The Taxable Income Elasticity: A Structural Differencing Approach[*]

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Abstract: We extend a standard taxable income model with its typical functional form assumptions to account for nonlinear budget sets. We show how to estimate the nonlinear consumption net-of-tax elasticity that is policy relevant unlike the typically estimated linear elasticity. Using the NBER tax panel for the US 1979-1990 and differencing techniques, we estimate an elasticity of 0.75 for taxable income and 0.20 for broad income respectively. These estimates are higher than those obtained by specifications based on linearizing the budget set at observed taxable income. Our approach circumvents the issues of the linearized budget set being endogenous to taxable income.

Keywords: taxable income, nonlinear budget sets, panel data

JEL classification: D11, H24, J22

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1. Introduction

Since the work of Feldstein (1995, 1998), the responsiveness of taxable income to changes in tax rates has been widely recognized as a research question of central importance in public finance. There is, by now, a substantial literature devoted to the estimation of the taxable income elasticity measured as the responsiveness of taxable income to a relative change in the marginal net-of-tax rate at the observed taxable income.[1] The standard method makes use of panel data and regresses log income changes on log of marginal net-of-tax rate changes, arguing that tax reforms provide exogenous sources of variation in tax rates. The main econometric difficulty is that the budget set is nonlinear with different marginal net-of-tax rates at different income levels, rendering the (change in) observed marginal net-of-tax rate endogenous to (the change in) taxable income.

The standard method addresses the endogeneity by instrumenting the log change in marginal net-of-tax rate with the log change in marginal net-of-tax rate at the base-year income level (Gruber and Saez, 2002). However, because the base-year income is part of the dependent variable, the instrument would be endogenous, e.g., if there is a temporary component of taxable income causing mean reversion where individuals with high income in one year tend to revert toward the mean in the next year. Including a base-year income control function, as typically done in the standard method to address this issue, is not a satisfactory solution as Weber (2014) shows. However, she argues that instruments based on income lagged several years are potentially valid instruments.

The issue of instrument validity has received a lot of attention in the literature and alternative related instruments based on some other income levels have been suggested (see, e.g., Caroll, 1998; Kopczuk, 2005; Blomquist and Selin, 2010; Weber, 2014). A more fundamental issue that the standard method assumes that behavior in a nonlinear budget set is essentially the same as behavior in a budget set that is linearized at the observed taxable income. This assumption is rather atheoretical regarding the individual decision problem in nonlinear budget sets rendering the observed marginal net-of-tax rate endogenous. It ignores, e.g., how to account for individuals that change tax brackets.

In the standard empirical double-log specification, the estimated taxable income elasticity corresponds to the elasticity of substitution parameter in a quasi-linear utility function when individuals face linear budget sets. In nonlinear budget sets, this linear elasticity only indirectly informs about the responsiveness to taxable income. It is, however, possible to define nonlinear policy-specific taxable income elasticities, e.g., based on a rotation of the budget frontier such as done in Blomquist et al. (2011, 2014). We define a similar nonlinear elasticity that corresponds to changing a flat consumption tax and relate it to the higher linear elasticity. We then investigate the standard approach of linearizing nonlinear budget sets at observed income levels and estimating the linear elasticity. The linear elasticity estimate is downward biased if there is preference heterogeneity but a valid instrumental variable could provide consistent estimates. However, we show that it is the reduced-form estimate of the instrument on taxable income rather than the structural estimate that corresponds to a nonlinear elasticity.

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[1] The net-of-tax rate is one minus the tax rate. See Saez et al. (2012) for a review of the literature.

Our main contribution is then to provide and to estimate a quite simple empirical specification of our nonlinear consumption net-of-tax elasticity guided by our model that accounts for nonlinear budget sets. We do this by deriving the taxable income function in nonlinear budget sets under similar functional form assumptions that lead to the double-log specification in the linearization approach. We show that the independent budget set variable should be a weighted average of the marginal net-of-tax rates at all income levels rather than at a single observed income level. The weight should be the unconditional income density function. Like in the standard method, we difference our structural specification in the empirical specification to make use of variation in tax rates provided by tax reforms. Our approach is structural regarding nonlinear budget sets and circumvents the issue of the observed marginal net-of-tax rate being endogenous to taxable income.

In our empirical application, we estimate the nonlinear consumption net-of-tax elasticity using U.S. data from the NBER tax panel for 1979-1981, which have been used several times previously for estimating the linear elasticity (e.g., Gruber and Saez, 2002). We find an estimated elasticity of 0.75 for taxable income and 0.20 for broad income respectively. The point estimates are statistically significant at the one percent level. In comparison, the comparable reduced-form estimates of the standard method are 0.27 for taxable income and 0.13 for broad income respectively. Accounting for nonlinear budget sets therefore increases the estimated elasticity. Unlike the standard method, our estimated elasticity is insensitive to controlling for the base-year income, indicating that we do not have problems with mean reversion or heterogeneous income growth over time and between groups with different income levels due to, e.g., income inequality trends. This result lends support to the view that the variation in tax rates provided by tax reforms is exogenous when characterizing this variation in a manner that appropriately accounts for nonlinear tax rates.

We are not the first to account for nonlinear budget sets. In the labor supply literature investigating hours of work as the outcome variable, Burtless and Hausman (1978) and Hausman (1985) proposed a method in which estimation is carried out using maximum likelihood. Dagsvik (1994) and Hoynes (1996) among others proposed an alternative method that discretizes the outcome variable and that focuses on estimating parameters of the utility function. Blomquist and Newey (2002) instead made advances by allowing nonparametric estimation with least squares. Ongoing work by Blomquist et al. (2011, 2014) adapt this method for estimation of taxable income. This paper differs from this literature by providing a method that allows using panel data differencing techniques like the main strand of the taxable income "linearization" literature. We do need to make similar arguably strong functional form assumptions as in the linearization approach, but a consequence is that we could isolate the influence of nonlinear budget sets and provide comparable estimates. The way we account for nonlinear budget sets with minimal modifications of the standard method keeps the empirical specification simple, transparent, and intuitive.

