

A Lifecycle Model with Human Capital, Labor
Supply and Retirement
(Preliminary and Incomplete)

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Abstract

We develop and estimate a Ben-Porath human capital model in which individuals make decisions on consumption, human capital investment, labor supply, and retirement. The model allows for both an endogenous wage process (which is typically assumed exogenous in the retirement literature) and an endogenous retirement decision (which is typically assumed exogenous in the human capital literature). We estimate the model using the Method of Simulated Moments to match the life-cycle profiles of wages and hours from the SIPP data. The model replicates the main features of the data.

KEYWORDS: human capital, Ben-Porath, labor supply, retirement

JEL Classification: J22, J24, J26

1. Introduction

The [Ben-Porath \(1967\)](#) model of lifecycle human capital production and the lifecycle labor supply model are two of the most important models in labor economics. While the former is the dominant framework used to rationalize wage growth over the life-cycle, the latter has been used to study hours worked over the life-cycle. Quite surprisingly, aside from the seminal work in [Heckman \(1976\)](#) there has been little effort integrating these two important paradigms. This paper attempts to fill this void by estimating a lifecycle model in which workers make human capital and labor supply decisions jointly. Workers acquire human capital on the job. Perhaps the most important aspect of our model is that we do not treat retirement as a separate decision. It occurs endogenously as part of the lifecycle labor supply decision. While most work to date on the lifecycle human capital model aims to explain the wage growth early in the lifecycle, there has been surprisingly little work combining human capital with labor supply and retirement. We estimate a model that is rich enough to explain both the lifecycle pattern of wages as well as the lifecycle pattern of labor supply-focusing on retirement and wage patterns at the end of working life.

The retirement literature typically takes the wage process as given and estimates the date of retirement. One typically sees wages fall substantially before retirement. Raw wages for people who work fall by over 25% between ages 55 and 65. In the retirement literature, this trend is critical in explaining retirement behavior. Lifecycle human capital models provide a very different perspective. They take the retirement date as given, but model the formation of the wage process. We merge these two literatures by allowing for both endogenous retirement and an endogenous wage process. In addition, we also incorporate health shocks to investigate how important health shocks are relative to human capital depreciation in rationalizing retirement decisions. While most work to date on the lifecycle human capital model aims to explain the wage growth early in the lifecycle, there has been surprisingly little work combining human capital with labor supply wherein the labor supply, wage and retirement choices are rationalized in one unified setting.

Specifically, we develop and estimate a Ben-Porath type human capital model in which workers make consumption, human capital investment, and labor supply decisions. We estimate the model using the Method of Simulated Moments (MSM), matching the wage and hours profiles from the Survey of Income and Program Participation (SIPP). With a parsimonious lifecycle model in which none of the parameters explicitly depend upon age or experience, we are able to replicate the main features of the data. In particular we match the large increase in wages and very small increase in labor supply at the beginning of the lifecycle as well as the small decrease in wages but very large

decrease in labor supply at the end of the lifecycle. The key to our ability to fit both parts of the lifecycle is human capital depreciation. In a simple model without human capital or depreciation, there would be no a priori reason for workers to concentrate their leisure at the end of the lifecycle. However, as soon as we bring in the possibility that human capital might depreciate, this is no longer the case. If workers take time off in the middle of their career, their human capital will fall substantially and they will make much less when they return to the labor market. However, if this period of nonworking occurs at the end of the career this is no longer the case. Another way to see the same point is that if a middle age worker stops working for a while, they will generally return to the labor market and build up their wages. However, as a result of the shorter time horizon, it might not be worth it for an older worker to re-enter at a lower wage so they continue to stay out of the labor market. Allowing for exogenous human capital accumulation across the lifecycle without depreciation will not be enough to explain the patterns. If the tastes for leisure does not vary across the labor market the standard model cannot simultaneously reconcile the small increase in labor supply and large increase in wages at the beginning of the lifecycle and the small decrease in wages and large decrease in labor supply at the end. Of course if one exogenously allowed both wages and labor supply to depend upon age in a completely flexible way one could easily fit the joint pattern. Moreover, it is not clear that this model would have any testable implications so we cannot reject it. The goal of this paper is to try to fit the profiles without resorting to arbitrary taste preferences and exogenous wage variation.

