How Short-Time Work Stabilizes Employment In Times Of Crisis

- preliminary version -

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The extensive use of short-time work is given credit to be a major cause for the extraordinary stability of the German labor market during the recession in 2009. Although there are numerous empirical analyses on the incidence and the extent of short-time compensation programs, the attempts to capture the effects of this kind of subsidy and its various designs on the decision making processes of the individual firm on a theoretical level are still largely insufficient. The model at hand exhibits short-time work from an investment point of view. In an exogenous uncertain market environment, implemented through a Geometric Brownian Motion, a firm optimizes its factor input of labor by variations of the amount of labor force and operational working hours. Dependent on its parameter respective strategy, the firm has to face hiring and firing cost or remanence cost. Utilizing methods of stochastic control the firm can continuously maximize its expected future profits. Based on numerical results the present framework gives insights to the respective extent of labor hoarding and the way the trade-off between hours of work reduction and dismissals is affected by the presence and design of a short-time compensation program in times of a cyclical downturn.

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1 Introduction

As a preventative measure against unemployment short-time work has been an established remedy of economic policy in times of economic recession for roughly 85 years in Germany.[1] This kind of subsidy on a reduction of operational working time through a state short-time compensation (STC) program, as a reaction to a temporary cyclical downturn in sales, is not indisputable in research. In their empirical analysis Brenke/

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[1]See Flechsenhar (1980), p. 14-18.

Rinne/Zimmermann (2011) find that the impact of the economic crisis of 2009 on the German labor market might have doubled the increase in unemployment, if no STC had been available. Three out of four short-time workers are guessed to have skipped unemployment, following Deeke (2009). An international comparison of STC programs in 19 OECD countries in Hijzen/Venn (2011) grants a high effectiveness to the German "Kurzarbeitergeld".[2] In contrast its role is seen as rather minor in Boysen-Hogrefe/ Groll (2010), who underline the advantage of internal flexibility of working time, while Burda/Hunt (2011) give credit to the dilatory hirings in the preceding boom period. Boeri/Bruecker (2011) finally stress the dependence of a running STC program on the leverage effect of other employment stabilizing policies such as job protection.

In consequence of its frequent application in the economic crisis around the year 2009 short-time work has drawn attention to itself. Still research to its effectiveness is mostly descriptive in nature. Theoretic work, like Burdett/Wright (1989) or Göcke (2009) is infrequent and rudimentary. In the following approach, a sustainable profound theoretical framework is developed to promote a coherent comprehension of the effects of STC as an instrument of labor market policy.

The featured model sheds light on a firms trade-off between the introduction of short-time work and layoffs. A firm's decision to hire or fire workers is seen as an investment. Given uncertain demand for a homogenous product, evolving through continuous time, the firm can realize profits under perfect competition, adapting its respective equilibrium employment-working-hours allocation of factor input to the expected present value of future production. The firm makes use of a simple production function with decreasing returns to scale. Costs of labor include wages and fixed costs for firing or hiring workers. Demand is seen as a stochastic process, taking the form of a Geometric Brownian Motion with an exponential drift, reflecting growth in demand, and normally distributed increments, to account for uncertainty.[3] The history of the exogenous demand curve can be observed in the sense of a filtered probability space, but, given the process has the Markov property[4], the future development of the trajectory is solely defined by the current state and the parameters of the underlying probability function. The firm will adjust its labor input to meet variations in demand using stochastic control techniques, resulting in a regulated stochastic process. [5] The economic agent regulates the labor process with respect to costly labor turnover. Hirings and layoffs only take place, when the expected shadow value of the forfeit production exceeds the turnover costs, hoarding a certain amount of labor. The labor hoarding effect depends on the magnitude of the fixed hiring and firing costs and on the remanence cost of reduced working hours. In this model, the firm has to choose between keeping a worker, paying his wages and non-wage labor cost, with the future prospect of recovering sales, or a layoff, meaning immediate certain firing costs, if the option value of waiting exceeds the boundaries of inactivity, defined by the transaction costs of labor adjustment.

The used techniques of stochastic control are closely related to Harrison (1985) with

[2] Germany is rated second right behind Japan.
[3] See Mikosch (2008), p. 139; Shreve (2004), p. 106.
[4] See Karatzas/Shreve (1988), p. 71–79.
[5] See Harrison (1985).

respect to the more comprehensive revision in Stokey (2009) referring to economic applications to transaction costs. Yet the most influential mathematical foundation of this work is based on the pioneering works of Bentolila/Bertola (1990) and Dixit/Pindyck (1994). Applications from a finance point of view may be found in Pham (2009) and Sethi/Thompson (2005). For a similar approach to labor market policies in general see Faia/Lechthaler/Merkl (2011).

2 Dynamic demand

A firm is defined to produce a homogeneous good, using exclusively the input factor of labor. The production function is defined as

$$Q_t = A_t \cdot {L_t}^{\mu} \cdot {h_t}^{\gamma}, \quad \text{where} \quad \mu + \gamma \leq 1. \tag{1}$$

Q_t is the produced output at the present time t. Labor productivity is denoted by A_t, and L_t is the present work force. Note that the partial output elasticities μ and γ promote the assumption of decreasing returns to scale. μ and γ reflect the idea of heterogeneous labor, where μ represents a labor market, where expanding business activity necessitates the utilization of continuously less productive and successively lower qualified work force, whereas γ means a reduction in productivity, the more workers are exhausted by increasing working hours. The factor input of labor, i.e. the volume of labor, is not necessarily utilized entirely for production. Nonetheless the factor costs for the currently available volume of labor as a product of work force L and an hours of work factor h, are to be paid in full by the firm. In this model $h = 1$ is the standard measure of full-time work, whereas smaller values represent short-time work of varying extent. As stock keeping is not considered within the given framework, the firm can only produce output at a maximum of the current sales. Furthermore the firm is a price taker under perfect competition, thus incapable of controlling the market conditions. It therefore can only adapt its level of production to the continuously fluctuating price P_t of the produced good. Note that $\{P_t\}$ is a stochastic process, modeling random variations in demand. The process itself is defined by the stochastic differential equation

$$dP_t = P_t \vartheta_p dt + P_t \sigma_p dB_t, \tag{2}$$

which means that $\{P_t\}$ takes the form of a Geometric Brownian Motion, solely depending on the parameters of the underlying distribution and the history of the process:

$$P_t = P_{t-s} + P_{t-s}\vartheta_p dt + P_{t-s}\sigma_p dB_t. \tag{3}$$

Since time is regarded as continuous, the equation can be broken down recursively [6]

$$P_t = P_0 e^{\left(\left(\vartheta_p - \frac{\sigma_p^2}{2}\right)t + \sigma_p B_t\right)}. \tag{4}$$

[6] See Mikosch (2008), p. 139; Shreve (2004), p. 106. In this equation the exponential effect of the geometrical drift on the stochastic increments of the infinitesimal time interval $[t;\tau]$ is considered to be negligible. The drift and the underlying Wiener process are not interacting.

The stochastic component of these functions is defined by $\{B_t\}$, an "ordinary" Brownian motion, also called a Wiener process, embedded into the Geometric Brownian Motion.[7] The expected value of price with respect to time is[8]

$$E_t\left(P_\tau\right) = P_t e^{\left(\vartheta_p - \frac{1}{2}\sigma_p{}^2\right)(\tau - t)}. \tag{5}$$

3 The optimization calculus

The firm will optimize its profits, the difference of its turnover and its labor cost, by variation of labor input. Such a variation may be gained by adjusting the overall hours of work h_t within the restrictions of any labor agreement or else, if that is impossible or would be inefficient, by a transfer of work force dX_t. The latter will result in additional transaction cost. Within an instantaneous cycle of production X_t equals the present work force L_t. However a formal distinction is required comparing different points in time, as the firm experiences a continuous attrition of labor, caused by retirement and worker initiated migration. So costly turnover dX_t may be necessary to compensate for the attrition rate δ:

$$dL_t = dX_t - \delta L_t dt. \tag{6}$$

The firm operates in perfect competition, so under given prices, revenue R is

$$R_t\left(L_t, h_t\right) = P_t \cdot A_t L_t{}^\mu h_t{}^\gamma. \tag{7}$$

To realize this level of production, labor costs need to be paid in the amount of the product of work force L_t and wages W. According to the expression

$$C_t\left(L_t, h_t\right) = W\left(h_t\right) L_t = \left(wh\left(1+s\right) + w(1-h_t)s(1-k) + f\right) L_t \tag{8}$$

labor costs contain fixed costs f and variable costs.[9] Variable costs further consist of the given wage rate w and a surcharge s, which represents non-wage labor costs related to the actual working hours per worker. The fixed costs parameter f contains payments like a vacation bonus. It may also be interpreted as operating expenses affiliated with each single worker. These may be any kind of equipment and asset that is required for

[7] The term Wiener process is henceforth used synonymously for the regular standard Brownian Motion, in order to support the demarcation between the regular and the geometric Brownian Motion. A Wiener process, the continuous equivalent to a random walk by Pearson, is defined by stationary and normally distributed increments: $dB_t = \varepsilon\sqrt{\Delta_t}$, where $\varepsilon \mapsto N\left(0;1\right)$. The variance of the increments is constant for uniform time intervals $t-s$ and increases proportional to the length of the time interval. The standardized increments are modified by the variance σ_p. Besides its stochastic property the process is determined by a drift of growth rate ϑ_p.

[8] See Wiersema (2008), p. 105-108; Etheridge/Baxter (2002), p. 88; Ross (2007), p. 631. Deviant of the suggestion in eq. (3), the process in this form is not expressed recursive via its past realizations but as a prognosis from the present state of time t into the indefinite future τ.

[9] In Crimmann/Wießner/Bellmann (2010) short-time work is modeled complementary to regular working hours. To account for any possible variation in working time, the more flexible adaption by a time factor was chosen.

production and independent of the amount of working hours.[10] Remanence costs are reduced by the factor k, to account for subsidies implemented into a public short-time work program.[11]

This profit maximization constraint is extended by an indicator function, to incorporate costly labor turnover. If $dX_t > 0$, the firm has to pay the fixed one-time hiring costs H to employ a new worker. These hiring costs contain expenses for recruitment and qualification of the new employee and may be further affiliated with means of production. If a slope in demand necessitates a layoff ($dX_t < 0$), the one-time firing costs F will be applied. If no transfer takes place, both indicator functions are removed.[13] The now modified revenue function takes the form

$$V_t \equiv \max_{\{x_t\},\{h_t\}} E_t \left\{ \int_t^\infty e^{-r(\tau-t)} \left[\left(P_\tau A_\tau {L_\tau}^\mu {h_\tau}^\gamma - W L_\tau \right) d\tau - \right.\right.$$
$$\left.\left. - \left(1_{[dX_\tau > 0]} H - 1_{[dX_\tau < 0]} F \right) dX_\tau \right] \right\}. \quad (9)$$

Note that the realizations of this revenue function are reiterated through continuous time, thus future realizations are discounted to present value. The discount factor r can be seen as the rate of interest of an alternative risk-free investment. Following the principle of continuous pricing as a geometric Brownian Motion with drift, the model makes use of an exponential discount rate.

dX_t depends on dL_t. The work force nevertheless is dependent on the progress of prices dP_t, once transfers of labor are the dominant strategy or, after working time arrangements are exhausted, become the last resort. As dP_t is adapted to the stochastic $\{P_t\}$-process, so then are dL_t and dX_t as well as dh_t. So $\{X_t\}$ and $\{h_t\}$ are likewise adapted to the filtration of $\{P_t\}$, and therefore are Markov processes as well, only dependent on the present information.[14] Other than the given information, $\{L_t\}$ is restricted to nonnegative values.[15]

3.1 Adjustment strategies with respect to the restrictions

The transaction costs for adjustments of work force H and F can be interpreted as barriers of labor turnover. The economically efficient firm will change its work force, once

[10]Within the present framework, no formal distinction is made, whether these fixed costs are beneficial to the employees or just upkeep costs of a work station.