Another difference with both the taxable income linearization literature and most of the nonlinear budget set literature is that we define and estimate a nonlinear elasticity that is policy relevant and more informative like in Blomquist et al. (2011, 2014), Liang (2012) and Blundell and Shephard (2012).

The next section provides the theoretical analysis. Section 3 describes the empirical specifications. Section 4 reports the elasticity estimates. The last section concludes.

2. Theoretical model

2.1 Linear budget sets

We work in a framework where the individual taxable income choice problem is a static two-dimensional problem. The individual chooses (c, y) to maximize utility $u(c, y)$ subject to a budget constraint $c(y)$, where c is consumption which is also the net income, and y is taxable income. The budget set is the area below the budget constraint.

The double-log taxable income function typically estimated corresponds to assuming a utility function that is quasi-linear and has a constant elasticity of substitution for each individual. In a linear budget set, the decision problem is:

$$\max_{c,y} u = -\frac{\beta_l^i e^{-\alpha_l^i/\beta_l^i}}{\beta_l^i + 1} y^{\frac{\beta_l^i+1}{\beta_l^i}} + c \quad \text{s.t.} \tag{1}$$

$$c = \tau y + c_0. \tag{2}$$

α_l^i and β_l^i are parameters of the utility function. β_l^i is also the elasticity of substitution. The superindex i is used to denote that the parameters are allowed to differ between individuals, and the subindex l is used to denote that the parameters could be estimated when the budget set is linear. $\tau = c_y$ is the slope (consumption derivative with respect to taxable income) and c_0 is the intercept of the linear budget set. The slope is the marginal net-of-tax rate of the income sources that we define as taxable income. The intercept is the net income from remaining residual sources of income which may be taxed or untaxed, and it is assumed to be exogenous. In the case one wants to investigate the effect of taxes on earned income rather than taxable income, it would be suitable to set the intercept to net unearned income which would include net capital income.

The taxable income choice y^* for an interior solution is given by the first-order condition. Solving for y^* gives:

$$\ln y^* = \alpha_l^i + \beta_l^i \ln \tau. \tag{3}$$

The resulting taxable income choice does not depend on the intercept and the income effect is therefore zero. We observe that β_l^i could be interpreted as the linear taxable income elasticity.

Now, let us generalize the model to allow for unobserved individual preference heterogeneity to capture differences in tastes for work and optimization errors to capture that individuals may not be able to fine-tune their taxable income, e.g., because of job availability issues, and often have to choose among a limited set of taxable income options:

$$\alpha_l^i = e_1 \alpha_l, \tag{4}$$

$$\beta_l^i = e_2 \beta_l, \tag{5}$$

$$y = e_3 y^*. \tag{6}$$

The e:s are error terms that vary between individuals and have a mean of one conditional on the budget set. We omit the i index on them for notational simplicity. e_3 could also represent measurement errors.

The expectation of taxable income with the error terms is:

$$E(\ln y) = \alpha_l + \beta_l \ln \tau. \tag{7}$$

When allowing for individual heterogeneity that can be controlled for, z, we only require independence of the error terms conditional on z. In the empirical differencing estimation setting that we later apply, individual-fixed heterogeneity is differenced away, relaxing the independence assumption considerably.

The taxable income functions in Eqs. (3) and (7) were obtained analytically by solving for y^* from the first-order condition. Let us present an equivalent but more tedious iterative procedure of obtaining the taxable income function which is to increase y in steps and at each step check whether the first-order condition holds. Using j to denote the running variable index, the taxable income function can then be expressed as:

$$E(\ln y) = E(\ln y^*) = \iiint \ln y_j \, f(\ln y_j \mid \ln \tau, e_1, e_2) f(e_1) f(e_2) d(\ln y_j) de_1 de_2, \tag{8}$$

$$f(\ln y_j \mid \ln \tau, e_1, e_2) = f(\ln y_j = e_1 \alpha_l + e_2 \beta_l \ln \tau). \tag{9}$$

where $f(\ln y_j \mid \ln \tau, e_1, e_2)$ is a conditional probability density function. Note that we have made use of $f(\ln y_j, e_1, e_2 \mid \ln \tau) = f(\ln y_j \mid \ln \tau, e_1, e_2) f(e_1) f(e_2)$. For Eq. (8) to equal Eq. (7), we require:

$$\int \ln y_j \, f(\ln y_j \mid \ln \tau, e_1, e_2) d(\ln y_j) = \int (e_1 \alpha_l + e_2 \beta_l \ln \tau) f(\ln y_j) d(\ln y_j). \tag{10}$$

Expressing the functional form assumption in this way is useful later when we want to extend the model to allow for nonlinear budget sets.

2.2 Nonlinear budget set elasticity

Now, real budget constraints are typically nonlinear. Real tax and transfer systems often, but not always, produce piecewise-linear continuous convex budget sets because tax rates are typically progressive. Let us begin with the case without preference heterogeneity or optimization errors. One way to approach the problem is to linearize the budget set around the observed taxable income choice to obtain the observed marginal net-of-tax rate and proceed to estimate the linear taxable income elasticity as if behavior is the same as in a linear budget set. A rationale for this procedure is that the optimal choice on the linearized budget set is the same as the optimal choice on the nonlinear budget set if preferences and budget set are convex. Varying the marginal net-of-tax rate of the linearized and nonlinear budget sets therefore has the same effect on behavior as long as the individual stay within the same tax bracket after the variation.