An interesting aspect of our model is that even though the preference for leisure does not vary systematically over the lifecycle, we do find that measured "labor supply elasticities" do vary over the lifecycle. In our dynamic model, the shadow cost of not working much higher early in the lifecycle (as pointed out by [Imai and Keane \(2004\)](#)) but in our model it is also lower for older workers as opposed to peak earners. We find that early in the lifecycle the measured labor supply elasticity is low for younger workers, around 0.3. However, workers around standard retirement age are more sensitive to wage fluctuations with elasticities around 0.8. Once the model has been estimated, we can use it to measure the impacts of Social Security policy. The U.S. government currently faces a huge debt and it is no understatement to say that addressing this problem is one of the primary issues facing policy makers in Washington. Moreover, with the aging of the baby boom generation this problem is going to get worse. The amount that we spend on Social Security and Medicare is not sustainable without major increases in funding for them. Given this, many programs have been proposed to address these problems and eventually something must be done.

Much serious work has been developed to quantitatively estimate the economic con-

sequences of aging population and evaluate the remedy policies (Rust and Phelan, 1997; French, 2005; French and Jones, 2011). Haan and Prowse (2012) directly estimate the effect of increasing life expectancy on retirement decisions. They model retirement as a result of declining wages and increasing actuarial unfairness of the Social Security and pension system. However, from our model one can see that there is a major issue in the previous retirement literature. They typically take the wage process as given and focus on the retirement itself. For example, when conducting the counterfactual experiment of delaying Social Security Normal Retirement Age (NRA) from age 65 to age 67, all previous literature takes the same age-wage profile as in the baseline model where the NRA is at age 65 and re-estimates the retirement behavior under the new environment where the NRA is 67.

As the wage has already been declining significantly approaching the previous NRA of 65, under the new policy of NRA at 67 working is not likely attractive for many workers since the wage further declines between age 65 and 67. It is assumed away that such policy change will also affect the age-wage profile. If one is expecting a NRA at age 67 instead of age 65 and retiring later is optimal, then she or he will certainly try to keep the wage high during the extra working time. Omitting such channel will likely generate bias in the counterfactual policy experiments. After estimating the baseline model, we conduct two sets of counterfactual policy experiments. In the first experiment we remove the Social Security earnings test which is effective for age 62 to 70. In the second experiment we delay the NRA two years (from age 65 to 67). We find in both counterfactual policy experiments workers invest more in their human capital at old ages, which leads to at least more than 20% increase in wages near retirement. Omitting such channel will almost surely bias the results otherwise.

The remainder of the paper is organized as follows. Section 2 reviews relevant literature in human capital models and retirement. Section 3 develops the Ben-Porath model with labor supply and retirement. Section 4 describes the estimation strategy and data. Section 5 presents parameter estimates in the baseline model matching the SIPP data. Section 6 discusses model implications and Section 7 simulates the effects of several policies. Section 8 concludes.

2. Relevant Literature

(This section is rough and incomplete)

There is a large and growing literature on many aspects of retirement. In these models, typically retirement is induced either by increasing utility toward leisure (Gustman and Steinmeier, 1986) or similarly increasing disutility toward labor supply (Blau, 2008).

[Gustman and Steinmeier \(2006\)](#) and [Gustman and Steinmeier \(2009\)](#) estimate the impact of Social Security policies on retirement in structural models with increasing utility toward leisure. [Haan and Prowse \(2012\)](#) estimate the extent to which the increase in life expectancy affects retirement. [Blau \(2008\)](#) evaluates the role of uncertain retirement ages in the retirement-consumption puzzle.

