which implies the estimation could be imprecise (Brock and Durlauf, 2003). Existence of instruments could possibly reduce the correlation between $E(e|X,D_1,\ldots,D_R)$ and other control variables and could increase the precision of the estimation. In particular, the number of common hobbies is considered as an instrument for social-based networks.

A valid instrument in our model has to be correlated with network formation but does not affect the exam scores directly and we argue that the number of common hobby is a valid instrument for social-based networks. One may argue that students who are have a certain type of hobby, say playing musical instruments, are generally better students. Our model allows for a such possibility; we catergorize the hobbies into three groups, "playing musical instruments", "playing sports", and "participating youth movements". The types of hobby students have is one of the individual characteristics included in X_{self} and the effect is captured by the coefficients of these regressors.[18] Notice that the variation of the number of common hobbies will not vanish even if we controlled for the types of hobbies due to the detailed hobby information in the data. The exogeneity of the instrument relies on the assumption that students who play piano do not generally perform better in exams than those who play violin, rather than that playing musical instrument itself is exogeneous to exam score.

However, the number of hobbies in common can only be an instrument for social-based networks and they do not work for the seatmates network. It is because the seating plan is usually decided by the teachers instead of the students. Having more common hobbies may not increase the chance for students to sit close to each others in class depends on the teachers' seating arrangement policy.[19] Some teachers may just randomly assign seats but it is possible that the teachers would take into account student preference or assign the seats based on their impression of students' academic performance. Teacher can put good students together or pair up good and bad students. We

[18] In other words, the validity of the instruments could be sensitive to how we categorize the hobbies. Therefore, we explore various ways of categorization in Section 6.1. Our results are robust to these alternative specifications.

[19] In the survey data we see a significant amount of student claiming that seats are assigned by the teacher . Also need to explain this more because in the United States the seating arrangment is not the same as in asian countries.

do not know their rules, neither can we claim that the assignment is random. To be on the safe side, our estimation allows the seatmates network to be endogenously formed and we include height difference as an instrument. Teachers may have different rules in assigning seats, but a very common practice is to assign seats according to height because tall students will block the view of shorter students if they are sitting at the front. As a result, students with similar height are more likely to sit next to each others and this is confirmed in later sections by the estimates in Table 6.

3.3 Computation

We estimate the model using a Bayesian approach. The likelihood function of the model is difficult to be computed as the latent error e_i appears in the outcome equation Y_i and the equations of the network formation which involves i ($D_{r,i,j}$ $\forall j \neq i$ and r). Hsieh and Lee (2013a) and Goldsmith-Pinkham and Imbens (2013) use similar estimation strategy to tackle the peer selection problem. The major difference between our model and their work is that we allow the students to have more than one endogenous network. In addition, our model uses a probit link instead of a logit link for network formation to take into account possible random effect between networks. The details of the likelihood function and the estimation procedure are documented in Appendix III.

4 Formation of Networks, Instruments, and Own Effects

We report our results in two sections. We begin with discussions on the network formation and own effect estimations. In the next section, we report the estimated peer effects and discuss the implications.

The focus of empiricial results is on the peer effects instead of the network formation parameters and own effects. However, it is very helpful to look at these estimates before discussing peer effects. Since the empirical work of this paper is based on a new survey dataset, sensible own effects and network formation parameters makes our peer effects estimates more convincing and more comparable to other studies based on well-known datasets. Moreover, the own effect estimates can give a clearer idea on which covariates are significant factors in determining academic outcomes. The magnitude of them also serves as a good benchmark to for comparison when we are evaluating whether the peer effects estimated are economically significant.

4.1 Relevance of Instruments and Selection Effects

The network formation parameters associated with the number of common hobbies and height differences are shown in Table 6. Among all the network formation parameters estimated, those associated with number of common hobbies and height differences are highlighted because they are considered instruments as mentioned in Section 3.2. The estimates show that the number of hobbies in common between a student pair is strongly positively correlated with the probability that the student pair are friends or studymates. Involving in the same category of hobby also improve the probability of connection. This is reasonable since a student who plays piano is more likely to be a friend of another student who plays violin, but the effect is not as strong as if both students are violin players. Height differences are negatively related to the seatmates network as expected. This confirms our instruments' relevancy and also show that hobby is one of the major factors in the formation of social-based networks.

Table 6: Peer Selection Effects and Selected Estimates for Network Formation Parmeters

Mathematics Exam Score Network		Friends	Studymates	Seatmates
Peer Selection Effects		0.101***	0.174***	0.483***
		(0.038)	(0.051)	(0.154)
Number of Hobbies in Common				
Music(IV) [2]		0.060***	0.055***	-0.014
		(0.012)	(0.015)	(0.014)
Sports(IV) [2]		0.036***	0.028**	0.017
		(0.011)	(0.014)	(0.012)
Youth Movements(IV) [2]		0.035***	0.022**	0.014*
		(0.007)	(0.009)	(0.008)
Height	Level	0.039**	-0.005	0.022
		(0.017)	(0.021)	(0.018)
	Difference(IV) [2]	-0.070***	-0.070***	-0.104***
		(0.014)	(0.017)	(0.016)
Conscientiousness	Level	-0.003	0.121***	0.045
		(0.019)	(0.023)	(0.019)
	Difference	-0.029**	0.002	-0.041
		(0.013)	(0.016)	(0.015)
Agreeableness	Level	0.044**	0.072***	0.010
		(0.018)	(0.023)	(0.019)
	Difference	0.021	0.005	0.007
		(0.013)	(0.016)	(0.014)
Extraversion	Level	0.053***	0.060***	0.009
		(0.018)	(0.022)	(0.019)
	Difference	-0.047***	-0.024	0.005
		(0.013)	(0.016)	(0.015)
Same Class		1.004***	1.172***	1.996***
		(0.022)	(0.030)	(0.139)
Different Gender		-1.063***	-0.712***	-0.035
		(0.033)	(0.033)	(0.025)

[1] *,** and *** denote statistical significance at the 10%, 5% and 2% level, respectively.
[2] These variables are considered as instruments.
[3] Other controls include family characteristics, cognitive ability, neuroticism, openness, and difference in type of hobby played,

In addition to the instruments, our network formation equation also includes other covariates. Class and gender are the most important factors in network formation. Being in the same class or having the same gender increases the probability that a student pair is friends or studymates. The effect from class is almost the same magnitude as that from gender in friendship formation, but for studymates, being in the same class is relatively more important. This is because even though students in the same grade have the same general syllabus, they have the same homework and quiz schedule only if they are in the same class. Having studymates in the same class makes it easier for them to discuss schoolwork.

Personality traits also play an important role in the formation of the social-based networks. Agreeable students have a higher probability of obtaining more friends and studymates. Extraversion has very similar effects to agreeableness. Differences in the effects of agreeableness and extraversion are made apparent when one looks at the differences in the traits between a pair of students. Students with similar level of extraversion are more likely to be friends or studymates but they care less about the difference in agreeableness. Conscientious students have more studymates but not friends. However, students want to make friends with someone with similar conscientiousness and we do not have evidence that they care about this difference in conscientiousness when they are looking for studymates. Utilizing pair-wise comparison of students' personalities allows us to explore more into how personalities work in network formation in more detail instead of only examining which factors increase or decrease the number of friends.