Assume for now that the budget frontier is differentiable for simplicity[2], the first-order condition in Eq. (3) of the optimization problem in Eq. (1) for the linearized budget set in Eq. (2) is then both sufficient and necessary for an optimum. The complication is that in a nonlinear budget set, the marginal net-of-tax rate function is:

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[2] Most budget sets are piece-wise linear in which case the budget frontier is not differentiable. Individuals may then want to locate at kinks which are the main complications with nonlinear budget sets. The linearization approach ignores such complications. Assuming that the nonlinear budget frontier is smooth simplifies the analysis and still illustrates the essential complications with nonlinear budget sets.

$$\ln \tau (\ln y) = \ln c_y(\ln y). \tag{11}$$

The chosen linearized marginal net-of-tax rate needed to obtain the taxable income in Eq. (3) is therefore not a constant; it is endogenous to taxable income itself. The chosen taxable income y^* and marginal net-of-tax rate τ^* are the solutions to the equation system given by Eqs. (3) and (11).[3]

The first question one may ask is whether β_l^i is still an interesting parameter when the budget set is nonlinear. In linear budget sets, β_l^i directly informed about the marginal net-of-tax rate effect which is of policy interest. In nonlinear budget sets, both chosen taxable income and marginal tax rate are outcome variables, and there are many marginal tax rates. There are many ways to vary the marginal net-of-tax rate function, and it is possible to define a nonlinear taxable income elasticity for each type of variation. We note that the linear taxable income elasticity, among other things, corresponds to a tax schedule change where marginal net-of-tax rates are changed proportionally at all taxable income levels. Such an overall tax schedule change that proportionally rotates the budget frontier is possible also when the tax schedule is nonlinear as noted by Blundell and Shephard (2012). The rotation corresponds to changing a proportional flat consumption tax. We define the nonlinear consumption net-of-tax elasticity of taxable income in this way as:

$$\beta_{nl}^i = \frac{d \ln y^*}{d \ln \tau (.)}, \quad \beta_{nl} = E\left(\beta_{nl}^i\right) \tag{12}$$

This nonlinear elasticity is similar in spirit to the nonlinear elasticity defined by Blomquist et al. (2011, 2014) that rotates the budget frontier upwards in absolute terms.[4]

Let us investigate how β_{nl} relates to β_l when the budget set is nonlinear. Plugging Eq. (11) into Eq. (3) and differentiating with respect to $\ln \tau(.)$ assuming that $\ln \tau(.)$ is differentiable and solving for β_{nl}^i gives:

$$\beta_{nl}^i = \frac{\beta_l^i}{1 - \beta_l^i \ln \tau_{\ln y}} \le \beta_l^i. \tag{13}$$

We see that the nonlinear elasticity is lower than the linear elasticity when the tax system is progressive since $\ln \tau_{\ln y} < 0$ in this case. The intuition is that increasing marginal net-of-tax rates has a direct positive effect on taxable income at the linearized budget set. However, the individual may enter new tax brackets with higher tax rates, leading to a subsequent counteracting negative effect on taxable income. The difference between the linear and nonlinear elasticity is positively related to the linear elasticity and the progressivity of the tax system.[5]

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[3] The nonlinear budget set model in Burtless and Hasusman (1978) and Hausman (1985) is equivalent to our Eqs. (3) and (11) for the case with piecewise-linear budget sets.

[4] Their absolute rotation corresponds to changing a linear local tax. In linear budget sets, absolute or relative rotations are identical up to a scaling factor. In nonlinear budget sets, this is not the case. An absolute rotation has theoretical attractive features by keeping the intercept income effects of any linearized budget sets fixed. However, generalizing a constant linear elasticity leads to a nonlinear elasticity that corresponds to a relative rotation.

[5] Blomquist et al. (2011, 2014) also show that their nonlinear elasticity is lower than the linear elasticity when the budget frontier is piecewise linear and quasi-concave. They also conject that welfare effects are more closely related to a nonlinear elasticity than the linear elasticity.

2.3 The linearization approach

From Eq. (12), we see that we could estimate the nonlinear consumption net-of-tax elasticity if the variation in the taxable income outcome is only generated by variation in a consumption tax by regressing taxable income against the consumption net-of-tax rate. On the other hand, the correlation between the chosen taxable income and marginal-net-of tax rate outcomes is:

$$\frac{d \ln y^*}{d \ln \tau^*}|_{\ln \tau(.) \text{ varies}} = \frac{\ln y^*_{\ln \tau}}{\ln \tau^*_{\ln \tau}} = \frac{\dfrac{\beta_l^i}{1 - \beta_l^i \ln \tau_{\ln y}}}{\dfrac{1}{1 - \beta_l^i \ln \tau_{\ln y}}} = \beta_l^i. \tag{14}$$

Using the linearization approach estimating Eq. (3) therefore provides the linear elasticity of interest. This is because, as the individual enters tax brackets with higher tax rates, there is a counteracting effect on the chosen marginal net-of-tax rate (denominator in Eq. (14)) which is of the same size as the counteracting effect on the chosen taxable income. The two effects cancel out. It can be shown that any other type of budget set variation can be used to estimate the linear elasticity.

However, preference heterogeneity may also affect the correlation between chosen taxable incomes and marginal net-of-tax rates. Let us introduce preference heterogeneity according to Eq. (5). The correlation due to such heterogeneity is:

$$\frac{d \ln y^*}{d \ln \tau^*}|_{e_2 \text{ varies}} = \frac{\ln y^*_{e_2}}{\ln \tau^*_{e_2}} = \frac{\dfrac{\beta_l^i \ln \tau}{1 - e_2 \ln \tau_{\ln y}}}{\dfrac{\ln \tau_{\ln y} \beta_l^i \ln \tau}{1 - e_2 \ln \tau_{\ln y}}} = \ln \tau_{\ln y} \leq 0 \tag{15}$$

The intuition is that individuals with a high taste for work choose higher taxable incomes and lower marginal net-of-tax rates due to the progressivity of the tax system creating a negative correlation. The correlation between y^* and τ^* observed in a data set typically reflects a mixture of the variations in preferences and budget sets, whereas we are interested in sorting out the effect of preferences which introduces a negative bias.