Retirement can also be induced by declining wages at old ages and/or fixed costs of working. [Rust and Phelan \(1997\)](#) estimate a dynamic life-cycle labor supply model with endogenous retirement decisions to study the effect of Social Security and Medicare in retirement behavior. [French \(2005\)](#) estimates a more comprehensive model including saving to study the effect of Social Security and pension as well as health in retirement decisions. [French and Jones \(2011\)](#) evaluate the role of health insurance in shaping retirement behavior. [Casanova \(2010\)](#) studies the joint retirement decision among married couples. [Prescott et al. \(2009\)](#) and [Rogerson and Wallenius \(2010\)](#) present models where retirement could be induced by a convex effective labor function or fixed costs.

In all the literature listed above—theoretical or empirical, the wage process is assumed to be exogenous. That is, even when the environment changes while conducting counterfactual experiments, for example changing the Social Security policies, the wage process is kept the same and only the response in the retirement timing is studied.

On the other hand, human capital models have been accepted widely to explain the life-cycle wage growth as well as the labor supply and income patterns. In his seminal paper, [Ben-Porath \(1967\)](#) develops the human capital model with the idea that individuals invest in their human capital "up front." In what follows we use the two terms—"human capital model" and "Ben-Porath model" —interchangeably. [Heckman \(1976\)](#) further extends the model and presents a more general human capital model where each individual makes decisions on labor supply, investment and consumption. In both papers, each individual lives for finite periods and the retirement age is fixed. In their recent paper, [Manuelli et al. \(2012\)](#) extend the Ben-Porath model to include the endogenous retirement decision. All three theoretical models are deterministic.

Relative to the success in theory, there hasn't been as much work empirically estimating the Ben-Porath model. [Mincer \(1958\)](#) derives an approximation of the Ben-Porath model and greatly simplifies the estimation with a quadratic in experience, which is used in numerous empirical papers estimating the wage process ([Heckman et al. \(2006\)](#) survey the literature). Early work on explicit estimation of the Ben-Porath model was done by [Heckman \(1976\)](#); [Haley \(1976\)](#), and [Rosen \(1976\)](#). [Heckman et al. \(1998\)](#) is a more recent attempt to estimate the Ben-Porath model. They utilize the implication of the standard Ben-Porath model where at old ages the investment is almost zero. However, this implication does not hold any more when the retirement is uncertain, where each individual

always has incentive to invest a positive amount in human capital. Other more recent work includes [Taber \(2002\)](#) who incorporates progressive income taxes into the estimation and [Kuruscu \(2006\)](#) who estimates the model nonparametrically. [Browning et al. \(1999\)](#) survey much of this literature.

Another type of human capital model, the learning-by-doing model, draws relatively more attention in the empirical work. In the learning-by-doing model human capital accumulates exogenously as long as an individual works-thus they only impact their human capital accumulation through the work decision. In these models the total return from labor supply is not only the direct wage income at current time, but also includes all the extra wage income from the augmented human capital in all future time. [Shaw \(1989\)](#) is among the first to empirically estimate the learning-by-doing model, using the PSID model and utilizing the Euler equations on consumption and labor supply with translog utility. [Keane and Wolpin \(1997\)](#) and [Imai and Keane \(2004\)](#) are two examples of many that directly estimate a dynamic life-cycle with learning-by-doing. These papers assume an exogenously fixed retirement age. [Wallenius \(2009\)](#) points out that such a learning-by-doing model does not fit the pattern of wages and hours well at old ages.

3. Model

We present and estimate a Ben-Porath human capital model with endogenous labor supply and retirement in which individuals make decisions on consumption, human capital investment, and labor supply (including retirement as a special case). For simplicity we suppress the individual subscript i for all variables.