As shown in Table 6, selection effects are positive in all three networks. The effect for seatmates is positive, which could represent the teachers' tendency to put good students together during seat assignment. For social-based peers, the selection effects are positive and the effect for studymates is generally larger than that of friends.[20] Since selection effect is positive when students choose their peers according to their academic achievements, it is not surprising that students are choosing studymates according to the academic achievements more than when they are choosing friends.[21]

Given that network formation has been taken into account in the estimation, the peer effects estimates in Section 5 and Section 6 will not be polluted by the endogeneity problem caused by the self-selected peer groups.

4.2 Own Effects

The own effects are estimated under all different network specifications. Selected estimates are shown in Table 7. We focus on reporting how student characteristics affect their Mathematics exam score and will thereafter compare the results to those of other outcome variables.

Males are better at Mathematics. This is significant even with controls for their differences in cognitive ability and personality traits. Both cognitive ability and conscientiousness are important

[20] The difference in the selection effect for friends and studymates is even more significant if we take into account both networks simultaneously.

[21] The network formation paramters for multiple network models are very similar to those in single network models.

characteristics for student learning effectiveness. They both have very strong positive correlation with mathematics exam score and cognitive ability is relatively more important than conscientiousness. Extraversion and agreeableness are negatively associated with mathematics exam score. Almlund et al. (2011) also report similar effects from these student characteristics using data collected from New York City middle schools and data source from Poropat (2009)[22], except for the effect of openness. We do not find the significant positive effect from openness which appears in Almlund et al. (2011). However, since openness is positively correlated with artistic subjects like visual arts or music as expected, instead of mismeasurement of our survey, we tend to explain this by the emphasis on memorization and routine procedures instead of other personalities like creativity for the junior secondary school mathematics syllabus in Hong Kong.

Table 7: Selected Own Effects Estimates: Estimated Under Different Networks

Mathematics Exam Score					
Network	OLS	Multiple Networks	Single Social-based Network		Proximity-based Network
	(No Network)	(Friends and Studymates)	Friends	Studymates	Seatmates
Own Effects					
Cognitive Ability	0.279***	0.265***	0.271***	0.274***	0.293***
	(0.032)	(0.032)	(0.032)	(0.031)	(0.033)
Conscientiousness	0.173***	0.163***	0.207***	0.161***	0.220***
	(0.034)	(0.034)	(0.034)	(0.034)	(0.035)
Openness	-0.053	-0.065**	-0.068**	-0.063*	-0.068*
	(0.033)	(0.033)	(0.034)	(0.033)	(0.035)
Agreeableness	-0.114***	-0.108***	-0.099***	-0.105***	-0.092***
	(0.033)	(0.033)	(0.034)	(0.032)	(0.034)
Neuroticism	-0.043	-0.049	-0.021	-0.052	-0.024
	(0.035)	(0.034)	(0.035)	(0.034)	(0.036)
Extraversion	-0.090***	-0.080**	-0.074**	-0.081**	-0.072**
	(0.033)	(0.033)	(0.034)	(0.032)	(0.034)
Male	0.240***	0.153**	0.140*	0.149**	0.130*
	(0.070)	(0.077)	(0.078)	(0.071)	(0.071)

[1] *,** and *** denote statistical significance at the 10%, 5% and 2% level, respectively.
[2] Other controls include height, family characteristics, hobby type, and class, school, grade fixed effects.

Notice that estimates under different network specifications are not significantly different from their corresponding OLS estimators as shown in the first column of Table 7. In other words, while we would like to show that the peer effects are important in determining academic achievements, we have no evidence to show that the traditional research on academic achievements is biased because of the absence of peer effect considerations. Since network specifications are found to have not significant effect on the own effect estimates, we picked only the estimations with the friends network for illustration and compare them across different outcome variables. Estimates are shown in Table 8.

Conscientiousness is positively correlated to both academic subjects and behavioral outcomes while cognitive ability is only positively correlated with academic subjects. The effect of cognitive ability is relatively larger for Mathematics scores than for English scores while Conscientiousness

[22] For agreeableness, Almlund et al. (2011) find very small positive correlation to course grades using data from Poropat (2009), insignificant effect for New York City public school achievements test and significant negative association to the comprehensive testing program in their private school sample.

Table 8: Selected Own Effects Estimates: Compare Different Subjects

Network	Friends		
	Mathematics	English	Conduct Grade
Own Effects			
Cognitive Ability	0.271***	0.132***	-0.019
	(0.032)	(0.032)	(0.036)
Conscientiousness	0.207***	0.289***	0.208***
	(0.034)	(0.033)	(0.037)
Openness	-0.068**	-0.081**	-0.054
	(0.034)	(0.033)	(0.037)
Agreeableness	-0.099***	-0.089***	0.045
	(0.034)	(0.033)	(0.037)
Neuroticism	-0.021	0.067*	0.048
	(0.035)	(0.034)	(0.038)
Extraversion	-0.074**	-0.057*	-0.108***
	(0.034)	(0.033)	(0.037)
Male	0.140*	-0.222***	-0.226***
	(0.078)	(0.078)	(0.088)

[1] *,** and *** denote statistical significance at the 10%, 5% and 2% level, respectively.
[2] Other controls include height, family characteristics, hobby type, and class, school, grade fixed effects.

is relatively more important for English scores. This seems to be consistent with conventional understanding that Mathematics relates more to whether a student can do logical deduction well, and for language subjects, it is relatively more important for them to have discipline in study. Gender comparison also yield expected results. Male are better in Mathematics but worse in language subjects like English and are less well-behaved.

5 Empirical Results on Peer Effects

Peer effect estimation is the focus of our empirical analysis. There are many kinds of peer effects but we mainly focus on the spillover effects and contextual effects as defined in Section 3 In this section, we give insights gleaned from our estimates mainly by comparing the spillover effects and contextual effects across different network specifications.

5.1 Peer Types Matters

Different networks produce peer effects of different size and significance. This can be shown even if we only refer to the estimates of the single network models as shown in the last three columns in Table 9. The contextual effects estimates are quite different for different type of peers. While no significant contextual effects are found for seatmates, statistically significant contextual effects of certain characteristics are found for friends and studymates. Comparing the effects from friends and studymates, we observe that contextual effects for cognitive ability is positive for studymates but a similar effect is not found for friends.