An empirical approach to isolate and estimate the tax effect on taxable income is to use an instrumental variable driven by variation in tax schedules only. The direct reduced-form effect of the instrument on taxable income provides a nonlinear elasticity, whereas the structural estimate provides the linear elasticity which could be seen from Eq. (14). When the instrument is weak, the nonlinear elasticity could still be consistently estimated, whereas the linear elasticity would suffer from weak-instrument bias. The estimated nonlinear elasticity is policy-specific to the type of tax-schedule variation that the instrument is based on. To obtain the consumption net-of-tax elasticity requires an instrument driven by variation in a consumption tax.

Introducing optimization or measurement errors cause additional complications. Not only would the observed taxable income differ from the taxable income choice, but also the marginal net-of-tax rate at observed taxable income that is used to proxy the marginal net-of tax rate at desired taxable income would contain an error. For the estimation of a nonlinear elasticity, the error only enters the dependent variable and does not cause a bias. For the linear elasticity estimated using the linearization approach, the error enters an independent variable

resulting in an attenuation bias if the error is normally distributed, unless an instrument that is uncorrelated with the error is used.

Even if the more complicated estimation of the linear elasticity could be done consistently, it is less useful for predicting taxable income responses to tax rate changes. With individual heterogeneity, we could only estimate the average linear taxable income elasticity in Eq. (7) and not the individual-specific elasticities in Eq. (3) needed for predictions. To evaluate a consumption tax reform, we are, e.g., interested in $\beta_{nl} = E\big(\beta_l^i/(1 - \beta_l^i \ln \tau^i{}_{\ln y})\big) \neq E\big(\beta_l/(1 - \beta_l \ln \tau^i{}_{\ln y})\big)$, which is a strong argument for focusing on nonlinear elasticities.

2.4 Taxable income function in nonlinear budget sets

In this subsection, we derive a taxable income function in nonlinear budget sets from which it is possible to estimate the nonlinear consumption net-of-tax elasticity in Eq. (12). We do this within the outlined model given the same functional form assumptions as typically made in the linear budget set case. At the optimum, the first-order condition and taxable income outcome are now (allowing for preference heterogeneity):

$$\ln y^* = e_1 \alpha_l + e_2 \beta_l \ln \tau^* \tag{16}$$

where τ^* is the slope of the linearized budget set at the chosen y^*.[6]

We can derive the taxable income function using the same procedure as for linear budget sets by increase y in steps and at each step check whether the first-order condition holds in which case Eq. (16) also holds. This is possible because at a fixed y_j, τ_j is exogenously to the extent that the budget set is exogenous. We then obtain:

$$E(\ln y) = E(\ln y^*) = \iiint \ln y_j f\big(\ln y_j \mid \ln \tau_j, e_1, e_2\big) f(e_1) f(e_2) d\big(\ln y_j\big) de_1 de_2, \tag{17}$$

$$f\big(\ln y_j \mid \ln \tau_j, e_1, e_2\big) = f\big(\ln y_j = e_1 \alpha_l + e_2 \beta_l \ln \tau_j\big), \tag{18}$$

which is a straightforward generalization of the expression for linear budget sets in Eqs. (8) and (9). Making the same type of functional form assumption as in Eq. (10) we get:

$$\int \ln y_j f\big(\ln y_j \mid \ln \tau_j, e_1, e_2\big) d\big(\ln y_j\big) = \int \big(e_1 \alpha_{nl} + e_2 \beta_{nl} \ln \tau_j\big) f\big(\ln y_j\big) d\big(\ln y_j\big), \tag{19}$$

which gives the taxable income function:

$$E(\ln y) = \alpha_{nl} + \beta_{nl} \int \ln \tau_j f\big(\ln y_j\big) d\big(\ln y_j\big). \tag{20}$$

The taxable income function depends here on a weighted average of marginal net-of-tax rates on the budget frontier rather than the marginal net-of-tax rate at a single point. The weight is the unconditional taxable income probability density function. For estimation, we could discretize the budget set and numerically integrate over points on the budget frontier,

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[6] Kink points can be handled by modifying the first-order condition so that we require $FOC > 0$ at $y^* - \epsilon$ and $FOC < 0$ at $y^* + \epsilon$. Corner solutions can be handled in a similar manner. In Eq. (16), we could replace $\tau^* = \tau(y^*)$ by $\tau^* = \tau(y = \lim_{\epsilon \to 0+} y^* + \epsilon)$ or $\tau^* = \tau(y = \lim_{\epsilon \to 0-} y^* + \epsilon)$. It can be shown in a more fullfledged nonlinear budget set model that the two specifications bound the true estimate, where the bounds decrease with the spread of the optimization errors. In practice, both specifications give almost identical estimates.

and we could approximate the probability density function by the sample probability distribution.[7]

Note that the estimated parameters, α_{nl} and β_{nl} in Eqs. (19) and (20) are not equal to α_l and β_l in Eqs. (16) and (18), unless the budget sets are linear. Our nonlinear budget set equations nests the linear budget set equations. It is easy to verify that β_{nl} is the nonlinear consumption net-of-tax elasticity in Eq. (12) as the regressor increases by one if the entire tax schedule $\ln \tau(.)$ increases by one.

Note also that, although the taxable income function depend on the function $\ln \tau(.)$ which may require a large number of parameters to characterize unlike a linear budget set that only required two parameters (τ and c_0), the taxable income function is an integral of a two-dimensional term (τ_j and j) and therefore itself two-dimensional. Dimensionality therefore only increases by one compared to the linear budget set case (that only depends on τ). This is because the structure of the first-order condition is the same at every point on the budget frontier where the optimality of every point only depends on the marginal net-of-tax rate of that point.