[11]Note that $k \in [0;1]$, where $k = 1$ represents the case of full absorption of additional non-wage labor cost[12], as they depend on the full-time equivalent wages. The presented wage structure depicts the most complex case. It can easily be shown that given full absorption of non-wage labor cost differentials, full-time work or the absence of fixed labor costs, the wage composition is gradually simplified to the basic product of working time and the wage rate.

[13]The work force then still decreases by the attrition rate δ.

[14]This information is the past development, leading to the present state P_t and the properties of the underlying distribution function determinant of its potential trajectories. See Seppelfricke (1996), p. 181.

[15]Given a sufficiently negative progress of $\{P_t\}$, even a negative amount of labor might be a mathematically optimal reaction. Under a realistic choice of the set of parameters however production will almost never reach zero level and is then defined to be suspended.

the marginal revenue product of its factors of production deviates from their marginal costs. The marginal revenue product of work force (MRPW) is henceforth denoted by η_T. It is derived as the partial derivative of the revenue function (7) with respect to the work force L:

$$\frac{\partial R_t}{\partial L_t} = \mu P_t A_t L_t^{\mu-1} {h_t}^{\gamma} = \eta_t. \tag{10}$$

Changes in working hours h_t on the other hand affect φ, the marginal revenue product of hours of work (MRPH):

$$\frac{\partial R_t}{\partial h_t} = \gamma P_t A_t L_t^{\mu} {h_t}^{\gamma-1} = \varphi_t. \tag{11}$$

A variation of work force affects the marginal costs of production[16]

$$\frac{\partial C_t\left(L_t, h_t\right)}{\partial L_t} = W_t(h_t) = wh\left(1+s\right) + w(1-h_t)s(1-k) + f, \tag{12}$$

equivalent to the total labor costs of each single employee.[17] If the firm readjusts its hours of work instead, production costs vary according to

$$\frac{\partial C_t\left(L_t, h_t\right)}{\partial h_t} = \left(w\left(1+sk\right)\right) L_t. \tag{13}$$

If the marginal revenue exceeds the marginal costs of labor, an increase in the volume of labor is remunerative, if it is lower, employees are being laid off or hours of work are being reduced. If such transactions are costless waiting is irrational. The firm will immediately react to any changes. That is the case, if the strategy of working hour adjustments is chosen or if transfers are free of costs in terms of $H = 0$ and $F = 0$.

3.1.1 Absence of transaction cost

To simplify matters the special case of no transaction cost will be analyzed first, so for this brief section it shall be assumed that $H = 0$ as well as $F = 0$.[18] Given the preceding definition the optimal ${h_t}^*$hours of work, according to section 6.2 are

$$h_t^* = \frac{\left(\gamma-\mu\right)\left(ws\left(1-k\right)+f\right)}{\mu\left(w\left(1+sk\right)\right)}. \tag{14}$$

As they are solely dependent on the wage rate w, the fixed costs f, the surcharges s, which are all defined as constants, and the likewise unvarying economies of scale, equilibrium hours of work must be constant as well. A trade-off between a change of

[16]For explicit comments to this statement see section 6.1.

[17]The term "‘compensation of employees"’ should be avoided here, as no statement has been made about the benefit of the fixed costs of labor f to the employee.

[18]This case does not necessarily represent a laissez faire policy as transaction cost also contains cost of recruitment and qualification, which are primarily determined by the present state of the underlying labor market.

working hours or work force then does not exist and the equilibrium amount of labor force ${L_t}^*$ is according to[19]

$$ {L_t}^*(P_t, A_t) = \left[\frac{\mu^{\gamma-1} \left(w\left(1+sk\right)\right)^{2-\gamma}}{\gamma P_t A_t \left[\left(\gamma-\mu\right)\left(ws\left(1-k\right)+f\right)\right]^{\gamma-1}} \right]^{\frac{1}{\mu-1}} \tag{15} $$

determined by the price P_t and the current level of productivity A_t. As no working time adjustment takes place and the labor input is independent of the constant working hours, it can already be shown that short-time work does not have any retarding effect on layoffs.

3.1.2 hiring and firing effects

If on the contrary transaction costs become due when hiring or firing takes place, results may differ considerably. If the MRPW falls below the marginal costs, causing negative marginal profits, the firm may still hold on to the work force at risk. This stand-by state is practiced as long as the expected present value of future losses exceeds the critical shadow value – the firing costs F. On the other side a firm will delay recruiting new staff even if it experiences positive marginal profits, if the costs of hiring h are yet unreasonably high. The thus spanned window of inactivity widens, the higher the barriers of the transaction costs are in relation to the alternative operating costs of production. Formally it can be stated that[20]

$$ dX_t < 0 \text{ if} \qquad E_t \left\{ \int_t^{\infty} \left(\eta_\tau - W\right) e^{-(r+\delta)(\tau-t)} d\tau \right\} = -F, \tag{16} $$

$$ dX_t = 0 \text{ if} \qquad -F < E_t \left\{ \int_t^{\infty} \left(\eta_\tau - W\right) e^{-(r+\delta)(\tau-t)} d\tau \right\} < H \tag{17} $$

and

$$ dX_t > 0 \text{ if} \qquad E_t \left\{ \int_t^{\infty} \left(\eta_\tau - W\right) e^{-(r+\delta)(\tau-t)} d\tau \right\} = H. \tag{18} $$

So the firm is evaluating expectations for any future point in time τ in the present instant of time t and optimizes its cumulative present values of future profits by instantaneous regulation of labor input. In the presence of hiring costs H and firing costs F, the firm will adjust its factor input, once the opportunity costs – the present values of the unrealized marginal profits or the tolerated marginal losses – pass the barriers determined by H and F. As shown in section 6.4, the equations (16) and (18) can be rewritten to

$$ dX_t < 0 \text{ if} \qquad E_t \left\{ \int_t^{\infty} \eta_\tau e^{-(r+\delta)(\tau-t)} d\tau \right\} = \frac{W}{(r+\delta)} - F $$

and

$$ dX_t > 0 \text{ if} \qquad E_t \left\{ \int_t^{\infty} \eta_\tau e^{-(r+\delta)(\tau-t)} d\tau \right\} = \frac{W}{(r+\delta)} + H. $$

[19]See section 6.2 for an explicit derivation of this function.
[20]See section 6.4.

3.1.3 The option of short-time work

If in conformity with a potential labor agreement short-time work is an option, its introduction can be beneficial to the firm. The basic acceptance of short-time work by the employee representation in times of economic crisis is assumed within the model framework.[21] Working time is then regulated, to keep marginal profits at the break even level, which is accomplished by maintaining the strategy[22]

$$h_t{}^*(P_t, A_t, L_t) = \left[\frac{w\,(1+sk)}{\gamma P_t A_t L_t{}^{\mu-1}}\right]^{\frac{1}{\gamma-1}}. \tag{19}$$

If this adaption is costless, an immediate response to any fluctuation in pricing is possible. This strategy will be chosen, if transfers of work force are impossible or, in the presence of high transaction costs would result in disproportionate losses. If, as henceforth will be assumed, both strategies – hiring and firing as well as short-time work – are possible, the firm will follow a more sophisticated regulation.

3.1.4 Dominant strategies

Which of both strategies is chosen depends on the specific costs of adaption. As in times of crisis layoffs and reduction of working hours are predominant, the focus of the following treatment will be on cutbacks, as shown in case (16).[23] A downward change of operational hours of work is equivalent to a cost reduction in the amount of

$$\frac{\frac{\partial C_t(L_t,h_t)}{\partial h_t}}{r+\delta} = \frac{(w\,(1+sk))\,L_t}{r+\delta}.$$

The above expression is the cumulative variable part of wages per instant of time, paid for each employee over an infinite time of operation. The perpetuity $r+\delta$ again supports the continuous nature of the framework, giving present values of all future wages. This difference in working hours is saved by the firm at all times in the future, its value however is discounted by the risk-free interest rate r and the exogenous deterministic labor attrition rate δ, which stands for the cumulative relative frequency that wages are saved anyway, because the marginal work force at risk has already quit or has retired. If on the other hand a layoff is considered, then the cumulative wages

$$\frac{\frac{\partial C_t(L_t,h_t)}{\partial L_t}}{r+\delta} - F = \frac{wh\,(1+s) + w(1-h_t)s(1-k) + f}{r+\delta} - F$$

are saved. Comparing these savings to the ones obtained through short-time work, the benefit of layoffs is that the fixed part of labor costs f as well as the additional non-wage

[21] A corresponding optimization calculus for the employee's decision-making process is neglected, to maintain simplicity.

[22] See appendix 6.3.

[23] A quite similar application to expanding business activity, resulting in recruitment and even overtime work, can be done, but will not be considered in this model, to support comprehensibility.

labor costs of canceled hours of work can be avoided. The however disadvantageous aspect of firing is, that the one-time firing cost F is applied. So initially the trade-off is in preference to short-time work, if cost reduction of a working time adjustment exceeds the difference of cumulative total labor costs of a single employee and the firing costs. In the simple case of full absorption of additional non-wage labor cost through a state STC program (i.e. $k = 1$), short-time work is preferred, as long as

$$\frac{(w\,(1+s))\,L_t \Delta h_t}{r+\delta} \geq \frac{w\,(1+s)\,h \Delta L_t + f}{r+\delta} - F,$$

resp.

$$(w\,(1+s))\,L_t \Delta h_t \geq w\,(1+s)\,h \Delta L_t + f - F(r+\delta). \tag{20}$$

Given the feasible supposition, that either way of adjustment is related to the same amount of a total volume of labor, or, to put it differently, that $L_t \Delta h_t = h \Delta L_t$, it can already be checked, that short-time work is advantageous, as long as $F > \frac{f}{r+\delta}$, then the sum of future fixed labor costs is less than the immediate firing costs. Short-time work then again is unremunerative, if costs of dismissal are rather small or nonexistent or if the fixed upkeep costs of a work station or fixed benefits of a labor agreement are quite extensive.

This brief examination however is an initial oversimplification of matters, as it does not consider the way the expected cumulative MRPW (henceforth abbreviated to ecMRPW), that is to say the option value of the otherwise forfeit future production, is shifted by the presence and conditions of the window of inactivity. As shown in (10), the correlation between the MRPW and the operational labor force is negative. While the ecMRPW can easily be forecast using the drift rate of prices and the constants r and δ in the absence of transaction costs, this is no longer coherent, if transfers are costly. This is particularly relevant, if transaction costs are asymmetrical. If the barrier of inactivity in one direction of adjustment is significantly closer to the present allocation of work force and working hours, in relation to the other barrier, the probability is higher that this first barrier is overrun more frequently as time progresses than the barrier in the opposite direction. Overrunning a barrier however will cause transactions, thus resetting the ecMRPW. This more probable incidence causes a bias in the expected value. Furthermore, MRPW levels beyond this barrier will not be realized and their corresponding trajectories are no longer elements of the filtration of the underlying MRPW process. The probability distribution of the process is affected by the presence of these barriers.[24] Keeping in mind this intuitive idea, the difference of the ecMRPW of both strategies of adjustment must be incorporated into the framework. Thus in the following section, a modified ecMRPW in terms of a regulated stochastic process will be derived.