Table 9: Comparison of Selected Estimates for Different Network Specifications

Mathematics Exam Score					
Network	Multiple Networks[4] (Friends and Studymates)		Single Social-based Network		Proximity-based Network
			Friends	Studymates	Seatmates
Contextual Peer Effects					
	Friends	Studymates			
Cognitive Ability	0.041	0.109**	0.084	0.126**	-0.014
	(0.070)	(0.055)	(0.066)	(0.051)	(0.051)
Conscientiousness	0.161**	0.078	0.223***	0.148***	-0.005
	(0.073)	(0.061)	(0.067)	(0.054)	(0.056)
Degree	0.017	0.109***	0.060***	0.119***	0.058**
	(0.016)	(0.018)	(0.015)	(0.016)	(0.024)
Spillover Effects	0.022	0.075	0.094*	0.077*	-0.049
	(0.054)	(0.047)	(0.049)	(0.042)	(0.049)
Peer Selection Effects	0.079**	0.156**	0.101***	0.174***	0.483***
	(0.037)	(0.061)	(0.038)	(0.051)	(0.154)
Network Formation Random Effect	0.938***				
	(0.009)				

[1] *,** and *** denote statistical significance at the 10%, 5% and 2% level, respectively.
[2] Other personality traits are included in the estimation but omitted in the table. They do not produce contextual effects which are statistically significant.
[3] Other controls include height, family characteristics, hobby type, and class, school, grade fixed effects.
[4] See footnote 23

However, the postive estimate of network fomration random effects shown in Table 9 shows that the formation of friends and studymates networks are positively correlated. This is reasonable since it is not uncommon for students to pick their studymates among friends, and discuss schoolwork to enhance friendship. The estimates of peer effects from one type of peer can be diluted with the peer effects from other peers if we do not control for the peer effect from different type of peer. When the effects from different networks are tangled, we could potentially misinterpret differences between them compare to if we only stick to a single network model. Therefore, to illustrate the difference more explicitly and alleviate the bias caused by the related network formations, we estimates the multiple networks model for friends and studymates and the results are shown in the first two columns in Table 9.[23]

Taking into account multiple networks, we observe that the contextual effects of cognitive ability have the same sign as those estimated in the single network models but the magnitude is smaller.

[23] We also estimate the multiple network models with the seatmates network. The estimates for the seatmate network is very close to the single network estimates. Despite the small but positive correlation between the formation of seatmates and social-based peers, the estimates for friends and studymates are not altered by including the seatmates network. These results are omitted in Table 9 and are reported in Appendix II Table 17.

Smart studymates positively affect a student's mathematics exam score. Although there is not necessarily a causal relationship between own effect and contextual effects, one could imagine that given such a strong own effect for cognitive ability on mathematics exam score, cognitive ability is a very important attribute in studying mathematics. Therefore, it is fair to say that smart peers being conducive to learning mathematics is not a surprising result. However, notice that this effect only appears for studymates but not friends. In other words, having smart studymates is helpful but we have no evidence to say that having smart friends will do the same.

Conscientiousness is another personality trait that produces positive contextual effects. The estimates from the single network models show that both friends and studymates have contextual effects through conscientiousness. However, only the contextual effect from friends survive after we include both friend and studymate networks. Having conscientious friends improves mathematics exam scores, but we have no evidence of the same for conscientious studymates. We observe no significant contextual effects from other personality trait.

We highlight the significance of these results in three ways. Firstly, on top of much evidence in existing literature, this provide further evidence that high quality peers can improve academic achievement. The effects are not only statistically significant, but they are strong. As the variables are normalized, the estimates can be interpreted in the unit of standard deviation. In some channels like contextual effect through conscientious friend, the size of the effect is comparable to two-thirds of the corresponding own effect. It means that one standard deviation in peers' conscientiousness has almost the same effect as two-thirds standard deviation increase in the students' own conscientiousness.

Secondly, different networks produce peer effects through different characteristics. It is misleading to refer all kinds of peers as being the same. It is not always valid to proxy one peer type with another in a network, even though sometimes they seems closely related to each other. For example, one may be interested in whether a studymate's average cognitive ability has influence on a student's mathematics exam score. Quoting from a study that use a friend-based social network and conclude that the "peer effect" is not significant can be misleading. Studymate and friends are not interchangeable in this instance. Similarly, it is misleading to say there is no significant peer effects of any type because of the evidence from seatmates.

Thirdly, although strictly speaking these estimates only show the relationship between the characteristics of the peers and academic achievement, they also hint how different types of peer produce peer effects. Studymates tend to affect their peers by teaching them directly. For teaching mathematics, being smart is important. On the other hand, friends are often considered to be influencing each others by creating a social environment with positive learning attitude. In this case, whether someone have an habit to adhere to their study plans or have discipline in doing a thorough job matters. This is consistent with our findings about contextual effects from conscientious friends. Further evidence supporting these claims is shown in Section 5.3.

5.2 Discussion on the Number of Peers

In Section 3, we constructed the weighted adjacency matrix W to compute the average charateristics of peers. Our construction of W implicitly assumes that students have a fixed amount or social interactions and only the quality of social interaction matters. In some studies, the model uses the adjacency matrix D instead (Zimmerman (2003)) because it captures the potentially significant effect of having more peers and more social interactions. In our setup, all peer effects estimates are based on the weighted adjacency matrix W. However, the effect of having more peers are captured by including the "degree" variable in X_{self}. This variable is constructed by counting the number of connections a student has for a particular network. Notice that peer connection in this paper are reciprocal so having more peers not only shows that the students are more willing to get peer acceptance, they also have more mutually agreed matches with their schoolmates.

Table 9 shows the estimates of the coefficient of the degree covariate. Mathematics exam score are positively correlated with the number of studymates, but number of friends has no significant effect after controlling for studymates. Number of seatmates has positive effect on mathematics exam score too, but the effect is relatively smaller than that of studymates. We can interpret this in the following ways. Literally, it means that number of studymates matters in studying mathematics. Degree is also a measure of centrality known as "degree centrality", which means that those students who are closer to the center of the network perform better. This interpretation is particularly insightful for the seatmates network. Students who are sitting in the middle of the classroom more often will have higher measure in degree centrality. The positive estimates implies that these students are performing better in the mathematics exam.

5.3 Interactions Between Personality Traits

We have discussed how student characteristics affect students' own exam score and how they affect their peers in previous sections. While some characteristics, such as conscientiousness and cognitive ability, are particularly important in the determination of academic outcomes and producing peer effects, quite a number of other characteristics do not show significant contextual effects or even own effect in our estimation. However, some of these characteristics can magnify or dampen the effect from contextual effects. In this section, we aim to obtain insights about the importance of these characteristics by investigating the coefficients of the interaction terms. Selected estimates of these coefficients are shown in Table 10.

We found that certain contextual effects can be magified by the same characteristics of the students themselves. While students can benefit from having studymates with higher cognitive ability, the positive effect is even larger if they have high cognitive ability themselves. Similar effect can be found in the contextual effect of the conscientiousness of friends.