3. Empirical estimation

3.1 Regression specifications

It is possible to estimate the linear and nonlinear consumption net-of-tax elasticities β_l and β_{nl} in Eqs. (12) and (14) using a single cross-section. However, the independence assumption of the budget set is unlikely to hold because budget sets typically correlates with demographic variables which may have their own effects on taxable income. To the extent that these variables are individual-specific, their effects can be differenced away if panel data is available. We could then estimate the basic specification:

$$\Delta \ln y_{i,t} = \alpha_{i,t} + \beta_{nl} \sum_j \Delta \ln \tau_{j,i,t} \Pr(\ln y_j) + \varepsilon_{i,t}, \tag{21}$$

where i index individuals, t index year, Δ is the difference operator, $\alpha_{i,t}$ is a constant and $\varepsilon_{i,t}$ is an idiosyncratic error term. In our application, j indexes 200 income intervals where the first 199 each cover 1,000 dollars, and the 200th covers the open interval above 199,000 dollars. $\Pr(\ln y_j)$ is the observed probability of taxable income y being in interval j. $\tau_{j,i,t}$ is the average marginal net-of-tax rate in j.

In the basic estimation, we use three-year differences. Obviously, identification requires at least one tax reform that had different effects on different individuals' budget sets. To the extent that these tax reforms lead to exogenous budget set changes, the elasticities would be consistently estimated. In our empirical application, we stack three-year differences from eight different years and make use of several tax reforms. Because we have single filers (singles and couples that choose to file separately) and joint filers (couples), we also control for filing status.

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[7] We set $\tau_j = (c_{j+1} - c_j) / (y_{j+1} - y_j)$ in the numerical integration.

Note that constant relative changes in taxable income over years and between individuals due to, e.g., productivity growth is accounted for by the constant in Eq. (21). To remove any possible correlation between the timing of the different tax reforms and differences in productivity growth between years, we also include year dummies.

In comparison, the standard method is to linearize the budget set at the observed taxable income and estimating the effect of the log change in the observed marginal net-of-tax rate on the log change in taxable income. The endogenous change in marginal net-of-tax rate is instrumented by the predicted change in marginal net-of-tax rate at the base-year income level. Our specification in Eq. (21) is similar to the structural equation, but replaces the log change in observed marginal net-of-tax rate by the log change in the weighted average marginal net-of-tax rate, which avoids the endogeneity of the observed marginal net-of-tax rate to taxable income.

As discussed in Subsection 2.3, the obtained nonlinear consumption net-of-tax elasticity with our method is, however, more comparable to the reduced-form estimate in the standard specification that also provides a nonlinear elasticity (whereas the structural equation provides the linear elasticity). We therefore compare our estimates with the estimates from the reduced-form equation of the standard method:

$$\Delta \ln y_{i,t} = \alpha_{i,t} + \beta_{nl}\Delta \ln \tau_{J,i,t} + \varepsilon_{i,t}, \tag{22}$$

where the net-of-tax rates are evaluated at is the base-year income level y_J. Even when comparing our specification with this reduced-form specification, the difference is that we use changes in marginal net-of-tax rates across income levels rather than at a single income level. Year dummies are also usually included in the standard specification. The difference between the estimates in Eqs. (21) and (22) reflects the bias from not accounting for nonlinear budget sets in the estimation, although both provide elasticities informing about taxable income responses in nonlinear budget sets.

An issue with the standard instrument ($\Delta \ln c'_{J,i,t}$) based on the observed base-year income is not valid because the observed base-year income is correlated with factors that may have their own direct effects on taxable income. This is the case if there is a temporary component of taxable income causing mean reversion where individuals with high income in one year tend to revert toward the mean in the next year. Another issue is heterogeneous productivity growth between individuals with different background characteristics who also have different income levels. The standard method to address both mean reversion and heterogeneous income growth is to control for either log of or spline in log of base-year income. Because the instrument and this control function both rely on variation in base-year income, separate identification could be problematic, and possibly needs to rely heavily on functional form assumptions. The availability of data spanning several reforms may help identification by providing variation in the change in marginal net-of-tax rate at the same income level for different base years. Of course, this only helps if the effect of base-year income is the same for different base years.

Weber (2014) investigates whether the standard instrument and other related instruments that are functions of taxable income are valid. She finds that they cannot overcome the mean reversion type of issue, even when including a base-year income control function. She shows, however, that income lagged several years back may be a valid

instrument in the limit, because the temporary component of taxable income dies out over time. For the issue with heterogeneous productivity growth, including an income control function is more promising, although again, the base-year income is endogenous. Weber's suggestion is, again, to use longer income lags.

Because our independent variable is not a function of taxable income, there is no automatic correlation with factors that correlate with taxable income in any year. To the extent that tax reforms are exogenous and not correlated with taxable income, there is no need to include an income control function.

It is, of course, still possible that tax reforms may correlate with other groups-specific trends. Reforms may, e.g., target groups with certain background characteristics, which may have different trends in taxable income. This could be addressed by including demographic control variables. We do not have such variables. Base-year taxable income could, however, be a proxy for such heterogeneity and solve the issue to the extent that such controls correlate with taxable income. We include a base-year taxable income control function to check the sensitivity of our estimates and to assess the exogeneity of tax reforms assumption. We also include such control functions in our standard reduced-form specification.

3.2 Data

We use data from the NBER panel of tax returns over the 1979-1990 period also known as the Continuous Work History File, which is the data used by Gruber and Saez (2002) and Weber (2014). The data contains detailed administrative information on tax and income variables but does not contain any demographic background variables. See Gruber and Saez for a detailed description of the data set.

We investigate the two most commonly examined measures of taxable income in the literature: actual taxable income (almost exactly as technically defined in the tax forms) and broad income. Broad income is an extensive definition of gross income and includes, among other things, wage income, interest income, dividends, and business income. We exclude capital gains, however. Taxable income is broad income minus a number of deductions. We use the definition in 1990 and include all adjustments that can be computed from the data for all sample years. Our definitions are consistently defined over the entire sample period and are identical to the ones used by Gruber and Saez (2002).