[24] To promote comprehensibility, a brief example shall be illustrated: In the case of very high hiring costs and low firing costs, the MRPW trajectory, following the characteristics of a Geometric Brownian Motion with drift, will most likely underrun the firing barrier more frequently than it overruns the hiring barrier. As the underrunning process is reflected at the boundary at an early stage, even lower levels of the MRPW are never being realized, thus the ecMRPW is positively biased compared to a state of no regulation.

3.2 Formal solution

If hiring and firing are costly, the firm will no longer react to minor price changes and consequently changes in revenues. In the absence of transaction costs, $\{X\}$ is a direct mapping of $\{P\}$ solely defined by the constant μ and the variables W and A_t. As $\{P\}$ is a stochastic process, so is its mapping $\{X\}$. In the presence of transaction costs $\{X\}$ will react buffered as predefined environmental conditions occur, i.e. in this specific model as a certain amount of opportunity cost has accumulated. The instant time of transaction is following the principle of a stopping time.[25] The resulting process can be interpreted as a regulated stochastic process, generated from $\{P\}$ utilizing techniques of stochastic control theory. For that purpose a regulated process is defined as a function of the underlying unregulated stochastic process and a regulator. In the present framework the regulated stochastic process of the ecMRPW, which is controlled by labor adjustments, shall exclusively become active when the present MRPW, determined by the unregulated process of pricing, is about to underrun or overrun the window of inactivity, that is when dD_t or dU_t takes place. Then dD_t and dU_t quantify the extent of the underrun or overrun within the instant of time.[26] Dependent on dD_t and dU_t, which are accumulated to D_t and U_t in terms of the filtered progression through time, lower and upper barrier values d and u respectively evolve for each present MRPW. In order to determine these barriers in general and their specific values, a generic regulator needs to be defined. In the case of a Geometric Brownian Motion with drift, following Bentolila/Bertola (1990), the regulator takes the form

$$\xi_t = \zeta_t \frac{D_t}{U_t}. \tag{21}$$

The thereby defined regulated process shall meet the following properties:

1. $\{\zeta_t\}$ is a stochastic process in terms of a Geometric Brownian Motion. It equals $d\zeta_t = \zeta_t \vartheta dt + \zeta_t \sigma dB_t$, where ϑ and σ are constant drift and variance parameters and dB_t are the increments of a standard Wiener process. For the initial value of the regulated process ξ_0 the condition $d \leq \xi_0 \leq u$ holds true,

2. $\{U_t\}$ and $\{D_t\}$ are increasing and continuous processes with initial values $D_0 = U_0 = 1$,

3. $\{D_t\}$ increases exclusively and yet always, when $\xi_t = d$. Likewise $\{U_t\}$ increases exclusively and yet always, when $\xi_t = u$. Furthermore d and u meet the condition $d, u \in \mathbb{R}^+$,

4. $d \leq \xi_t \leq u \quad \forall\ t \geq 0$.

[25] See Koralov/Sinaj (2007), p. 187 for an illustrative approach to stopping times. A more formal definition of a stopping time with respect to a filtered probability space, can be found in Athreya/Lahiri (2006), p. 405-406 and Shorack (2000), p. 305.

[26] In continuous time the quantification must be seen as a mathematical formality, as regulation takes place at an infinitesimal level.

Taking the logarithm, the regulated process can be reformed to a differential, where, following Harrison (1985), p. 22, $\{D_t\}$ and $\{U_t\}$ are uniquely defined. According to the product rule of Itô processes,[27] eq. (21) can be expressed as the differential

$$d\xi_t = \frac{D_t}{U_t} d\zeta_t + \frac{\zeta_t}{U_t} dD_t - \frac{\zeta_t D_t}{U_t^2} dU_t. \tag{22}$$

Higher-order derivatives are ignored here.[28]

Applying the Itô formula in Harrison (1985), p. 74, to $dg(\xi_t)$ yields the generic form:

$$\begin{aligned} dg(\xi_t) &= g'(\xi_t)\, d\xi_t + \frac{1}{2}\, g''(\xi_t)\,(d\xi_t)^2 \\ &= \left[\vartheta g'(\xi_t)\,\xi_t + \frac{\sigma^2}{2}\, g''(\xi_t)\,\xi_t^2\right] dt + \\ &\quad + \sigma g'(\xi_t)\,\xi_t dB_t + g'(\xi_t)\,\frac{\zeta_t}{U_t} dD_t - g'(\xi_t)\,\frac{\zeta_t D_t}{U_t^2} dU_t. \end{aligned} \tag{23}$$

According to eq. (21) the term ζ_t can be expressed by ξ_t as well. As D_t only increases, when the sample path reaches the lower barrier, i.e. $\xi_t = d$, and D_t only increases, when $\xi_t = u$, in the last two summands ξ_t can be substituted for d and u:[29]

$$\begin{aligned} dg(\xi_t) = &\left[\vartheta g'(\xi_t)\,\xi_t + \frac{\sigma^2}{2}\, g''(\xi_t)\,\xi_t^2\right] dt + \\ &+ \sigma g'(\xi_t)\,\xi_t dB_t + dg'(d)\,\frac{dD_t}{D_t} - ug'(u)\,\frac{dU_t}{U_t}. \end{aligned} \tag{24}$$

Integration by parts leads to[30]

$$\begin{aligned} e^{-\lambda t} g(\xi_t) = g(\xi_0) + &\int_0^t e^{-\lambda\nu}\left[\vartheta g'(\xi_\nu)\,\xi_\nu + \left(\frac{\sigma^2}{2}\right) g''(\xi_\nu)\,\xi_\nu^2 - \lambda g(\xi_\nu)\right] d\nu + \\ &+ \int_0^t e^{-\lambda\nu}\sigma g'(\xi_\nu)\,\xi_\nu dB_t + dg'(d)\int_0^t e^{-\lambda\nu}\left(\frac{dD_t}{D_t}\right) - \\ &- ug'(u)\int_0^t e^{-\lambda\nu}\left(\frac{dU_t}{U_t}\right). \end{aligned} \tag{25}$$

Given $\lambda > 0$, $dg'(d) = 0$ and $ug'(u) = 0$, for $t \to \infty$

$$\begin{aligned} 0 &= g(\xi_0) + E_0\left\{\int_0^\infty e^{-\lambda\nu}\left[\vartheta g'(\xi_\nu)\,\xi_\nu + \left(\frac{\sigma^2}{2}\right) g''(\xi_\nu)\,\xi_\nu^2 - \lambda g(\xi_\nu)\right] d\nu + \right\} \\ -g(\xi_0) &= E_0\left\{\int_0^\infty e^{-\lambda\nu}\left[\vartheta g'(\xi_\nu)\,\xi_\nu + \left(\frac{\sigma^2}{2}\right) g''(\xi_\nu)\,\xi_\nu^2 - \lambda g(\xi_\nu)\right] d\nu + \right\} \end{aligned} \tag{26}$$

[27] See Wiersema (2008), p. 78; Harrison (1985), p. 72; Deck (2006), p. 96.

[28] Wiersema (2008), p. 76 states, that the higher-order derivatives at an infinitesimal level are minor compared to the first-order derivatives and therefore considered negligible to promote simplicity.

[29] See section 6.5.

[30] See section 6.6. For an explicit elaboration see Harrison (1985), p. 73.

holds true.

This is equivalent to the differential equation

$$-\xi_\nu = \vartheta g'(\xi_\nu)\,\xi_\nu + \left(\frac{\sigma^2}{2}\right) g''(\xi_\nu)\,\xi_\nu^2 - \lambda g(\xi_\nu)\,, \tag{27}$$

to which the general solution is[31]

$$\begin{aligned}
&E_0\left\{\int_0^\infty \xi_t e^{-\lambda t} dt; \xi_0, u, d\right\}\\
&= \frac{1}{\lambda-\vartheta}\left(\xi_0 + \frac{{\xi_0}^{\alpha_1}\left(u^{\alpha_2}d - ud^{\alpha_2}\right)}{\alpha_1\left(u^{\alpha_1}d^{\alpha_2} - u^{\alpha_2}d^{\alpha_1}\right)} + \frac{{\xi_0}^{\alpha_2}\left(ud^{\alpha_1} - u^{\alpha_1}d\right)}{\alpha_2\left(u^{\alpha_1}d^{\alpha_2} - u^{\alpha_2}d^{\alpha_1}\right)}\right)\\
&\equiv g\left(\xi_0; u, d, \vartheta, \sigma, \lambda\right).
\end{aligned} \tag{28}$$

In the framework of this model, the MRPW denotes the current observation of the regulated process, i.e. the present MRPW, so that $\xi = \eta$. The continuous discount rate λ stands for the sum of risk-free interest rate r and labor attrition rate δ, thus $\lambda = r + \delta$. The rate of growth ϑ_η of the η process and its standard deviation σ_η are gained through their causal parameters:[32]

$$\vartheta_\eta = \vartheta_a + \vartheta_p + \delta(1-\mu) \tag{29}$$

resp.

$$\sigma_\eta = \sigma_p. \tag{30}$$

Replacing the generic parameters with the ones previously specified, will yield the regulated ecMRPW process as a mapping of the current MRPW and the lower and upper barrier values d and u, determined by the transfer costs. These are the critical parameter values of the MRPW, the values that, if observed at any time, will result in an instantaneous transfer action, which then is the optimal reaction, given future expectations.

$$\begin{aligned}
&E_t\left\{\int_t^\infty \eta_\tau \cdot e^{-(r+\delta)(\tau-t)} d\tau; \eta_t, u, d\right\}\\
&= \frac{1}{r+\delta-\vartheta_\eta}\left(\eta_t + \frac{{\eta_t}^{\alpha_1}\left(u^{\alpha_2}d - ud^{\alpha_2}\right)}{\alpha_1\left(u^{\alpha_1}d^{\alpha_2} - u^{\alpha_2}d^{\alpha_1}\right)} + \frac{{\eta_t}^{\alpha_2}\left(ud^{\alpha_1} - u^{\alpha_1}d\right)}{\alpha_2\left(u^{\alpha_1}d^{\alpha_2} - u^{\alpha_2}d^{\alpha_1}\right)}\right).
\end{aligned} \tag{31}$$

To solve this equation, at least two specific values of the MRPW, that are feasible under the given set of parameters, must be known. For lack of specific information about each

[31] See section 6.7.
[32] See section 6.8.