The effect of cognitive ability and conscientiousness are highly related to each other. Although we find no significant effects from the cognitive ability of friends, friends who are both conscientious

Table 10: Estimation with Interaction between Characterstics

Mathematics Exam Score			
Network	Multiple Networks (Friends and Studymates)		Single Network Seatmates
	Friends	Studymates	
Self Cognitive Ability × Peer Cognitive Ability	-0.025	0.117*	0.008
	(0.071)	(0.070)	(0.047)
Self Conscientiousness × Peer Conscientiousness	0.141**	-0.039	-0.012
	(0.069)	(0.054)	(0.052)
Peer Cognitive Ability × Peer Conscientiousness	0.191*	-0.041	-0.025
	(0.104)	(0.064)	(0.065)
Peer Agreeableness × Peer Cognitive Ability	0.034	0.210***	-0.090
	(0.064)	(0.060)	(0.062)
Peer Openness × Peer Cognitive Ability	0.004	-0.075	0.060
	(0.072)	(0.060)	(0.073)
Self Extraversion × Peer Cognitive Ability	-0.090	0.077	0.036
	(0.069)	(0.056)	(0.050)
Number of Peer × Peer Cognitive Ability	-0.017	0.018	0.064*
	(0.033)	(0.040)	(0.038)
Number of Peer × Peer Conscientiousness	0.060*	0.014	-0.013
	(0.035)	(0.043)	(0.042)
Number of Elder Siblings × Peer Cognitive Ability	0.025	-0.206***	0.086
	(0.076)	(0.075)	(0.054)

[1] *,** and *** denote statistical significance at the 10%, 5% and 2% level, respectively.
[2]

and smart produce extra positive effect on mathematics score, in addition to the effects from their conscientiousness. On the other hand, for studymate, agreeableness can magnify their positive impact through cognitive ability. This shows that it is more beneficial to learn from a studymate who is agreeable and smart, even though being an agreeable studymate itself is not very helpful.

We have shown in Section 5.2 that having more studymates is associated with a high mathematics score, but the number of friends does not matter. The coefficients of the interaction terms show that the number of friends magnify the effect of conscientious friends, but number of studymates has no significant effect on increasing the effect of smart studymates. We mentioned in the previous sections that one possible interpretation of the difference in contextual effects for friends and studymates is that studymates influence usually come from discussion or teaching and friends influence more by their attitude towards studying. The evidence from interactions terms is consistent with this explanation. It is conceivable that a students is influenced more by a large amount of conscientious friends then a small amount of friends with the same average conscientiousness. However, it is unclear that a larger amount of studymates will help more for the same level of average cognitive ability.

Our estimates show that the number of siblings itself does not produce any significant contextual effects. However, the number of elder brother or sisters makes the effect of a studymate's cognitive ability less significant. This is again consistent with the interpretation that studymates produce influence on their peers through teaching. If a student has an elder brother or sister, they can get help from their elder siblings and rely less on discussion with studymates. For a sanity check, we do not see the such an effect from younger brothers or sisters.

5.4 Comparing Spillover Effects

Apart from contextual effects, the model also estimates spillover effects. Table 9 shows a comparison of spillover effects under different network specifications. We observe positive spillover effect from friends and studymates only when we estimate them in separated single network models. None of the spilloever effects survive after controlling for multiple networks. Also, we do not find any significant spillover effect from seatmates in either single or multiple network estimation.

Spillover effects are interpreted as how peers outcome variable affects a student's outcome variable. However, the structure of the econometric model in this paper provide another interpretation. Since peer effects controlled for selection can only go through either contextual effect or spillover effect, the spillover effect part can be considered as all the effects that are not captured by the pre-determined characteristics included in the estimation (X_{peers}). In other words, the insignificant spillover effect estimates indicates that the peer effects on mathematics exam score are well captured by the contextual effects.[24]

5.5 Peer Effect On Other Subjects And Behavioral Impact

Our discussion focuses on the mathematics exam score, but it is also interesting to see how peer effects impact other subjects or behavioral outcomes. We focus on comparing our results in mathematics exam scores to those with English exam scores and conduct grades. We choose to focus on English exam because language classes could be very different from mathematics classes in terms of the learning methods and thus students' characteristics play very different roles in determining student outcomes. Conduct grades are measuring how well-behaved a student is. It is reasonable to believe that peer effects could work differently on academic subjects to behavioral outcomes.

Table 11 shows the comparison of the estimation of spillover effects, selection effects and selected contextual effects for English exam Score and behavioral outcome. Selected own effects estimates are included for comparison.

As mentioned in previous sections, spillover effects for mathematics is not statistical significant after controlling for friends and studymates networks. However, for English exam scores, we still observe significant effects for studymates. Compared to mathematics, studymates have more positive effects on a peer's English score through spillover effects, which means studymates do affect

[24] Significant spilloever effects are found in the estimation of peer effects for conduct grades.

Table 11: Peer Effect Estimates for English and Conduct Grade

Subject	English			Conduct Grade		
Network	Multiple Networks (Friends and Studymates)		Single Network Seatmates	Multiple Networks (Friends and Studymates)		Single Network Seatmates
Own Effects						
Cognitive Ability	0.121***		0.154***	-0.020		0.003
	(0.030)		(0.033)	(0.034)		(0.037)
Conscientiousness	0.244***		0.304***	0.182***		0.228***
	(0.033)		(0.035)	(0.036)		(0.039)
Contextual Peer Effects						
	Friends	Studymates		Friends	Studymates	
Cognitive Ability	-0.036	0.137***	-0.017	0.055	0.065	0.058
	(0.065)	(0.052)	(0.056)	(0.072)	(0.060)	(0.057)
Conscientiousness	0.123*	0.117**	-0.012	0.154*	0.073	0.147**
	(0.071)	(0.060)	(0.052)	(0.079)	(0.067)	(0.073)
Spillover Effects	0.056	0.088*	0.011	-0.070	0.214***	0.017
	(0.053)	(0.049)	(0.019)	(0.063)	(0.053)	(0.021)
Degree	0.044***	0.112***	0.081***	0.052***	0.086***	0.009
	(0.015)	(0.017)	(0.024)	(0.018)	(0.019)	(0.055)
Peer Selection Effects	0.074**	0.269***	0.447***	0.164***	0.154***	0.147**
	(0.035)	(0.058)	(0.156)	(0.051)	(0.059)	(0.073)
Network Formation Random Effect	0.939***			0.942***		
	(0.008)			(0.009)		

[1] *,** and *** denote statistical significance at the 10%, 5% and 2% level, respectively.

each other in the English subject but the effect are not captured by the student characteristics we included. This effect is also found in conduct grades where the effect is even larger.

Studymates with higher cognitive ability or more conscientious friends improves a student's English exam score just like what we observed for mathematics. However, conscientious studymates also improve English exam scores. Therefore, conscientiousness is not only more important in language subjects, it also affect peers through more different channels. Finally, conscientious friends improve a student's conduct grades, however there is no significant contextual peer effect estimates. Notice that since we observe significant spillover effect from studymates for conduct grade, insignificant contextual effect estimates does not mean that studymates are not important in determining a student's conduct grade. It only means that studymates affect a student's conduct grade, but not in a way measured by the predetermined personality measures included in our model.