We use the tax schedule on taxable income. This schedule could be described by a function that only depends on taxable income which is what our theoretical framework could handle. We obtain the schedules by varying taxable income in steps of 1000 USD and compute the marginal net-of-tax rates from NBER-TAXSIM. Deduction/itemization rules that apply to broad income are applied before the construction of the tax schedule. To the extent that these rules correlate with the constructed budget sets that enter the estimation and affect deduction behavior, our estimates would be biased. Because tax reductions often take place with (and is sometimes partially financed by) tax base broadening through less generous deduction rules, the bias could be substantial.

Another related issue is income shifting between taxable income and other components of broad income. To the extent that deduction behavior and composition of income is endogenous to the constructed budget sets, the taxable income estimates would be biased. The

broad income estimates should, however, be less problematic in this regard. The taxable income estimates do, however, include the effects of tax avoidance, which is also a behavioral effect of interest for policy and welfare evaluations.[8]

Our sample selection criteria are similar to the ones in Gruber and Saez (2002). We drop filers that change filing status and observations with abnormally large (top and bottom one percent of the sample) changes in (weighted average) marginal net-of-tax rates since these observations are more likely to be driven by variable construction errors. We also drop observations where we could not compute either taxable income or broad income. However, we do not truncate our data, unlike Gruber and Saez. We use the log of income plus one as the outcome variable to enable inclusion of observations that involve individuals with zero income. Sample statistics can be found in Appendix.

4. Results

4.1 Taxable income

In Table 1, we report estimates of the nonlinear taxable income elasticities in Eqs. (21) and (22). In the first rows, we report the estimates of the log change in average marginal net-of-tax as defined in Eq. (21) which is our specification that fully accounts for nonlinear budget sets. In the last rows, we report the estimates of the log change in the standard predicted marginal net-of-tax rate at base-year income level. This is the reduced-form estimate of the instrument on taxable income in the standard linearization method. In Column (1), no control variables are included. In subsequent columns, we subsequently add a dummy for filing status, year dummies, and then, log base-year income or a ten-piece spline in log base-year income.

Table 1. Nonlinear net-of-tax elasticity of taxable income

	(1)	(2)	(3)	(4)	(5)
Log change in	0.559**	0.600**	0.756**	0.742**	0.748**
average net-of-tax rate	(0.056)	(0.056)	(0.059)	(0.054)	(0.052)
Log change in	-0.146**	-0.110**	-0.082**	0.358**	0.269**
predicted net-of-tax rate	(0.025)	(0.025)	(0.026)	(0.023)	(0.023)
Filing status	No	Yes	Yes	Yes	Yes
Year dummies	No	No	Yes	Yes	Yes
Log base income	No	No	No	Yes	No
Spline log base income	No	No	No	No	Yes

Notes: Each cell is a nonlinear elasticity estimate from one regression. The log change in taxable income is the outcome variable. 3-year differences are used. The spline in base-year income contains ten pieces. Each regression contains 51,392 observations. * $p<0.05$; ** $p<0.01$.

Our estimated nonlinear consumption net-of-tax elasticity of taxable income that account for nonlinear budget sets in the estimation is 0.56 when no control variables are included and around 0.75 after controlling for filing status and year effects. The elasticity point estimates

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[8] To properly investigate income shifting effects and source-specific deductions would require a model that allows for several choice variables. The dimensionality of the budget set would then be the number of choice variables plus one.

are all statistically significant at the one percent level. The estimated elasticity does not change much when additionally controlling for the base-year income, either in the form of log base-year income or a spline in log base-year income. The evidence therefore does not contradict that the tax variation between individuals provided by tax reforms as represented by the log change in average marginal net-of-tax rate are exogenous. An elasticity of 0.75 implies that a one percent increase in net-of-tax rates at all income levels that could be carried out by decreasing the tax rate of a flat consumption tax leads to an increase in taxable income by 0.75 percent.

In comparison, the estimated nonlinear net-of-tax elasticity based on linearizing the budget set in the estimation is negative at first, likely reflecting a correlation that is at least partly caused by preference heterogeneity as shown in Eq. (15). Controlling for base-year income turns the estimated elasticity positive, indicating that mean reversion and/or heterogeneous productivity growth is an issue for the standard log change in predicted marginal net-of-tax rate instrument. Also, the exact point estimate is sensitive to the functional form of the base-year income control. The specification with the spline corresponds to the reduced-form estimates of the standard method and produces an elasticity of 0.27. Again, all elasticity point estimates are statistically significant at the one percent level. Comparing with our nonlinear consumption net-of-tax elasticity point estimates, we observe that accounting for nonlinear budget sets in the estimation increases the estimated elasticity by almost a factor of three.

Our linearized budget set estimates are comparable to Gruber and Saez's (2002) estimates on the same data but slightly different sample. Their taxable income elasticity point estimate is 0.40 in the specification that includes a spline and in this regard comparable to our estimates in Column (5). The main reason for the discrepancy is that is that they estimate the structural equation and therefore obtain the linear elasticity, that they weigh their regression by taxable income, and that their sample differs slightly. Their reduced-form estimate (that we computed) would have been 0.21 and statistically significant at the one percent level which is close to the 0.27 that we obtain.

Weber (2014) uses instruments based on longer income lags on the same data. She finds a linear taxable income elasticity that is between 0.86 and 1.36 in most specifications. Because she uses several instruments, there are several first-stage estimates and there is not a single reduced-form estimate for us to compare with. Rescaling her estimates by Gruber and Saez's first-stage point estimate, we would obtain something similar to a nonlinear elasticity between 0.43 and 0.68, which would still be smaller than our estimated elasticity.