potential process, these are the general values $\eta_t = u$ and $\eta_t = d$. Thus

$$\begin{aligned} & E_t\left\{\int_t^{\infty} u \cdot e^{-(r+\delta)(\tau-t)} d\tau; u, u, d\right\} \\ & = \frac{1}{r+\delta-\vartheta_\eta}\left(u + \frac{u^{\alpha_1}\left(u^{\alpha_2} d - u d^{\alpha_2}\right)}{\alpha_1\left(u^{\alpha_1} d^{\alpha_2} - u^{\alpha_2} d^{\alpha_1}\right)} + \frac{u^{\alpha_2}\left(u d^{\alpha_1} - u^{\alpha_1} d\right)}{\alpha_2\left(u^{\alpha_1} d^{\alpha_2} - u^{\alpha_2} d^{\alpha_1}\right)}\right) \qquad (32) \end{aligned}$$

resp.

$$\begin{aligned} & E_t\left\{\int_t^{\infty} d \cdot e^{-(r+\delta)(\tau-t)} d\tau; d, u, d\right\} \\ & = \frac{1}{r+\delta-\vartheta_\eta}\left(d + \frac{d^{\alpha_1}\left(u^{\alpha_2} d - u d^{\alpha_2}\right)}{\alpha_1\left(u^{\alpha_1} d^{\alpha_2} - u^{\alpha_2} d^{\alpha_1}\right)} + \frac{d^{\alpha_2}\left(u d^{\alpha_1} - u^{\alpha_1} d\right)}{\alpha_2\left(u^{\alpha_1} d^{\alpha_2} - u^{\alpha_2} d^{\alpha_1}\right)}\right). \qquad (33) \end{aligned}$$

Furthermore to both cases specific values for the dependent variable, that is the present value of the ecMRPW, need to be assigned. The value of the modified marginal profit function $\mathcal{G}_t^*(u)$ at both designated allocations equals zero, as, at the infinitesimal state of indifference between transaction and waiting, the marginal revenue equals marginal costs, that is the entity of future wages plus the transaction costs, as referred to in eq. (57). So

$$\mathcal{G}_t^*(u) = E_t\left\{\int_t^{\infty} u \cdot e^{-(r+\delta)(\tau-t)} d\tau; u, u, d\right\} - \frac{W_t(h_t)}{(r+\delta)} - H = 0.$$

Likewise $\mathcal{G}_t^*(d)$ depicts the case of the ecMRPW specified in eq. (56), which, if underrun, would mean unreasonable factor costs of labor. This would result in a deficit exceeding the firing costs. An immediate layoff of a worker is initiated:

$$\mathcal{G}_t^*(d) = E_t\left\{\int_t^{\infty} d \cdot e^{-(r+\delta)(\tau-t)} d\tau; d, u, d\right\} - \frac{W_t(h_t)}{(r+\delta)} + F = 0.$$

Inside the perimeter of inaction the firm can buffer fluctuation of prices utilizing working time adjustment. The positive correlation of the MRPW and the time factor h is already shown in eq. (10). As by definition $u \geq d$, consequently $h_t(u) \geq h_t(d)$ and, due to the positive correlation of hours of work and the total labor costs of an employee, following eq. (12), $W_t(h_t(u)) \geq W_t(h_t(d))$ must hold true. The more voluminous the time adjustment strategy is applied in the window of inaction, the more the window will be spread. As shown in section 6.9, the total labor costs of an employee at the barrier value are equivalent to

$$W_t(d) = \frac{\gamma}{\mu} \cdot d + ws(1-k) + f \qquad (34)$$

and

$$W_t(u) = \frac{\gamma}{\mu} \cdot u + ws(1-k) + f. \qquad (35)$$

Insertion of both conditions into the marginal profit functions, given constant parameters, the regulator is reduced to the yet unknown values d and u.

$$\mathcal{G}_t^*(u) = \frac{1}{r+\delta-\vartheta_\eta}\left(u + \frac{u^{\alpha_1}(u^{\alpha_2}d - ud^{\alpha_2})}{\alpha_1(u^{\alpha_1}d^{\alpha_2} - u^{\alpha_2}d^{\alpha_1})} + \frac{u^{\alpha_2}(ud^{\alpha_1} - u^{\alpha_1}d)}{\alpha_2(u^{\alpha_1}d^{\alpha_2} - u^{\alpha_2}d^{\alpha_1})}\right) - \\ - \frac{\frac{\gamma}{\mu}\cdot u + ws(1-k) + f}{(r+\delta)} - H = 0 \tag{36}$$

resp.

$$\mathcal{G}_t^*(d) = \frac{1}{r+\delta-\vartheta_\eta}\left(d + \frac{d^{\alpha_1}(u^{\alpha_2}d - ud^{\alpha_2})}{\alpha_1(u^{\alpha_1}d^{\alpha_2} - u^{\alpha_2}d^{\alpha_1})} + \frac{d^{\alpha_2}(ud^{\alpha_1} - u^{\alpha_1}d)}{\alpha_2(u^{\alpha_1}d^{\alpha_2} - u^{\alpha_2}d^{\alpha_1})}\right) - \\ - \frac{\frac{\gamma}{\mu}\cdot d + ws(1-k) + f}{(r+\delta)} + F = 0. \tag{37}$$

The simultaneous equations now are ready to be solved numerically. With the parameter specific values of d and u the conditions for a transfer of labor force are known. As both boundaries are solely defined by constant parameters, d and u are unvarying as well under each given set of parameters at any point in time and for every trajectory as an elementary event of the underlying probability space. Once these boundaries of inaction are determined, the optimal allocation of work force and hours of work can be assigned to every possible market development, i.e. every change in prices.

3.3 work force-working-time-allocations within the boundaries of inactivity

The optimal operational working time with respect to a given present MRPW may take values $h_t(\eta_t) \in [h_t(d); h_t(u)]$. As shown in section 6.9, it is nonetheless defined by

$$h_t{}^*(\eta_t) = \frac{\gamma}{\mu}\frac{\eta_t}{w(1+sk)},$$

as a function of η_t. As the boundaries d and u are constant, the factor of working time too can vary between the steady constant values $h_t{}^*(d)$ and $h_t{}^*(u)$, following the equations (70) and (71). So for any h_t a corresponding optimal work force exists, in terms of eq. (10), which, according to the mapping $\eta_t \mapsto h_t{}^*(\eta_t)$ can be expressed via

$$L_t{}^*(P_t, \eta_t) = (P_t A_t)^{\frac{1}{1-\mu}} \cdot \left(\frac{\mu}{\eta_t}\right)^{\frac{1-\gamma}{1-\mu}} \cdot \left(\frac{\gamma}{w(1+sk)}\right)^{\frac{\gamma}{1-\mu}} \tag{38}$$

as a dependence of L_t on the MRPW.[33] Utilizing the critical values of inactivity $\eta_t = d$ as well as $\eta_t = u$ then yields the critical amount of work force

$$L_{t,F}{}^*(P_t, d) = (P_t A_t)^{\frac{1}{1-\mu}} \cdot \left(\frac{\mu}{d}\right)^{\frac{1-\gamma}{1-\mu}} \cdot \left(\frac{\gamma}{w(1+sk)}\right)^{\frac{\gamma}{1-\mu}} \tag{39}$$

and

$$L_{t,H}{}^*(P_t, u) = (P_t A_t)^{\frac{1}{1-\mu}} \cdot \left(\frac{\mu}{u}\right)^{\frac{1-\gamma}{1-\mu}} \cdot \left(\frac{\gamma}{w(1+sk)}\right)^{\frac{\gamma}{1-\mu}}, \tag{40}$$

[33] See section 6.10.

where ${L_{t,F}}^*$ represents the tolerated maximum of work force at a specific currently observed price level, which, if theoretically exceeded, would cause immediate layoffs to the resetting allocation ${L_{t,F}}^*$. A firm on the contrary, that experiences an underrun of critical low employment ${L_{t,H}}^*$, will instantaneous start recruiting new staff, unhesitantly paying hiring costs h, until employment is back up to level ${L_{t,H}}^*$, where further hirings are suspended for the moment.[34] The distance between ${L_{t,H}}^*$ and ${L_{t,F}}^*$ can be interpreted as the range of inaction.

Finally, a brief perspective approach to the individual firm seems called for. The previous comprehension allows for a statement considering the extrema of employment at given price level in an average firm within a homogeneous branch. Nonetheless the position of a single firm is one of an initial work force. This work force is then adapted to changes in revenues of the firm, as time progresses. Given its staff, the firm can buffer a certain range of fluctuation in demand, sitting out smaller setbacks utilizing short-time work, until a severe slump may still necessitate firing. The price range that can be hibernated with present personnel can be shown, once the equations (39) and (40) are reformed:[35]

$$
{P_{t,F}}^* \left(L_t, d\right) = {L_t}^{1-\mu} {A_t}^{-1} \cdot \left(\frac{d}{\mu}\right)^{1-\gamma} \cdot \left(\frac{w\left(1+sk\right)}{\gamma}\right)^{\gamma} \tag{41}
$$

resp.

$$
{P_{t,H}}^* \left(L_t, u\right) = {L_t}^{1-\mu} {A_t}^{-1} \cdot \left(\frac{u}{\mu}\right)^{1-\gamma} \cdot \left(\frac{w\left(1+sk\right)}{\gamma}\right)^{\gamma}. \tag{42}
$$

4 Results

Using static parameter sets, representing feasible market scenarios, the strategic behavior of a firm can be analyzed numerically. The firm here will adapt its course of action to the design characteristics of a state STC program, utilizing the previously described techniques of stochastic control.

The baseline scenario is associated with a very restrictive design of a STC program that leaves the whole additional non-wage labor cost to be paid by the firm. The alternative scenario in comparison will introduce a short-time work policy that is more benevolent and grants full absorption of additional non-wage labor cost by a state authority. Both scenarios are further split into two cases. Case A describes a market environment of still moderate uncertainty of future price development, whereas case B, doubling standard deviation of the price process, puts future revenues under substantial risk. So case B can be interpreted as a phase of severe economic recession, implying increasing uncertainty of

[34] This scenario does not necessarily mean an increase in market prices, but can also be a consequence of the deterministic attrition δ, which may cause the operational labor force to drop below the tolerated minimum.

[35] See section 6.10. To avoid misunderstanding it has to be stressed that the resulting prices ${P_{t,H}}^*\left(L_t, u\right)$ and ${P_{t,F}}^*\left(L_t, d\right)$ do not suggest a dependence of the prices on the level of production, but instead mark the critical prices, that if their incidence is observed, will force activity in terms of hiring or firing.

Table 1: The examined market scenarios.

Parameters	Parameter values			
	Scenario A1	Scenario A2	Scenario B1	Scenario B2
ϑ_p	0.01	0.01	0.01	0.01
ϑ_a	0.01	0.01	0.01	0.01
σ_p	**0.1**	**0.1**	**0.2**	**0.2**
μ	0.65	0.65	0.65	0.65
γ	0.35	0.35	0.35	0.35
δ	0.05	0.05	0.05	0.05
r	0.05	0.05	0.05	0.05
f	0.8	0.8	0.8	0.8
F	1	1	1	1
H	0.166	0.166	0.166	0.166
w	1	1	1	1
s	0.2	0.2	0.2	0.2
k	**0**	**1**	**0**	**1**

expectations of future production level. The set of parameter values is shown in table 1. [36]

If a firm is taking into consideration the option of short-time work, the presence and extent of remanence cost is a crucial factor in decision making. In the baseline scenario A1 the firm at any ratio of short-time work still has to pay the full-time equivalent of non-wage labor costs. As these in this case are completely independent of the hours of work factor, they take the property of fixed cost, increasing in weight in relation to the wage rate per hour the more the operational hours of work are reduced. As a consequence the increasing unit labor cost does at a relative high level of working hours, i.e. a very moderate use of short-time work, not allow for a further decrease in working hours, resulting in layoffs at an early stage of recession.