Finally, the number of studymates is positively related with the score of both academic subjects and conduct grades. While for mathematics, number of friends does not have significant effect after controlling for the studymate network, this effect survives for English exam scores and Conduct Grades.

Despite the specific differences mentioned, peer effect works similarly across subjects in many ways. For example, we do not observe any significant contextual effect from other personalities such as openness and agreeableness. The direction of peer selection effects are the same for all of them too. Studymates selection is more significant for academic subjects, but it is equally important as friends for conduct grades.

5.6 Friends Network in Traditional Network Survey

Our empirical discussion has aimed at separating the effects of friends, studymates, and seatmates. This is possible largely due to the rich network information from the SESHK. The SESHK explicitly ask students about their friend and studymate networks. However, caution must be taken when we compare our results with studies using the friends network from other survey data because information carried by the data can be different. In particular, when a student is asked for different types of peers, they are instructed to catergorize their peers into different types like friends or studymates. However, if friendship is the only peer information the students are asked to provide the whole survey, it is conceivable that they will tend to put all types of peers onto the list of friends. Therefore, the friends network in our study may not be directly comparable to the friends network in other studies even if they use the exact question to elicit this particular piece of information.

To facilitate better comparison to other studies, we construct a network named "Any Social". This network is constructed by combining friends and studymates. In other word, two students are connected in the "Any Social" network if they are either friends or studymates. This more closely simulate the situation where the students were asked only about their friends and they are including multiple types of social connections as friends. The results of the estimation using this combined network are shown in Table 12. The "Any Social" network yields strong positive peer effects through cognitive ability and conscientiousness. It is expected because the network is constructed by mixing friends and studymates. It also shows strong spillover effect, even after we control for the peer selection effect and other peer characteristics.

We have to emphasize that our study is by no means a testimonial to say that the friendship network we collected is more correct. For example, one can argue that studymates should be defined as a particular type of friends and we are not objecting that. Nevertheless, this belief can coexist with the results that studymates produce different contextual effects than regular friends. A study with only friendship network could produce results similiar to Table 12, which is a mixture of different effects.

These results also facilitate discussion of whether we can proxy one peer type by another. Although we have shown in previous sections that different peer types produces difference effects through different channels, we cannot deny that these are some common features for peer effects of different networks. In particular, we see peers with higher quality, as measured by cognitive ability and conscientiousness, are conducive to learning. If an econometrician is only interested in whether having high quality peers are beneficial for learning or not, they will get the same sign whether they use friends or studymates or a mixture of both. In this case, it could be reasonable to use one type of peers as a proxy measure of another type. This is often very useful if the data of one type of peer is readily available but the type of interest is not. However, as shown in our estimation, the

Table 12: Peer Effects of the "Any Social" Network

Mathematics Exam Score Network	Any Social (Either Friends or Studymates)
Contextual Peer Effects	
Cognitive Ability	0.172**
	(0.075)
Conscientiousness	0.246***
	(0.075)
Spillover Effects	0.125**
	(0.054)
Peer Selection Effects	0.126***
	(0.041)

[1] *,** and *** denote statistical significance at the 10%, 5% and 2% level, respectively.

contextual effects can be quite different for different peer types or for different outcome variables. The validity of the strategy "proxying one peer type by another" depends on the research question.

6 Sensitivity Analysis

In the analysis in the previous sections, we have made several choices about the empirical specifications. For example, we focus on reciprocal peers, we group hobbies according to their types. While these choices and assumptions could be reasonable, it is useful to also explore whether our empirical results survive possible alternative specifications as a robustness check of our results. We illustrates the results for a refined definition of hobby and explore non-reciprocal peers here. Other robustness checks are shown in the appendix.

6.1 Refined Definition of Hobby

Using the number of hobbies in common as instrumental variables is one of the key elements of our empirical analysis. Hobbies are grouped into three categories namely music, sports, and youth movement. We allow different groups of hobbies to have different effects on exam scores but hobbies in the same group are assumed to have the same direct effect. In other words, the exogeneity assumption required is weaker if we include more groups of hobbies since we allow the hobbies from different groups to have their own direct effect on the outcome variable. Appropriate categorization of hobbies can provide more information and strengthen the instrument. However, one can imagine there are more than one valid categorization of hobbies and argue that our categorization in Section 3.2 may not the most reasonable one. Therefore, we further examine our results by introducing more detailed categorizations.

Table 13 shows selected contextual and spillover effects of the multiple network model for mathematics exam scores. Categorization 1 and 2 are two alternative groupings of hobbies. Details descriptions of each categorizations are shown in appendix. Since the dataset also provides information about when the students start to learn the musical instruments they play, we can construct a stricter definition of hobby which only includes musical instruments as a hobby and only if the student started the hobby before they entered secondary school. Finally, we estimated the model without using common hobby in the network formation. All our results on contextual effects, endogenous effects, and selection are preserved under these specifications.

Table 13: Peer Effect Estimates with Alternative Hobby Definition

Mathematics Exam Score						
Specification	Music Only		Alternative Hobby grouping			
			Catergorization 1		Catergorization 2	
Network	Multiple Networks (Friends and Studymates)		Multiple Networks (Friends and Studymates)		Multiple Networks (Friends and Studymates)	
Contextual Peer Effects						
	Friends	Studymates	Friends	Studymates	Friends	Studymates
Cognitive Ability	0.041	0.110**	0.039	0.110**	0.033	0.114**
	(0.070)	(0.055)	(0.070)	(0.055)	(0.070)	(0.055)
Conscientiousness	0.160**	0.081	0.159**	0.083	0.162**	0.090
	(0.073)	(0.061)	(0.073)	(0.061)	(0.073)	(0.061)
Degree	0.017	0.110***	0.016	0.108***	0.016	0.107***
	(0.016)	(0.018)	(0.016)	(0.018)	(0.016)	(0.018)
Spillover Effects	0.025	0.071	0.023	0.073	0.022	0.067
	(0.054)	(0.048)	(0.054)	(0.048)	(0.054)	(0.049)
Peer Selection Effects	0.076**	0.163**	0.077**	0.150**	0.075**	0.158**
	(0.036)	(0.066)	(0.036)	(0.067)	(0.036)	(0.070)

[1] *,** and *** denote statistical significance at the 10%, 5% and 2% level, respectively.

6.2 Non-reciprocal Peers

As described in our setup, only reciprocal peers are considered. We stick to those definitions because we would like to investigate friends or studymates which are acknowledged by both sides. While this definition is a reasonable definition of peer, it does not imply that other definitions are unreasonable.

For example, one could argue that when a student claims another student as a friend, he or she receives the peer influence already no matter if the other side says the same thing or not. Also, since everyone has different definition of what they mean by friends, one student may not make it into the other's list if they are not close enough friends. In this case, non-reciprocal data can still indicate a mutual influencing relationship, just that it may be weaker than the mutual agreement case. In short, the mutual agreement definition is by no means the only peer definition that is worth investigating. In some situations, it may not even be the only definition that capture all the relationships which are mutually acknowledged. We show the estimation of peer effects for mathematics exam score with non-reciprocal peers in Table 13.