To illustrate our estimates and the influence of base-year taxable income, we plot the log change in taxable income variable against the two different log change in net-of-tax budget set variables in Figure 1 and the budget set variables against log base-year income in Figure 2. We use both a local polynomial and a linear fit to the data in Figure 1, whereas we only use the local polynomial fit in Figure 2.
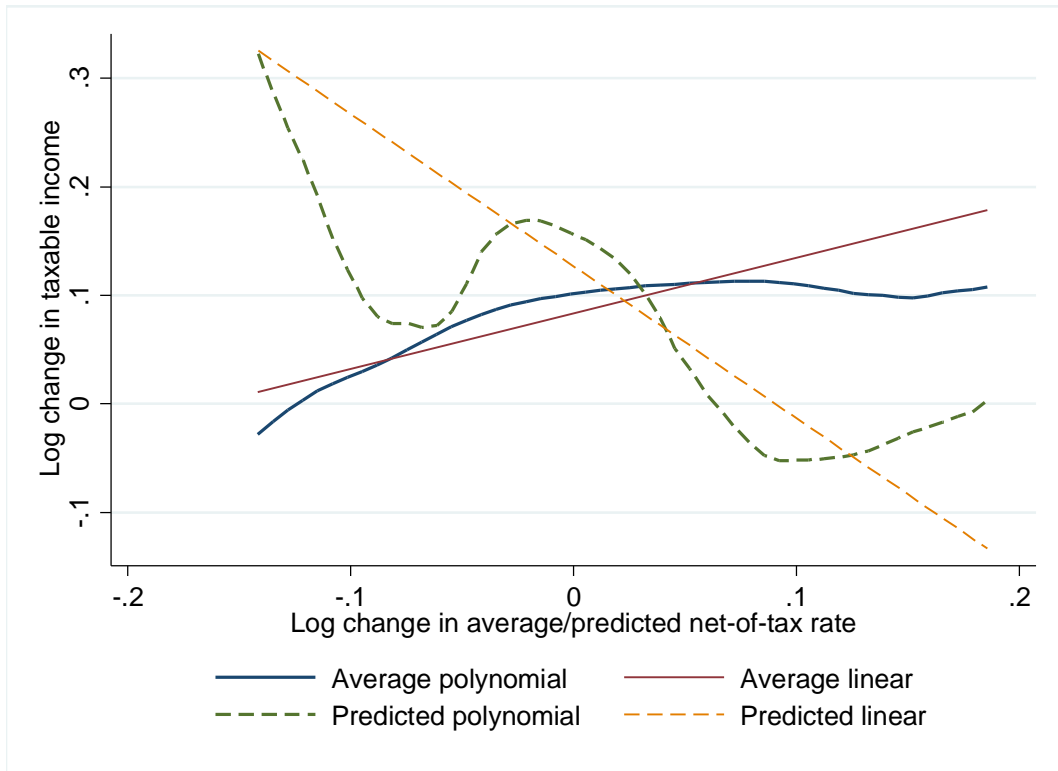
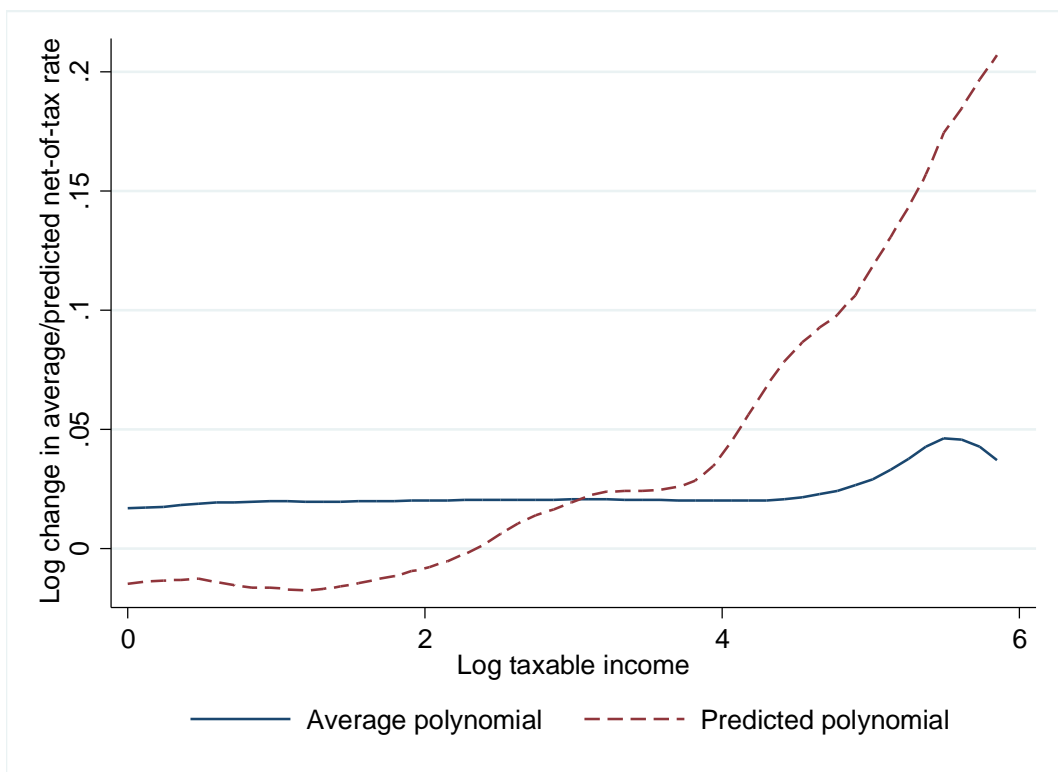Figure 1. Graphs of the outcome variable against the net-of-tax rate variables

Figure 2. Graphs of the net-of-tax variables against the base-year income

The correlation between the outcome variable and the change in average marginal net-of-tax rate is positive, whereas the correlation between the outcome variable and the change in

predicted marginal net-of-tax rate is negative in Figure 1. The base-year income is uncorrelated with the change in average net-of-tax rate in Figure 2 supporting the view that variation in this change induced by tax reforms is exogenous. Base-year income is, however, positively correlated with the change in predicted net-of-tax rate. Tax reforms were therefore such that it gave individuals with higher base-year income a larger net-of-tax rate decrease. To sort out the effect of the change in net-of-tax rate at base-year income from the direct effects of the base-year income therefore relies on controlling for the independent effects of the base-year income properly. The endogeneity of the change in marginal net-of tax rate at the base-year income does, of course, not necessarily indicate that variation provided by tax reforms are endogenous. More likely, it is a result of the base-year income being endogenous.