The dependence of tolerated levels of work force on the market price of the produced good is illustrated in figure 1. The range between the graphs may be interpreted as the

[36] For better understanding concerning the choice of parameter values the following shall be stated: Early simulations have shown, that the variance of the Geometric Brownian Motion should not be weighted too disproportionate in relation to the drift, as then forecasts become unreliable. Hence the drift of the $\{P\}$-process and the deterministic growth rates are aimed to keep the drift of the $\{\eta\}$-process at a value of $\vartheta_\eta = 0.038$ and thus slightly underweight compared to the standard deviation. The parameters μ and γ are chosen to account for decreasing returns to scale, as otherwise exogenous prices would result in unbounded cumulative profits, rendering any transaction cost barrier useless. F equals one year of wages ($F = w$) and H with $H = 1/6 \cdot w$ equals two months of wages. This value is doubled compared to the original one in Bentolila/Bertola (1990), as by definition it contains additional cost of qualification. The non-wage labor costs are estimated to be 20 Prozent of the wages, a value very close to the actual rate in Germany. The parameter value of fixed labor cost f can not be specified, as it greatly differs among different branches. Thus it has been chosen for means of calibration to adjust the hiring barrier in scenario A2 to an approximate full-time equivalent.

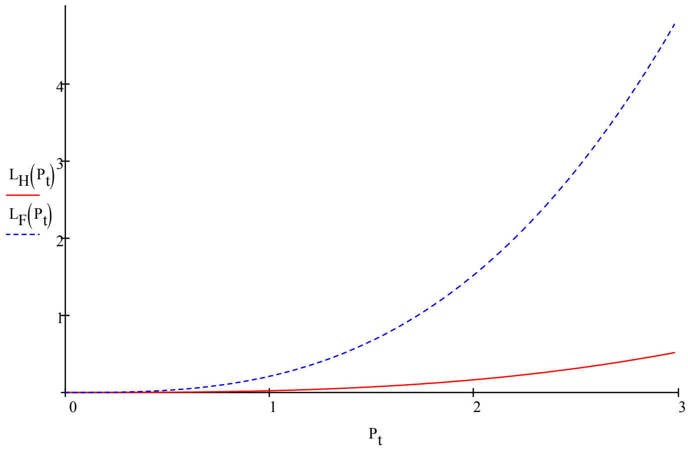

Figure 1: The barriers of inactivity at a given price level in Scenario A1.

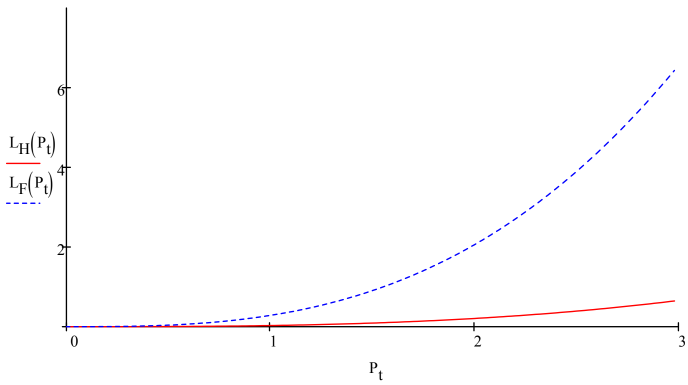

Figure 2: The barriers of inactivity at a given price level in Scenario A2.

corridor of inactivity, giving the set of all possible allocations of work force and hours of work. If the price increases or the attrition rate δ causes the labor force to drop below the L_H-curve, the firm will immediately increase employment by starting to hire. On a very abstract level the curves could also be interpreted as short-run reaction paths.[37] The L_F-curve then maps the allocations that are successively passed if, at a critically high state of employment, prices keep falling.

In comparison to figure 2 the full absorption of additional non-wage labor cost by the state increases the firing barrier and promotes labor hoarding. Under the given parameter values this effect is rather minor, as the fixed cost here is relatively high. Nonetheless the critical work force in scenario A2 is approximately 10 percent above

[37]The term "short-run" literally means an instant of time, as for any sufficiently long time interval the productivity A_t progresses. So any of the showcase pairs of boundary only applies to the given price for an infinitesimal period of time.

Parameters	Parameter values			
	Scenario A1	Scenario A2	Scenario B1	Scenario B2
$h_t(u)$	1.524	1.02	2.838	1.9
$h_t(d)$	0.461	0.296	0.346	0.223
$E_{L_H P}$	3.03%	2.899%	3.846%	3.077%
$E_{L_F P}$	2.889%	2.864%	2.888%	2.867%

Table 2: The variation of the optimal hours of work factor and of the price elasticities.

scenario A1 at given price levels. These 10 percent of workers are consequently laid off if, as in scenario A1, the difference in non-wage labor cost is added to the remanence cost.

The window of inactivity, i.e. the range between minimal and maximal tolerated work force in case A is already quite substantial given the yet moderate amount of uncertainty $\sigma_p = 0.1$.

Both scenarios differ even more in the amount of tolerated maximal labor. As can be extracted from table 2, the critical hours of work factor in both cases of uncertainty lies above the baseline scenario. In case A2 even a reduction of up to 70 percent of working hours may still be efficient compared to costly firing, hence the slope may still be hibernated with the complete initial staff. The utilization of work force must however be more than 50 percent higher in A1. If the sufficient factor of working hours drops below 46.1 percent, short-time work has lost its benefit, resulting in subsequent alternation to the firing strategy.

The hoarding of labor, promoted by transaction costs and a generous STC program is significantly intensified, if future pricing is increasingly stochastic and forecasts get even more undependable. As in case B the variance of the nondeterministic part of the increments of the price process is four times higher than in case A, the option value of waiting is so predominant, that even if the firm has to carry the whole additional non-wage labor cost, it still will hang on to short-time work up to a reduction of nearly two thirds of operational time. Thus a full-time equivalent of 34.6 percent can be realized until finally the certain and immediate payment of the firing cost is seen as advantageous. Evidently the highest extent of short-time work can be gained if under extreme uncertainty of future market conditions still another economic incentive is offered in terms of the STC program designed in Scenario B. Then the highest extent of short-time of all four showcases is given at a labor volume of but 22.3 percent.

The price elasticities of work force $E_{L_H P}$ and $E_{L_F P}$ shown in table 2 are not sufficient to back a significant influence of the remanence cost on the progression of dismissals, once the strategy of short-time work is abandoned during the downward economic slope. If a given STC program promotes the utilization of short-time work, layoffs are indeed slowed down, yet this effect is too small too allow for a strong statement. The even smoother elasticity concerning recruitment is due to the relatively high maximum of tolerated work force at low price levels and its convergence with rising prices.

5 Conclusion

Within the framework of the devised model the basic effectiveness of a subsidy on short-time work has been substantiated. Provided the compliance with a labor agreement, the window of inactivity may be widened considerably. Thus the stabilizing effect of positive long-term expectations that may be assumed for a competitive product and socially absorbable layoffs through retirement and job migration can be promoted. Severe business cycles can be hibernated with even massive cuts in working time, if then increasing fixed costs of labor are absorbed by a state program and hence the present value of uncertain future fixed costs is exceeded by certain immediate firing cost. Furthermore, layoffs can be retarded to a more severe state of recession.

The better future revenues can be forecast, the less inaction and a supposedly temporary reduction of labor time will be necessary. Yet if more weight is put on uncertainty as a key characteristic of times of crisis, an extended utilization of a STC program can be emphasized on a theoretical level.

The fact that increased uncertainty in the context of this approach has shown an employment stabilizing effect should not be seen indisputable. Yet it is obvious, that given the positive drift rate of the defined price process, long-term optimism still is an implication of this model, which per se delays transfers of costly labor, as the present value of the future prospect of production is sufficiently high.

6 Appendix

6.1 Marginal costs of labor

Consider the function (8):

$$C_t\left(L_t, h_t\right) = W\left(h_t\right) L_t = \left(wh\left(1+s\right) + w(1-h_t)s(1-k) + f\right) L_t.$$

The marginal costs of work force then are

$$\frac{\partial C_t\left(L_t, h_t\right)}{\partial L_t} = W = wh\left(1+s\right) + w(1-h_t)s(1-k) + f. \tag{43}$$

If eq. (8) on the other hand is partially differentiated with respect to working time, the marginal costs of hours of work then are

$$\begin{aligned}
\frac{\partial C_t\left(L_t, h_t\right)}{\partial h_t} &= \left(w\left(1+s\right) - ws(1-k)\right) L_t \\
&= \left(w + wsk\right) L_t \\
\frac{\partial C_t\left(L_t, h_t\right)}{\partial h_t} &= \left(w\left(1+sk\right)\right) L_t.
\end{aligned} \tag{44}$$

6.2 Optimal allocation of work force and working time in the absence of transaction costs

It shall be assumed that $H = F = 0$, i.e. that hirings as well as firings are costless. In this special case the maximization calculus is reduced to

$$V_t \equiv \max_{\{x_t\},\{h_t\}} E_t \left\{ \int_t^{\infty} e^{-r(\tau-t)} \left[\left(P_\tau A_\tau L_\tau{}^{\mu} h_\tau{}^{\gamma} - W L_\tau \right) d\tau \right] \right\}.$$

In every instant of time then profits $G(L_t, h_t) = P_t A_t L_t{}^{\mu} h_t{}^{\gamma} - W L_t$ are realized. As long as workers can be hired or fired free of cost, waiting is unreasonable. The firm will immediately optimize its work force-working-time-allocation. In perfect competition optimal factor input is given, if the price of the last unit of production equals its production cost. The marginal profit then equals zero:

$$\begin{aligned}
\frac{\partial R(L_t, h_t)}{\partial L_t} &= \frac{\partial C(L_t, h_t)}{\partial L_t} \\
\mu P_t A_t L_t^{\mu-1} h_t{}^{\gamma} &= wh(1+s) + w(1-h_t)s(1-k) + f \\
L_t^{\mu-1} &= \frac{h(w(1+s) - ws(1-k)) + ws(1-k) + f}{\mu P_t A_t h_t{}^{\gamma}} \\
L_t &= \left[\frac{h(w(1+s) - ws(1-k)) + ws(1-k) + f}{\mu P_t A_t h_t{}^{\gamma}} \right]^{\frac{1}{\mu-1}}. \qquad (45)
\end{aligned}$$

In perfect competition the same principle holds true for changes in hours of work:

$$\begin{aligned}
\frac{\partial R(L_t, h_t)}{\partial h_t} &= \frac{\partial C(L_t, h_t)}{\partial h_t} \\
\gamma P_t A_t L_t^{\mu} h_t{}^{\gamma-1} &= (w(1+sk)) L_t \\
\gamma P_t A_t L_t^{\mu-1} h_t{}^{\gamma-1} &= (w(1+sk)) \\
L_t^{\mu-1} &= \frac{w(1+sk)}{\gamma P_t A_t h_t{}^{\gamma-1}} \\
L_t &= \left[\frac{w(1+sk)}{\gamma P_t A_t h_t{}^{\gamma-1}} \right]^{\frac{1}{\mu-1}}. \qquad (46)
\end{aligned}$$