We observe no statistically significant contextual effects and spillover effects for non-reciprocal peers. Including the non-reciprocal peers includes more schoolmates into the definition of peers and this could make the measure of the average quality of peers more noisy. The effect from having a higher number of studymates is still positive, but the magnitude is smaller. This is consistent with the explanation that the non-reciprocal peers do not produce as clear of peer effects as reciprocal peers do.

Table 14: Robustness Checks for Alternative Network Specifications

Mathematics Exam Score		
Network	Multiple Networks Include Non-reciprocal Peers (Friends and Studymates)	
Contextual Peer Effects		
	Friends	Studymates
Cognitive Ability	0.189	0.164
	(0.119)	(0.131)
Conscientiousness	0.126	0.167
	(0.134)	(0.147)
Degree	-0.011	0.057***
	(0.011)	(0.007)
Spillover Effects	0.120	-0.004
	(0.099)	(0.103)
Peer Selection Effects	0.101**	0.077**
	(0.040)	(0.037)

[1] *,** and *** denote statistical significance at the 10%, 5% and 2% level, respectively.

7 Conclusion

Estimation of peer effects from multiple networks has never been a trivial task. Datasets with multiple network types are not as abundant and the estimation is often plagued by selection effects, correlation between formation of different networks, and missing data. In addition to estimating the size and significance of peer effects, we estimate peer effects arising from multiple networks. We compare the different in size of peer effects from different networks as well as through what channels peer effects is significant.

Consistent with the literature, we find that peers play a significant role in determining student exam scores and behavioural outcomes. Different types of peers produce very different effects and it is important to consider each separately. Focusing on the mathematics exam score, peer effects are stronger for social-based peers than seatmates. Conscientious friends positively affect peers but cognitive ability is more important for studymates. Similar results are observed in English exam scores except that Studymates' conscientiousness also matters. For behavioral outcomes, Friends' conscientiousness produces a positive effect. Studymates show significant spillover effects but no significant contextual effects are found for the personality traits we considered. These results are robust to various specifications. A summary of our main results and the results from different alternative specifications are shown in Table 15. Selection is carefully accounted for but we do not see significant selection bias.

Table 15: Summarizing Empirical Findings

	Our Model	Models for Comparison		Robustness Checks			Alternative Setups	
Model	Multi Network Models	Single Network Models	Exogenous Network Model	Alt Hobby Definition	Alt Set of Controls	Fixed effects on Network	Alt Definition of Network	Test
Mathematics Exam Score								
Conscientious Friends improve Math exam score but Conscientious Studymates do not	Yes	No	No	Yes	Yes	Yes		
Smart Studymates improve Math but Smart Friends do not.	Yes	Yes	Yes	Yes	Yes	Yes		
Studymates and Friends has no Significant Spillover effects	Yes	No	No	Yes	Yes	Yes		
Number of Studymates improves Math but number of Friends does not	Yes	No	No	Yes	Yes	Yes		
Seatmates have no significant peer effect	Yes	Yes	Yes	Yes	Yes	Yes		
English Exam Score								
Conscientious Friends or Studymates improve English exam score	Yes	Yes	Yes	Yes	Yes	Yes		
Smart Studymates improve exam score but Smart Friends do not.	Yes	Yes	Yes	Yes	Yes	Yes		
Friends has no Significant Spillover effects, but Studymates do	Yes	No	No	Yes	Yes	Yes		
Both Number of Friends or Studymates improves English exam score	Yes	Yes	Yes	Yes	Yes	Yes		
Seatmates have no significant peer effect	Yes	Yes	Yes	Yes	Yes	Yes		
Behavioral Outcome								
Conscientious Friends improve Behavior but Conscientious Studymates do not	Yes	No	No	Yes	Yes	Yes		
Cognitive Ability has no significant contextual effects	Yes	Yes	Yes	Yes	Yes	Yes		
Friends has no Significant Spillover effects, but Studymates do	Yes	Yes	Yes	Yes	Yes	Yes		
Both Number of Friends or Studymates improves Behavior	Yes	Yes	Yes	Yes	Yes	Yes		
Seatmates have no significant peer effect	Yes	Yes	Yes	Yes	Yes	Yes		
Other Results								
Sensible Own effects for All Outcome Variables	Yes	Yes	Yes	Yes	Yes	Yes		
Sensible Network Formation Parameters	Yes	Yes	NA	Yes	Yes	Yes		
Positive Selection effects for All Outcome Variables and All Networks	Yes	Yes	NA	Yes	Yes	Yes		

The study of peer effects is more than just whether or not there exists peer effects. Different peer types can yield different implications and they are not always substitutable. It would be useful to rethink previous studies of peer effects, which mostly focused on friends, would implies the same results if we brought other peer types into the picture.

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Appendix I Other Robustness Checks

We investigate estimation with alternative network formation utility. Also, with the framework developed in Section 3.1, we can allow for fixed effects for network formation. The results are robust to these alternative estimations and are shown in Table 16.

Table 16: Robustness Checks for Alternative Specifications

Subject	Mathematics Exam Score			
Specification	Including Peer Characteristics		Fixed Effects for Network Formation	
Network	Multiple Networks (Friends and Studymates)		Multiple Networks (Friends and Studymates)	
Contextual Peer Effects				
	Friends	Studymates		
Cognitive Ability	0.037	0.114**	0.041	0.109**
	(0.070)	(0.055)	(0.070)	(0.055)
Conscientiousness	0.161**	0.079	0.162**	0.080
	(0.073)	(0.061)	(0.073)	(0.061)
Degree	0.016	0.110***	0.017	0.110***
	(0.016)	(0.018)	(0.016)	(0.018)
Spillover Effects	0.030	0.059	0.024	0.071
	(0.055)	(0.048)	(0.054)	(0.048)
Peer Selection Effects	0.083**	0.201***	0.080**	0.169**
	(0.037)	(0.064)	(0.036)	(0.069)

Appendix II Multiple Network Estimation with the Seatmates Network

The multiple network model with seatmates are shown here in Table 17.

Appendix III Details of Estimation and Computation

Appendix III(i) Likelihood function

Let Θ be the set of all parameters, for each school the likelihood is given by

$$L(Y, D_1, ..., D_r|X, \Theta) = \int P(Y, D_1, ..., D_r|X, \Theta, e)dF_e(e) \tag{7}$$

$$= \int P(Y|D_1, ..., D_r, X, \Theta, e)P(D_1, ..., D_r|X, \Theta, e)dF_e(e) \tag{8}$$

Table 17: Comparison of Selected Estimates for Different Network Specifications: Multiple Network

Mathematics Exam Score				
Network	Multiple Networks			
	(Friends and Seatmates)		(Studymates and Seatmates)	
Contextual Peer Effects				
	Friends	Seatmates	Studymates	Seatmates
Cognitive Ability	0.078	-0.006	0.125**	-0.006
	(0.067)	(0.051)	(0.052)	(0.051)
Conscientiousness	0.211***	-0.014	0.148***	-0.014
	(0.069)	(0.055)	(0.055)	(0.055)
Degree	0.055***	0.046*	0.115***	0.046*
	(0.015)	(0.023)	(0.016)	(0.023)
Spillover Effects	0.100**	-0.057	0.086**	-0.057
	(0.050)	(0.048)	(0.043)	(0.048)
Peer Selection Effects	0.106***	0.478***	0.169***	0.478***
	(0.038)	(0.149)	(0.054)	(0.149)
Network Formation Random Effect	0.361***		0.379***	
	(0.033)		(0.034)	

[1] *,** and *** denote statistical significance at the 10%, 5% and 2% level, respectively.
[2] Other personality traits are included in the estimation but omitted in the table. They do not produce contextual effects which are statistically significant.
[3] Other controls include height, family characteristics, hobby type, and class, school, grade fixed effects.