4.2 Broad income

It is commonly believed that the taxable income elasticity to a great extent reflects income shifting due to deduction behavior. We therefore report nonlinear elasticity estimates for broad income in Table 2, which is similarly organized as Table 1. We observe that the estimated broad income elasticity that accounts for nonlinear budget sets in the estimation is in the region of 0.20 and still quite insensitive to including a base-year income control function. In comparison, the estimated elasticity based on linearizing the budget set is around 0.13 and depends on including the control function. All estimates are statistically significant at the one percent level. Even for broad income, accounting for nonlinear budget sets in the estimation therefore increases the estimated elasticity.

Table 2. Nonlinear net-of-tax elasticity of broad income

	(1)	(2)	(3)	(4)	(5)
Log change in average net-of-tax rate	0.126** (0.027)	0.145** (0.026)	0.173** (0.028)	0.202** (0.027)	0.199** (0.027)
Log change in predicted net-of-tax rate	-0.035** (0.012)	-0.017 (0.012)	-0.032** (0.012)	0.147** (0.012)	0.133** (0.012)
Filing status	No	Yes	Yes	Yes	Yes
Year dummies	No	No	Yes	Yes	Yes
Log base income	No	No	No	Yes	No
Spline log base income	No	No	No	No	Yes

Notes: Each cell is a nonlinear elasticity estimate from one regression. The log change in broad income is the outcome variable. 3-year differences are used. The spline in base-year income contains ten pieces. Each regression contains 51,392 observations. $^{*} p<0.05$; $^{**} p<0.01$.

In comparison, Gruber and Saez (2002) estimates of the linear elasticity is 0.12 and statistically insignificant which corresponds to a nonlinear elasticity of 0.06 that is statistically insignificant. Weber (2014) on the other hand obtains an estimated linear elasticity between 0.48 and 0.70 which corresponds to a nonlinear elasticity between 0.24 and 0.35. Our estimates lie therefore between the estimates in these two studies.

4.3 Difference length

The responsiveness of taxable income to changes in tax rates may be different in the short-run than in the long-run. It may take some time for individuals to react to tax changes. On the other hand income shifting between sources and over time is easier to do occasionally than permanently. In Table 3, we report nonlinear elasticity estimates that account for nonlinear budget sets in the estimation for taxable income and broad income using different difference lengths. We use observations with the same base-year which keeps the number of observations constant.

Table 3. Nonlinear net-of-tax elasticity with different difference lengths

	(1) 1-year differences	(2) 2-year differences	(3) 3-year differences
Taxable income	1.250**	0.788**	0.748**
	(0.048)	(0.046)	(0.052)
Broad income	0.215**	0.143**	0.199**
	(0.022)	(0.022)	(0.027)

Notes: Each cell is a nonlinear consumption net-of-tax elasticity estimate from one regression. The log change in income is the outcome variable. All specifications include controls for filing status, year dummies, and a spline in log base-year income. Each regression contains 51,392 observations. * $p<0.05$; ** $p<0.01$.

We see that the estimated taxable income elasticity decreases with difference length suggesting that there are temporary behavioral effects such as income shifting that decreases over time. The estimated broad income elasticity decreases first between one- and two-year differences, but increases between two- and three-year differences, suggesting that there are some effects with a longer response time.

5. Conclusions

We started out from a standard taxable income model with a quasi-linear and constant elasticity of substitution utility function. We showed that when allowing for nonlinear budget sets the nonlinear elasticity corresponding to a consumption tax change is a direct policy relevant extension of the linear elasticity. We then showed that to estimate the consumption nonlinear net-of-tax elasticity, we need to use a weighted average of the marginal net-of-tax rates in the entire budget set as the independent variable. The weight is the unconditional income density function. In comparison, the linearization specification uses the marginal net-of-tax rate at the observed income level. Furthermore, the structural estimate in a standard instrumental variables specification provides the linear elasticity, whereas the reduced-form estimate provides a nonlinear elasticity, given that the instrument is valid.

In our empirical application, we estimated the nonlinear consumption net-of-tax elasticity using U.S. data from the NBER tax panel for 1979-1981. We found an estimated elasticity of 0.75 for taxable income and 0.20 for broad income respectively. In comparison, the comparable reduced-form estimates of the standard method were 0.27 for taxable income and 0.13 for broad income respectively. Accounting for nonlinear budget sets therefore increased the estimated elasticity.

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Appendix

Table A1. Sample statistics

Variables	Mean	Std. dev.	Min	Max
Δ log taxable income	0.092	0.692	-4.022	4.275
Δ log broad income	0.040	0.329	-2.265	2.885
Δ average net-of tax rate	0.020	0.055	-0.142	0.186
Δ predicted net-of tax rate	0.017	0.124	-3.005	4.902
Log base-year taxable income	2.905	0.937	0.000	6.520
Log base-year broad income	3.571	0.606	1.752	6.596
Log base-year average net-of-tax rate	-0.431	0.061	-0.859	-0.175
Log base-year marginal net-of-tax rate	-0.312	0.152	-3.297	0.877
Base-year taxable income	25.367	24.108	0.000	677.657
Base-year broad income	41.657	29.345	4.768	730.991
Base-year average net-of-tax rate	0.655	0.039	0.424	0.847
Base-year marginal net-of-tax rate	0.739	0.093	0.037	2.404

Notes: Taxable income and broad income are in USD at the 1990 price level.