In an equilibrium both constraints must be met. Equalizing the equations (45) and (46) yields the optimal full-time factor:

$$\begin{aligned}
\left[\frac{w\left(1+sk\right)}{\gamma P_t A_t {h_t}^{\gamma-1}}\right]^{\frac{1}{\mu-1}} &= \left[\frac{h\left(w\left(1+s\right)-ws\left(1-k\right)\right)+ws\left(1-k\right)+f}{\mu P_t A_t {h_t}^{\gamma}}\right]^{\frac{1}{\mu-1}} \\
\frac{w\left(1+sk\right)}{\gamma {h_t}^{\gamma-1}} &= \frac{h\left(w\left(1+s\right)-ws\left(1-k\right)\right)+ws\left(1-k\right)+f}{\mu {h_t}^{\gamma}} \\
\frac{w\left(1+sk\right)}{\gamma {h_t}^{\gamma-1}} &= \frac{\left(w\left(1+s\right)-ws\left(1-k\right)\right)}{\mu {h_t}^{\gamma-1}} + \frac{ws\left(1-k\right)+f}{\mu {h_t}^{\gamma}} \\
\frac{1}{\gamma}\cdot w\left(1+sk\right) &= \frac{1}{\mu}\cdot\left(w\left(1+s\right)-ws\left(1-k\right)\right) + \frac{ws\left(1-k\right)+f}{\mu h_t} \\
\frac{ws\left(1-k\right)+f}{\mu h_t} &= \frac{1}{\gamma}\cdot w\left(1+sk\right) - \frac{1}{\mu}\cdot\left(w\left(1+s\right)-ws\left(1-k\right)\right) \\
\frac{\mu h_t}{ws\left(1-k\right)+f} &= \frac{\gamma}{w\left(1+sk\right)} - \frac{\mu}{\left(w\left(1+s\right)-ws\left(1-k\right)\right)} \\
\frac{\mu h_t}{ws\left(1-k\right)+f} &= \frac{\gamma}{w\left(1+sk\right)} - \frac{\mu}{w\left(1+sk\right)} \\
h_t^* &= \frac{\left(\gamma-\mu\right)\left(ws\left(1-k\right)+f\right)}{\mu\left(w\left(1+sk\right)\right)}. \qquad (47)
\end{aligned}$$

Recycling of this expression into eq. (46) shows the optimal amount of work force:

$$\begin{aligned}
L_t &= \left[\frac{w\left(1+sk\right)}{\gamma P_t A_t \left(\frac{(\gamma-\mu)(ws(1-k)+f)}{\mu(w(1+sk))}\right)^{\gamma-1}}\right]^{\frac{1}{\mu-1}} \\
{L_t}^* &= \left[\frac{\mu^{\gamma-1}\left(w\left(1+sk\right)\right)^{2-\gamma}}{\gamma P_t A_t\left[\left(\gamma-\mu\right)\left(ws\left(1-k\right)+f\right)\right]^{\gamma-1}}\right]^{\frac{1}{\mu-1}}. \qquad (48)
\end{aligned}$$

6.3 Optimum of working hours

The optimal full-time factor ${h_t}^*$ with respect to the labor force present L_t and the present price P_t is set, if the marginal profit of hours of work is zero, i.e.

$$\frac{\partial R\left(L_t, h_t\right)}{\partial h_t} = \frac{\partial C\left(L_t, h_t\right)}{\partial h_t}.$$

Equation (46) can be converted to:

$${h_t}^*(P_t, A_t, L_t) = \left[\frac{w\left(1+sk\right)}{\gamma P_t A_t {L_t}^{\mu-1}}\right]^{\frac{1}{\gamma-1}}. \qquad (49)$$

6.4 Optimal transaction calculus with respect to hiring and firing costs

The equations (16) and (18),

$$dX_t < 0 \text{ if } \qquad E_t\left\{\int_t^{\infty}(\eta_\tau - W)\, e^{-(r+\delta)(\tau-t)}d\tau\right\} = -F \tag{50}$$

and

$$dX_t > 0 \text{ if } \qquad E_t\left\{\int_t^{\infty}(\eta_\tau - W)\, e^{-(r+\delta)(\tau-t)}d\tau\right\} = H \tag{51}$$

can be rewritten to

$$dX_t < 0 \text{ if } \quad E_t\left\{\int_t^{\infty}\eta_\tau e^{-(r+\delta)(\tau-t)}d\tau\right\} - E_t\left\{\int_t^{\infty}W e^{-(r+\delta)(\tau-t)}d\tau\right\} = -F \tag{52}$$

and

$$dX_t > 0 \text{ if } \quad E_t\left\{\int_t^{\infty}\eta_\tau e^{-(r+\delta)(\tau-t)}d\tau\right\} - E_t\left\{\int_t^{\infty}W e^{-(r+\delta)(\tau-t)}d\tau\right\} = H. \tag{53}$$

The probability of total wages W being paid at future instants of time decays at a rate δ, which can be interpreted as the periodic probability of an employee quitting or retiring. Future expected value need to be further discounted by the inflation rate r. The present value of all future wages is then a perpetuity, thus both equations are

$$dX_t < 0 \text{ if } \qquad E_t\left\{\int_t^{\infty}\eta_\tau e^{-(r+\delta)(\tau-t)}d\tau\right\} - \frac{W}{(r+\delta)} = -F \tag{54}$$

and

$$dX_t > 0 \text{ if } \qquad E_t\left\{\int_t^{\infty}\eta_\tau e^{-(r+\delta)(\tau-t)}d\tau\right\} - \frac{W}{(r+\delta)} = H, \tag{55}$$

resp., after rewriting,

$$dX_t < 0 \text{ if } \qquad E_t\left\{\int_t^{\infty}\eta_\tau e^{-(r+\delta)(\tau-t)}d\tau\right\} = \frac{W}{(r+\delta)} - F \tag{56}$$

and

$$dX_t > 0 \text{ if } \qquad E_t\left\{\int_t^{\infty}\eta_\tau e^{-(r+\delta)(\tau-t)}d\tau\right\} = \frac{W}{(r+\delta)} + H. \tag{57}$$

So the present value of the expected cumulative marginal revenue product of work force at the state of hiring equals the costs of this new worker, which are the perpetuity of wages and the hiring cost. To simplify interpretation eq. (56) may also be rewritten to

$$dX_t < 0 \text{ if } \frac{W}{(r+\delta)} = E_t\left\{\int_t^{\infty}\eta_\tau e^{-(r+\delta)(\tau-t)}d\tau\right\} + F,$$

separating the benefits of the transaction to the left side and its cost to the right side of the equation, as done in eq. (57). In the incidence of a dismissal the saved future wages are then compared to the cumulative loss of production and the firing cost.

6.5 Modification of the barriers by an underrun or overrun

According to eq. (23) influence of the lower and upper boundary on the regulated process is

$$+g'(\xi_t)\frac{\zeta_t}{U_t}dD_t - g'(\xi_t)\frac{\zeta_t D_t}{U_t^2}dU_t.$$

Eq. (21) is equivalent to

$$\zeta_t = \xi_t \frac{U_t}{D_t}.$$

Insertion into the barrier terms yields

$$+g'(\xi_t)\frac{\xi_t U_t}{U_t D_t}dD_t - g'(\xi_t)\frac{\xi_t D_t U_t}{U_t^2 D_t}dU_t.$$

The process $\{D_t\}$ increases only if and yet every time the regulated process reaches the lower boundary. $\{U_t\}$ increases likewise, if ξ_t reaches the upper boundary. So if $\xi_t = d$, then $dD_t \neq 0$ and if $\xi_t = u$, then $dU_t \neq 0$. Substitution of ξ_t with d and u gives rise to

$$+dg'(d)\frac{dD_t}{D_t} - ug'(u)\frac{dU_t}{U_t}$$

and leads to eq. (24). The case, that $\xi_t \neq d$ or $\xi_t \neq u$ does not need to be considered, as the corresponding terms are multiplied to zero, if $dD_t = 0$ or $dU_t = 0$.

6.6 Integration by parts

For a generic stochastic process $\xi_t = \zeta_t \frac{D_t}{U_t}$, following the product rule of Itô processes, the differential

$$d\xi_t = \frac{D_t}{U_t}d\zeta_t + \frac{\zeta_t}{U_t}dD_t - \frac{\zeta_t D_t}{U_t^2}dU_t$$

applies, where $\zeta_t(B_t)$, $D_t(B_t)$ and $U_t(B_t)$ are stochastic processes in terms of respective mappings of a mutual Wiener process B_t. Integration by parts gives the expression in terms of integrals:

$$\xi_t = \xi_0 + \int_0^t \frac{D_t}{U_t}d\zeta_t + \int_0^t \frac{\zeta_t}{U_t}dD_t - \int_0^t \frac{\zeta_t D_t}{U_t^2}dU_t.$$

Similarly for the product $e^{-\lambda t}\xi_t$ the differential equation

$$d\left(e^{-\lambda t}\xi_t\right) = e^{-\lambda t}d\xi_t - \lambda e^{-\lambda t}\xi_t,$$

resp. the integral form[38]

$$e^{-\lambda t}\xi_t = \int_0^t e^{-\lambda t}d\xi_t - \lambda\int_0^t e^{-\lambda t}\xi_t$$

[38]See Harrison (1985), p. 73.

holds true. Integration by parts of $\left\{g\left(\xi_t\right)e^{-\lambda t}\right\}$ modifies eq. (24) to

$$\begin{aligned}
e^{-\lambda t}g\left(\xi_t\right) = g\left(\xi_0\right) &+ \int_0^t e^{-\lambda\nu}\left[\vartheta g'\left(\xi_\nu\right)\xi_\nu + \left(\frac{\sigma^2}{2}\right)g''\left(\xi_\nu\right)\xi_\nu^2 - \lambda g\left(\xi_\nu\right)\right]d\nu + \\
&+ \int_0^t e^{-\lambda\nu}\sigma g'\left(\xi_\nu\right)\xi_\nu dB_t + dg'(d)\int_0^t e^{-\lambda\nu}\left(\frac{dD_t}{D_t}\right) - \\
&- ug'(u)\int_0^t e^{-\lambda\nu}\left(\frac{dU_t}{U_t}\right).
\end{aligned}$$

Under the conditions

$$\begin{aligned}
\lambda &> 0, \\
-\infty < g\left(\xi_t\right) &< \infty, \\
-\infty < g'\left(\xi_t\right)\xi_t &< \infty, \\
dg'\left(d\right) &= 0 \\
\text{and } ug'\left(u\right) &= 0
\end{aligned}$$

the result for $t \to \infty$ is

$$\begin{aligned}
\overbrace{e^{-\lambda t}}^{\to 0}\overbrace{g\left(\xi_t\right)}^{\neq|\infty|} = g\left(\xi_0\right) &+ \int_0^t e^{-\lambda\nu}\left[\vartheta g'\left(\xi_\nu\right)\xi_\nu + \left(\frac{\sigma^2}{2}\right)g''\left(\xi_\nu\right)\xi_\nu^2 - \lambda g\left(\xi_\nu\right)\right]d\nu + \\
&+ \int_0^t e^{-\lambda\nu}\sigma\,\overbrace{g'\left(\xi_\nu\right)\xi_\nu}^{\neq|\infty|}\,dB_t + \overbrace{dg'(d)}^{=0}\int_0^t e^{-\lambda\nu}\left(\frac{dD_t}{D_t}\right) - \\
&- \overbrace{ug'(u)}^{=0}\int_0^t e^{-\lambda\nu}\left(\frac{dU_t}{U_t}\right).
\end{aligned}$$