Appendix III(ii) Conditional density of Y: $P(Y|D_1, ..., D_r, X, \Theta, e)$

Let $\Gamma = \mathbb{I} - \sum_{r=1}^{R} W_r\lambda_r$ and $\varepsilon = \Gamma Y - X\beta, \sum_{r=1}^{R} W_r X\theta_r, \alpha_g, \rho_y e,$ we have $\Gamma Y \sim N(X\beta + \sum_{r=1}^{R} W_r X\theta_r + \alpha_g + \rho_y e, \mathbb{I}\sigma_e^2)$.

To remove the group fixed effect, let $J = \left(\mathbb{I}, \frac{1}{N}1_N 1_N^{'}\right)$ be the annihilation matrix, $X^* = JX$ and $Y^* = JY$. The outcome equation can be written as

$$J\Gamma Y = X^*\beta, \sum_{r=1}^{R}(W_r X)^*\theta_r + \rho_y e^* + \varepsilon^* \tag{9}$$

After concentrating σ_e^2, the log likelihood of Y given on Θ and X is

$$-\frac{n-1}{2}\log(\frac{\varepsilon^{*'}\varepsilon^*}{n-1}) + \log(\det(\Gamma)), \log\left(1 - \sum_{r=1}^{R}\lambda_r\right) \tag{10}$$

Appendix III(iii) Conditional density of D: $P(D_1, ..., D_r|X, \Theta, e)$

Students i and j are connected in network r if $u_{r,i,j} > R_{r,i,j}$ and $u_{r,j,i} > R_{r,j,i}$, where $R_{r,i,j} \equiv -(Z'_{g,i,j}\gamma_r + |e_i - e_j|'\rho_r)$. For each i and j, the probability that they are connected in all networks is

$$P(D_{1,i,j} = 1, \ldots, D_{R,i,j} = 1) = P(u_{1,i,j} > R_{1,i,j}, \ldots, u_{R,i,j} > R_{R,i,j}) \times \quad (11)$$
$$P(u_{1,j,i} > R_{1,j,i}, \ldots, u_{R,j,i} > R_{R,j,i}) \quad (12)$$

The conditional probability is more complicated if they are not connected in some networks. For any pair of i and j, there are three possibilities for them to be disconnected in network r; either i has negative utility ($u_{r,i,j} < R_{r,i,j}$), or j has negative utility ($u_{r,i,j} < R_{r,i,j}$) or both. We need to sum over the probabilities of all possible situations. Consider $R = 2$, if i and j are connected in network 1 but not network 2, the probability is given by

$$\begin{aligned} P(D_{1,i,j} = 1, D_{2,i,j} = 0) = {} & P(u_{1,i,j} > R_{1,i,j}, u_{2,i,j} > R_{2,i,j}) \times P(u_{1,j,i} > R_{1,j,i}, u_{R,j,i} < R_{R,j,i}) + \\ & P(u_{1,i,j} > R_{1,i,j}, u_{2,i,j} < R_{2,i,j}) \times P(u_{1,j,i} > R_{1,j,i}, u_{R,j,i} > R_{R,j,i}) + \\ & P(u_{1,i,j} > R_{1,i,j}, u_{2,i,j} < R_{2,i,j}) \times P(u_{1,j,i} > R_{1,j,i}, u_{R,j,i} < R_{R,j,i}) \end{aligned} \quad (13)$$

Similar equations can be derived for cases where students are disconnected in both networks or with $R > 2$

Appendix III(iv) Full conditional distribution of parameters

The unobservable error e makes the likelihood function intractable. The EM algorithm or simulated maximum likelihood are not applicable due to the high dimensionality of e. Instead of obtaining the estimator by maximizing the likelihood function, we are using the Bayesian approach to compute the posterior distribution of the parameters. Asymptotically, the posterior distribution of the parameters would coincide the Maximium Likelihood estimator. We discuss the full conditional distribution of the parameters. The full conditional distribution of the parameters is the conditional distribution of a parameter given on other parameters and data. With the full conditional distribution of the parameters, we can obtain the posterior distribution of all parameters by the Gibbs Sampler.

For clarity of notation, let $\phi = (\beta, \theta, \rho)$, $\delta = (\gamma, \psi)$

Appendix III(v) Full conditional distribution of ϕ

Conditional on e, λ and D, the outcome equation can be written as a standard least square error. Under the weak information prior, the conditional distribution of ϕ is approximately [25] with mean and variance equal to the value and variance of least square estimates of

$$J\left(\mathbb{I}-\sum_{r=1}(W_r)\lambda_r\right)Y = JX\beta + \sum_r JW_rX\theta + \sum_r Je_r\rho_r + J\varepsilon \tag{14}$$

Appendix III(vi) Full conditional distribution of λ

The full conditional distribution of λ is:

$$\begin{aligned}\mathbb{P}(\vec{\lambda}^*|\Theta\backslash\vec{\lambda},Y,X,Z) &= \mathbb{P}(Y|\Theta\backslash\vec{\lambda},\vec{\lambda}^*,X)\times\mathfrak{pr}(\vec{\lambda}^*) &(15)\\ &\propto \det(\Gamma)\times\left(1-\sum_r\lambda_r\right)\times\mathfrak{pr}(\vec{\lambda}^*) &(16)\end{aligned}$$

Appendix III(vii) Full conditional distribution of $U_{r,i,j}$

Let $\tilde{\mu}_{r,i,j} = -R_{r,i,j} + \Psi_{r,-r}\Psi^{-1}_{-r,-r}(U_{-r,i,j}+R_{-r,i,j})$ and $\tilde{\sigma}^2_{r,i,j} = \Psi_{r,r},\Psi_{r,-r}\Psi^{-1}_{-r,-r}\Psi_{-r,r}$

Conditional on δ, Ψ, e, $U_{-r,i,j}$, $U_{r,j,i}$ and $D_{r,i,j}$, the conditional distribution of $U_{r,i,j}$ has 3 possible distributions.

1. Suppose $D_{r,i,j}=1$, then $U_{r,i,j}$ follows a truncated normal distribution with mean $\tilde{\mu}_{r,i,j}$ and variance $\tilde{\sigma}^2_{r,i,j}$ and a lower bound of 0.