6.7 Solution of the differential equation

The expected cumulative marginal revenue product of work force is given for $\{\xi_t\} = \{\eta_t\}$ according to the generic regulator

$$g(\xi) = \frac{\xi}{\lambda - \vartheta} + \frac{\xi^{\alpha_1}\left(u^{\alpha_2}d - ud^{\alpha_2}\right)}{\left(\lambda - \vartheta\right)\alpha_1\left(u^{\alpha_1}d^{\alpha_2} - u^{\alpha_2}d^{\alpha_1}\right)} + \frac{\xi^{\alpha_2}\left(ud^{\alpha_1} - u^{\alpha_1}d\right)}{\left(\lambda - \vartheta\right)\alpha_2\left(u^{\alpha_1}d^{\alpha_2} - u^{\alpha_2}d^{\alpha_1}\right)}$$

as a mapping of the $\{\xi\}$ process and the lower and upper boundaries d and u to the regulated process $g\left(\xi\right)$. It is given by the generic solution

$$g(\xi) = \frac{1}{\lambda - \vartheta}\left(\xi + B_1\xi^{\alpha_1} + B_2\xi^{\alpha_2}\right),$$

of the differential equation (25), to which the final form is determined by the constants of integration B_1 and B_2, being

$$B_1 = \frac{u^{\alpha_2} d - u d^{\alpha_2}}{\alpha_1 \left(u^{\alpha_1} d^{\alpha_2} - u^{\alpha_2} d^{\alpha_1}\right)}, \tag{58}$$

$$B_2 = \frac{u d^{\alpha_1} - u^{\alpha_1} d}{\alpha_2 \left(u^{\alpha_1} d^{\alpha_2} - u^{\alpha_2} d^{\alpha_1}\right)}, \tag{59}$$

and the general solutions α_1 and α_2 to the second degree polynomial equation

$$\left(\frac{\sigma^2}{2}\right)\alpha^2 + \left(\vartheta - \left(\frac{\sigma^2}{2}\right)\right)\alpha - \lambda = 0.$$

The solutions α_1 and α_2 are determined by the parameters of the distribution of the generic regulated process ξ and equal

$$\alpha_1 \equiv \left(\frac{1}{\sigma^2}\right)\left[\left(\left(\frac{\sigma^2}{2}\right) - \vartheta\right) + \sqrt{\left(\vartheta - \left(\frac{\sigma^2}{2}\right)\right)^2 + 2\sigma^2\lambda}\right] \tag{60}$$

resp.

$$\alpha_2 \equiv \left(\frac{1}{\sigma^2}\right)\left[\left(\left(\frac{\sigma^2}{2}\right) - \vartheta\right) - \sqrt{\left(\vartheta - \left(\frac{\sigma^2}{2}\right)\right)^2 + 2\sigma^2\lambda}\right]. \tag{61}$$

In this specific model these parameters are the parameters of the $\{\eta\}$ process, i.e. the MRPW:

$$\alpha_1 \equiv \left(\frac{1}{\sigma_\eta^2}\right)\left[\left(\left(\frac{\sigma_\eta^2}{2}\right) - \vartheta_\eta\right) + \sqrt{\left(\vartheta_\eta - \left(\frac{\sigma_\eta^2}{2}\right)\right)^2 + 2\sigma_\eta^2\,(r+\delta)}\right] \tag{62}$$

resp.

$$\alpha_2 \equiv \left(\frac{1}{\sigma_\eta^2}\right)\left[\left(\left(\frac{\sigma_\eta^2}{2}\right) - \vartheta_\eta\right) - \sqrt{\left(\vartheta_\eta - \left(\frac{\sigma_\eta^2}{2}\right)\right)^2 + 2\sigma_\eta^2\,(r+\delta)}\right]. \tag{63}$$

In this case the substitution with the specific parameters shows the regulator

$$E_t\left\{\int_t^\infty \eta_\tau \cdot e^{-(r+\delta)(\tau-t)} d\tau; \eta_t, u, d\right\}$$
$$= \frac{1}{r+\delta-\vartheta_\eta}\left(\eta_t + \frac{{\eta_t}^{\alpha_1}\left(u^{\alpha_2} d - u d^{\alpha_2}\right)}{\alpha_1\left(u^{\alpha_1} d^{\alpha_2} - u^{\alpha_2} d^{\alpha_1}\right)} + \frac{{\eta_t}^{\alpha_2}\left(u d^{\alpha_1} - u^{\alpha_1} d\right)}{\alpha_2\left(u^{\alpha_1} d^{\alpha_2} - u^{\alpha_2} d^{\alpha_1}\right)}\right).$$

6.8 The parameters of the distribution of the MRPW

The MRPW is a function of the stochastic $\{P_t\}$ process, hence $\{\eta_t\}$ too is a stochastic process. Its distribution parameters are defined by the determinant deterministic and stochastic variables. Following eq. (10) the MRPW is equal to

$$\eta_t = \mu P_t A_t L_t^{\mu-1} {h_t}^\gamma.$$

The standard deviation of η_t is simply given by

$$\sigma_\eta = \sigma_p \tag{64}$$

based on its direct proportionality. For the expected value of future instants of time τ the equivalent equations[39]

$$E_t(\eta_\tau) = \eta_t e^{(\tau-t)\left(\vartheta_\eta - \frac{1}{2}\sigma_\eta{}^2\right)} \tag{65}$$

$$E_t(\eta_\tau) = \mu \cdot E_t(A_\tau) \cdot E_t(P_\tau) \cdot E_t(L_\tau)^{\mu-1} \cdot E_t(h_\tau)^\gamma \tag{66}$$

hold true. The deterministically evolving labor productivity A_τ has the growth rate ϑ_a, thus in continuous time $A_\tau = A_t e^{\vartheta_a(\tau-t)}$. The hours of work have no independent trend: $E_t(h_\tau) = h_t$. The labor force is subject to a deterministic decay through retirement, extraordinary dismissal and job migration initiated by the employee. All these phenomena are combined to the attrition rate δ. The expected value is $E_t(L_\tau) = L_t e^{-\delta(\tau-t)}$. Eq. (66) can be expressed as

$$\begin{aligned} E_t(\eta_\tau) &= \mu \cdot \left(A_t e^{\vartheta_a(\tau-t)}\right) \cdot \left(P_t e^{\left(\vartheta_p - \frac{1}{2}\sigma_p{}^2\right)(\tau-t)}\right) \cdot \left(L_t e^{-\delta(\tau-t)}\right)^{\mu-1} \cdot h_t{}^\gamma \\ &= \left(\mu \cdot A_t P_t \cdot L_t^{\mu-1} \cdot h_t{}^\gamma\right) \cdot e^{\vartheta_a(\tau-t)} \cdot e^{\left(\vartheta_p - \frac{1}{2}\sigma_p{}^2\right)(\tau-t)} \cdot e^{-\delta(\tau-t)(\mu-1)}. \end{aligned}$$

The term in brackets is equal to the MRPW according to eq. (10), so that

$$\begin{aligned} E_t(\eta_\tau) &= \eta_t \cdot e^{\vartheta_a(\tau-t) + \left(\vartheta_p - \frac{1}{2}\sigma_p{}^2\right)(\tau-t) + \delta(\tau-t)(1+\mu)} \\ &= \eta_t \cdot e^{(\tau-t)\cdot\left(\vartheta_a + \vartheta_p - \frac{1}{2}\sigma_p{}^2 + \delta(1-\mu)\right)}. \end{aligned}$$

Inserting $E_t(\eta_\tau)$ from eq. (65) yields

$$\eta_t e^{(\tau-t)\left(\vartheta_\eta - \frac{1}{2}\sigma_\eta{}^2\right)} = \eta_t \cdot e^{(\tau-t)\cdot\left(\vartheta_a + \vartheta_p - \frac{1}{2}\sigma_p{}^2 + \delta(1-\mu)\right)},$$

and, after further simplification and logarithmic calculus

$$\vartheta_\eta - \frac{1}{2}\sigma_\eta{}^2 = \vartheta_a + \vartheta_p - \frac{1}{2}\sigma_p{}^2 + \delta(1-\mu).$$

Substitution of the standard deviation in eq. (64) eliminates the standard deviation terms in

$$\vartheta_\eta - \frac{1}{2}\sigma_p{}^2 = \vartheta_a + \vartheta_p - \frac{1}{2}\sigma_p{}^2 + \delta(1-\mu),$$

finally resulting in the drift of the MRPW

$$\vartheta_\eta = \vartheta_a + \vartheta_p + \delta(1-\mu). \tag{67}$$

[39]For the explicit solutions familiarize with Ross (2007), p. 631-632 and Wiersema (2008), p. 105-108.

6.9 Variation of total costs of work force in the window of inactivity

If working time adjustments are a locally dominant strategy the amount of work force and the hours of work factor, following eq. (46), are interdependent:

$$L_t(h_t) = \left[\frac{w\,(1+sk)}{\gamma P_t A_t {h_t}^{\gamma-1}}\right]^{\frac{1}{\mu-1}}.$$

The MRPW in eq. (10) may alternatively be expressed independent of L_t, solely as a function of h_t:

$$\begin{aligned}\eta_t(L_t,h_t) &= \mu P_t A_t L_t^{\mu-1} {h_t}^{\gamma}\\ \eta_t(h_t) &= \mu P_t A_t \left[\left[\frac{w\,(1+sk)}{\gamma P_t A_t {h_t}^{\gamma-1}}\right]^{\frac{1}{\mu-1}}\right]^{\mu-1} {h_t}^{\gamma}\\ \eta_t(h_t) &= \frac{\mu}{\gamma} w\,(1+sk)\,h_t.\end{aligned} \tag{68}$$

Rewriting yields the hours of work factor, that is the respective optimum for any individual allocation in the window of inaction

$${h_t}^{*}(\eta_t) = \frac{\gamma}{\mu}\frac{\eta_t}{w\,(1+sk)}, \tag{69}$$

which is now solely expressed in terms of the MRPW η_t. For the critical MRPW values d and u, marking the boundaries of inactivity, the corresponding hours of work factors are defined by

$${h_t}^{*}(d) = \frac{\gamma}{\mu}\frac{d}{w\,(1+sk)} \tag{70}$$

and

$${h_t}^{*}(u) = \frac{\gamma}{\mu}\frac{u}{w\,(1+sk)}. \tag{71}$$

Now eq. (69) is applied, to reform the total cost of employment

$$W_t(h_t) = wh_t\,(1+s) + w(1-h_t)s(1-k) + f,$$

which in eq. (12) depends on h_t, to a function solely defined by the MRPW:

$$\begin{aligned}W_t(\eta_t) &= w\left[\frac{\gamma}{\mu}\cdot\frac{\eta_t}{w\,(1+sk)}\right](1+s) + w\left(1-\left[\frac{\gamma}{\mu}\cdot\frac{\eta_t}{w\,(1+sk)}\right]\right)s\cdot(1-k)+f\\ &= \frac{\gamma}{\mu}\cdot\eta_t\cdot\frac{(1+s)}{(1+sk)} + ws(1-k) - \frac{\gamma}{\mu}\cdot\eta_t\cdot\frac{s(1-k)}{(1+sk)} + f\\ &= \frac{\gamma}{\mu}\cdot\eta_t\cdot\frac{1+s-s+sk)}{(1+sk)} + ws(1-k) + f\\ W_t(\eta_t) &= \frac{\gamma}{\mu}\cdot\eta_t + ws(1-k) + f.\end{aligned} \tag{72}$$

When again η_t is being substituted by the boundary values d and u, the critical total costs or to put it differently the marginal costs of the work force at the allocations of firing and hiring are found:

$$W_t(d) = \frac{\gamma}{\mu} \cdot d + ws(1-k) + f \tag{73}$$

and

$$W_t(u) = \frac{\gamma}{\mu} \cdot u + ws(1-k) + f. \tag{74}$$

6.10 Range of prices and employment within the boundaries of strategic optimality

Insertion of the optimal hours of work factor with respect to the MRPW[40]

$$h_t{}^*(\eta_t) = \frac{\gamma}{\mu} \frac{\eta_t}{w\,(1+sk)}$$

into eq. (10) yields the function of the optimal amount of labor force at any given MRPW:

$$\begin{aligned}
\eta_t &= \mu P_t A_t L_t^{\mu-1} \left[\frac{\gamma}{\mu} \frac{\eta_t}{w\,(1+sk)} \right]^{\gamma} \\
L_t^{1-\mu} &= \mu^{1-\gamma} P_t A_t \eta_t{}^{\gamma-1} \left(\frac{\gamma}{w\,(1+sk)} \right)^{\gamma} \\
L_t^{1-\mu} &= \left(\frac{\mu}{\eta_t} \right)^{1-\gamma} P_t A_t \left(\frac{\gamma}{w\,(1+sk)} \right)^{\gamma} \\
L_t{}^* \left(P_t, \eta_t\right) &= \left(P_t A_t\right)^{\frac{1}{1-\mu}} \left(\frac{\mu}{\eta_t} \right)^{\frac{1-\gamma}{1-\mu}} \left(\frac{\gamma}{w\,(1+sk)} \right)^{\frac{\gamma}{1-\mu}} .
\end{aligned} \tag{75}$$

Expressing the same statement in terms of P_t and replacing the generic variable η_t with the boundary specific values d and u finally shows the lower and upper barriers of prices

$$P_{t,F}{}^* \left(L_t, d\right) = L_t{}^{1-\mu} A_t{}^{-1} \cdot \left(\frac{d}{\mu} \right)^{1-\gamma} \cdot \left(\frac{w\,(1+sk)}{\gamma} \right)^{\gamma}$$

and

$$P_{t,H}{}^* \left(L_t, u\right) = L_t{}^{1-\mu} A_t{}^{-1} \cdot \left(\frac{u}{\mu} \right)^{1-\gamma} \cdot \left(\frac{w\,(1+sk)}{\gamma} \right)^{\gamma}$$

that, given initial labor force, will be realized without hiring or operational dismissals.

6.11 Expected effect on employment

To account for the effects of STW on

[40]See section 6.9, eq. (69).

Um Aussagen über die durchschnittliche Beschäftigung bzw. Beschäftigung eines durchschnittlichen Unternehmens innerhalb der durch die gegebenen Parameter skizzierten Branche treffen zu können, muss ein Personalniveau herangezogen werden, dass dem arithmetischen Mittel innerhalb der Barrieren entspricht. Der regulierte stochastische Prozess des Faktoreinsatzes L_t muss also mit Wahrscheinlichkeit $0,5$ über bzw. unter dem interessierenden Niveau $\bar{L}$ liegen:[41]

$$P\left(\bar{L} \leq L_F\right) = P\left(L_H \leq \bar{L}\right) = \int_{\bar{L}}^{L_H} f_L(L; P, A) dL = 0,5.$$

Bentolila/Bertola (1990), p. 289 f. folgend, ist die Dichtefunktion einer regulierten Geometrischen Brownschen Bewegung $\{\xi\}$

$$f(\xi) = \frac{(\varphi - 1)\xi^{\varphi-2}}{u^{\varphi-1} - d^{\varphi-1}} 1_{[d \leq \xi \leq u]}, \tag{76}$$

wobei allgemein $\varphi \equiv \frac{2\vartheta}{\sigma^2}$ bzw. in diesem Fall $\varphi = \frac{2\vartheta_\eta}{\sigma_\eta{}^2}$. Die bedingte Dichte von L ist weiterhin

$$f_L(L; P, A) = f_\eta\left[\eta(L; P, A)\right] \frac{\partial \eta(L; P, A)}{\partial L}$$
$$f_L(L; P, A) = f_\eta\left[\eta(L; P, A)\right] \cdot (\mu - 1)\mu P A h^\gamma L^{\mu-2},$$

unter Verwendung der partiellen Ableitung von eq. (10). Die Anwendung der Dichtefunktion (76) für $\xi \equiv \eta$ und anschließende Substitution von η gemäß eq. (10) ergibt:[42]

[41] Nachfolgende Bezeichnungen f und F stehen innerhalb dieses Abschnittes für Dichte- bzw. Verteilungsfunktion, nicht für die Modellparameter. Weiterhin muss gesondert darauf hingewiesen werden, dass sich die Verteilung von L, anders als üblich, vom oberen Wert L_F zum unteren L_H kumuliert, was darin begründet liegt, dass von η abgebildet wird, womit niedrige Werte von η auf hohe Werte von L abgebildet werden.

[42] Der Übersichtlichkeit halber wird nun auf den Gebrauch der Indikatorfunktion verzichtet, da die Möglichkeit, Werte jenseits der Barrieren zu erreichen ohnehin aufgrund der Regulierung ausgeschlossen werden kann.

$$
\begin{aligned}
f_L(L;P,A) &= f_\eta\left[\eta(L;P,A)\right]\cdot(\mu-1)\mu PAh^\gamma L^{\mu-2}, \\
&= \frac{(\varphi-1)\eta^{\varphi-2}}{u^{\varphi-1}-d^{\varphi-1}}\cdot(\mu-1)\mu PAh^\gamma L^{\mu-2} \\
&= \frac{(\varphi-1)\left(\mu P_tA_tL_t^{\mu-1}h_t\left(u\right)^\gamma\right)^{\varphi-2}}{u^{\varphi-1}-d^{\varphi-1}}\cdot(\mu-1)\mu PAh^\gamma L^{\mu-2} \\
&= \frac{(\varphi-1)\left(\mu P_tA_tL_t^{\mu-1}h_t\left(u\right)^\gamma\right)^{\varphi-2}}{u^{\varphi-1}-d^{\varphi-1}}\cdot(\mu-1)\left(\mu P_tA_tL_t^{\mu-1}h_t{}^*\left(u\right)^\gamma\right)L^{-1} \\
&= \frac{(\varphi-1)\left(\mu P_tA_tL_t^{\mu-1}h_t\left(u\right)^\gamma\right)^{\varphi-1}}{u^{\varphi-1}-d^{\varphi-1}}\cdot(\mu-1)L^{-1} \\
&= \frac{(\varphi-1)(\mu P_tA_th_t\left(u\right)^\gamma)^{\varphi-1}L^{(\mu-1)(\varphi-1)}}{u^{\varphi-1}-d^{\varphi-1}}\cdot(\mu-1)L^{-1} \\
&= \frac{(\mu-1)(\varphi-1)(\mu P_tA_th_t\left(u\right)^\gamma)^{\varphi-1}}{u^{\varphi-1}-d^{\varphi-1}}L^{(\mu-1)(\varphi-1)-1}
\end{aligned}
$$

Wie eingangs erwähnt wurde, gilt für $P\left(L_H\leq\bar{L}\right)=0,5$. Dann gilt weiterhin

$$
\begin{aligned}
E_t(L) &= \int_{L_F}^{L_H} Lf(L)dL \\
&= \int_{L_F}^{L_H} L\frac{(\mu P_tA_th_t\left(u\right)^\gamma)^{\varphi-1}}{u^{\varphi-1}-d^{\varphi-1}}(\mu-1)(\varphi-1)L^{(\mu-1)(\varphi-1)-1}dL \\
&= \left[\frac{(\mu P_tA_th_t\left(u\right)^\gamma)^{\varphi-1}}{u^{\varphi-1}-d^{\varphi-1}}\cdot\frac{(\mu-1)(\varphi-1)}{(\mu-1)(\varphi-1)+1}\cdot L^{(\mu-1)(\varphi-1)+1}\right]_{L_F}^{L_H} \\
&= \frac{(\mu P_tA_th_t\left(u\right)^\gamma)^{\varphi-1}}{u^{\varphi-1}-d^{\varphi-1}}\cdot\frac{(\mu-1)(\varphi-1)}{(\mu-1)(\varphi-1)+1}\cdot\left[L^{(\mu-1)(\varphi-1)+1}\right]_{L_F}^{L_H} \\
&= \left[\frac{(\mu P_tA_th_t\left(u\right)^\gamma)^{\varphi-1}}{u^{\varphi-1}-d^{\varphi-1}}\cdot\frac{(\mu-1)(\varphi-1)}{(\mu-1)(\varphi-1)+1}\right]\cdot\left({L_H}^{(\mu-1)(\varphi-1)+1}-L_F^{(\mu-1)(\varphi-1)+1}\right).
\end{aligned}
$$

The expression of the second fraction $\frac{(\mu-1)(\varphi-1)}{(\mu-1)(\varphi-1)+1}$

$$
\begin{aligned}
&= [\ldots] \cdot \left((P_t A_t)^{\frac{1}{1-\mu}} \cdot \left(\frac{\mu}{u}\right)^{\frac{1-\gamma}{1-\mu}} \cdot \left(\frac{\gamma}{w\,(1+sk)}\right)^{\frac{\gamma}{1-\mu}} - \right)^{(\mu-1)(\varphi-1)+1} \\
&- \left(\left((P_t A_t)^{\frac{1}{1-\mu}} \cdot \left(\frac{\mu}{d}\right)^{\frac{1-\gamma}{1-\mu}} \cdot \left(\frac{\gamma}{w\,(1+sk)}\right)^{\frac{\gamma}{1-\mu}} \right)\right)^{(\mu-1)(\varphi-1)+1} \\
&= [\ldots] \cdot \left(\left((P_t A_t)^{\frac{1}{1-\mu}}\, \mu^{\frac{1-\gamma}{1-\mu}} \cdot \left(\frac{\gamma}{w\,(1+sk)}\right)^{\frac{\gamma}{1-\mu}} \right) \left(u^{\frac{\gamma-1}{1-\mu}} - d^{\frac{\gamma-1}{1-\mu}} \right)\right)^{(\mu-1)(\varphi-1)+1} \\
&= \frac{(\mu P_t A_t h_t\,(u)^{\gamma})^{\varphi-1}}{u^{\varphi-1} - d^{\varphi-1}} \cdot \frac{(\mu-1)(\varphi-1)}{(\mu-1)(\varphi-1) + \frac{(\mu-1)}{(\mu-1)}} \cdot (L_H - L_F)^{(\mu-1)(\varphi-1)+1}
\end{aligned}
\tag{77}
$$

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