2. Suppose $D_{r,i,j}=0$ and $U_{r,j,i}+R_{r,j,i}<0$, then $U_{r,i,j}$ follows a normal distribution with mean $\tilde{\mu}_{r,i,j}$ and variance $\tilde{\sigma}^2_{r,i,j}$

3. Suppose $D_{r,i,j}=0$ and $U_{r,j,i}+R_{r,j,i}>0$, then $U_{r,i,j}$ follows a truncated normal distribution with mean $\tilde{\mu}_{r,i,j}$ and variance $\tilde{\sigma}^2_{r,i,j}$ with a upper bound of 0.

Appendix III(viii) Full conditional distribution of δ_r

Conditional on U, Ψ, e, D, the conditional distribution of δ_r is approximately[26] normal with mean and variance equal to the value and variance of least square estimates of

$$U_{r,i,j} = Z^{'}_{i,j}\gamma_r + |e_{r,i}-e_{r,j}|\psi_r + v_{r,i,j} \qquad \forall i,j=1,\ldots,N \tag{17}$$

[25] The differences are due to the prior which is negligible under the weak information prior

[26] The differences are due to the prior which is negligible under weak information prior

Let $\hat{\delta_r}$ be the OLS estimates of above model, then δ_r follows normal distribution with mean $\hat{\delta_r}$ and variance

Appendix III(ix) Full conditional distribution of $e_{r,i}$

Notice that since U contains all information from D, we can omit D from the conditional expectation. Thus,

$$
\begin{aligned}
\mathbb{P}(e^*_{r,i}|\Theta, e_{-r}, e_{r,-i}, Y, X, Z, U) &= \mathbb{P}(U_{r,i,1}, \ldots, U_{r,i,n}|\Theta, e^*_{r,i}, e_{-r}, e_{r,-i}, Z) \times \qquad (18) \\
&\quad \mathbb{P}(Y_i|U, \Theta, e^*_{r,i}, e_{-r}, e_{r,-i}, X) \times \mathfrak{pr}(e^*_{r,i}) \qquad (19)
\end{aligned}
$$

Appendix III(x) Full conditional distribution of Ψ

$$
\mathbb{P}(\Psi|\Theta\backslash\sigma^2_e, Y, X, Z) = \sum_{i,j} \mathbb{P}(U_{r,i,j}\forall r|\Psi) \times \mathfrak{pr}(\Psi) \qquad (20)
$$

$U_{1,i,j}+R_{1,i,j}, \ldots, U_{R,i,j}+R_{R,i,j}$ follows a normal distribution with mean 0 and variance Ψ. Since the diagonal elements of Ψ are always zero, we draw samples of Ψ using the Metropolis Hasting instead of an inverse-Wishart distribution.

Appendix III(xi) Computation

We use the combination of the Gibbs sampler and the Metropolis-Hasting algorithm to compute the posterior distribution of the parameters. For ϕ, U, δ, we directly draw from the conditional distribution. For λ, e and Ψ, we use the Metropolis-Hasting algorithm to draw samples. Then we obtain the posterior distribution of all parameters by the Gibbs sampler. In addition, we apply the adaptive MCMC algorithm to improve the sampling distribution in the Metropolis Algorithm. We run eight independent chains and stop the simulation if the ratios of the variance of between-chain mean and the mean of the within-chain variance for all parameters are smaller than 0.1.

We use the weakly informative prior for ϕ and δ, which is $N \sim N(0, 10000)$. The prior for $\vec{\lambda}$ is a uniform distribution with parameter -0.99 and 0.99. Priors for each elements in Ψ are uniform distributions with parameter -1 and 1.

Appendix III(xii) Conditional Expectation of Error given X and D

To derive the conditional expectation of error, we take the score formation equation:

$$
\begin{aligned}
Y &= X\beta + \sum_{r=1}^{R} W_r X \theta_r + \sum_{r=1}^{R} W_r Y \lambda_r + e && (21)\\
\left(\mathbb{I} - \sum_{r=1}^{R} W_r \lambda_r\right) Y &= X\beta + \sum_{r=1}^{R} W_r X \theta_r + e && (22)\\
\left(\mathbb{I} - \sum_{r=1}^{R} W_r \lambda_r\right) Y &= \left(\mathbb{I} - \sum_{r=1}^{R} W_r \lambda_r\right)^{-1} \left(X\beta + \sum_{r=1}^{R} W_r X \theta_r + e\right) && (23)\\
Y &= \left(\mathbb{I} - \sum_{r=1}^{R} W_r \lambda_r\right)^{-1} \left(X\beta + \sum_{r=1}^{R} W_r X \theta_r + e\right) && (24)\\
Y &= \left(\sum_{i}^{\infty} \left(\sum_{r=1}^{R} W_r \lambda_r\right)^{i}\right) \left(X\beta + \sum_{r=1}^{R} W_r X \theta_r + e\right) && (25)
\end{aligned}
$$

Taking conditional expectation on both side of equation 25 yields

$$
\begin{aligned}
E\left(Y|X, D_1, \ldots, D_R\right) &= E\left[\left(\sum_{i}^{\infty} \left(\sum_{r=1}^{R} W_r \lambda_r\right)^{i}\right) \left(X\beta + \sum_{r=1}^{R} W_r X \theta_r + e\right) \middle| X, D_1, \ldots, D_R\right] \\
&\quad + E\left[\sum_{i}^{\infty} \left(\sum_{r=1}^{R} W_r \lambda_r\right)^{i} e \middle| X, D_1, \ldots, D_R\right] && (26)
\end{aligned}
$$

Appendix IV List of Hobbies

Table 18 shows the lists of hobbies by hobby type.

Table 18: Lists of Different Types of Hobbies

Hobby Type	Hobby
Music	Violin, Piano, Guitar, Harp, Horn, Flute, Clarinet, Cello, Melodica, Bell, Erhu, Guzheng, Pipa, Yangqin, Chinese Flute, Yuan, Chinese Lo, Recorder, Xylophone, Keyboard, Electric Guitar, Harmonica, Africa Drum, Percussion, Saxophone, Drum, Ocarina
Sports	Basketball, Soccer, Pingpong, Badminton, Volleyball, Golf, Bowling, Tennis, Squash, Rugby, Dodgeball, Handball, Ropeskip, Mountaineer, Athletics, Swim, Skiing, Rowing, Hurdle, Archery, Cycling, Chinese Dance, Latin Dance, Dance, Ballet, Karate, Taekwondo, Yudo, Kungfu, Taiqi, Lion Dance, Run, Gun, Slideboard, Yoyo, Shotput, Gymnastics, Fencing, Climbing, Jianzi
Youth Movement	Boy Scout, Girl Guide, Boy Brigade, Girl Brigade, St John, Junior Police Call, Social Service Team, Community Youth, Road Safety Patrol, Red Cross, Flag Raising, Prefect, Teen, Leadership, Civil Aid, Cheerleading, Volunteering, Marching Band