3.1 Set-up

Each individual lives N periods, from t_0 to T . One period is defined as one year. At the beginning of the first period, each individual is endowed with an initial asset $A_{t_0} \in \mathbb{R}$, human capital level $H_{t_0} \in \mathbb{R}^+$ and health status $S_{t_0} \in \{0, 1\}$ with 1 being in good health and 0 in bad health.

The health status evolves exogenously according to a time-dependent Markov process. All stochastic shocks are realized at the beginning of each period before any decision is made.¹

At each period the individual decides how to use the time endowment which is normalized to one. His work choice is binary, either he works or stays home. If he chooses to work, an individual also makes decision on how much time, I_t , to invest in human

¹Literally, S_t , γ_t and ξ_t , as described later.

capital and spends the rest, $1 - I_t$, at effective (or productive) working from which the wage income is earned. If an individual chooses not to work, he enjoys leisure solely and cannot invest in human capital.

The flow utility at time t is

$$u_t(c_t, \ell_t) = \frac{c_t^{1-\eta_c}}{1-\eta_c} + \gamma_t \ell_t \quad (1)$$

where c_t is consumption and $\ell_t \in \{0, 1\}$ is leisure. The coefficient γ_t represents taste for leisure and is assumed to be a function of health status plus a stochastic shock that varies over time. We describe the exact process in the next subsection.

Human capital is produced according to the production function

$$H_{t+1} = (1 - \sigma) H_t + \zeta_t \pi I_t^\alpha H_t^\beta \quad (2)$$

where H_t is the human capital level at period t and I_t is the time investment. The ζ_t is an idiosyncratic shock to the human capital innovation. We assume it is i.i.d and follows a log-normal distribution,

$$\log(\zeta_t) \sim \mathcal{N} \left(-\frac{\log(\sigma_\zeta^2 + 1)}{2}, \log(\sigma_\zeta^2 + 1) \right) \quad (3)$$

which implies that ζ_t has mean 1 and variance σ_ζ^2 .

The labor market is perfectly competitive. The wage for the effective labor supply equals the rent of the human capital, wH_t , where w is the human capital rent. Thus pre-tax income at any point in time is $wH_t(1 - \ell_t)(1 - I_t)$.

The social security enrollment decision is a one time decision. That is once a person turns 62 they can start claiming social security. We will let sa_t be a binary decision variable indicating whether a person starts claiming at time t and let ss_t be a state variable indicating whether a person began claiming prior to period t . Thus $ss_{t_0} = 0$ and $ss_t = 1$ if $ss_{t-1} = 1$ or if $sa_{t-1} = 1$ and is zero otherwise. Claiming is irreversible, so once $ss_t = 1$ then sa_t is no longer a choice variable. An individual collects benefits ssb_t which are a function of claiming age and average indexed monthly earnings when $ss_t = 1$. The sa_t decision is independent of the labor force participation decision ℓ_t . That is, one can choose to receive the social security benefit while working (subject to applicable rules such as earnings test). The $AIME_t$ is the Average Indexed Monthly Earnings.

Each individual faces a budget constraint

$$A_{t+1} = (1 + r)A_t + Y_t(wH_t(1 - \ell_t)(1 - I_t), ssb_t) - c_t + \tau_t, \quad (4)$$

where A_t is asset and r is the risk free interest rate. $Y_t(\cdot)$ is the after-tax income which is a function of wage income, the social security benefit ssb_t , and the tax code. Each individual makes separate decisions for the labor supply and the social security benefit application. That is, she or he can choose to take social security benefit while working and subject to the Social Security earnings test if applicable. Government transfers, τ_t , provide a consumption floor \underline{c} as in (Hubbard et al., 1995) so

$$\tau_t = \max \{0, \underline{c} - ((1+r)A_t + Y_t)\}. \quad (5)$$

The life ends at period $T + 1$ and each individual values the bequest in the form of

$$b(A_{T+1}) = b_1 \frac{(b_2 + A_{T+1})^{1-\eta_c}}{1-\eta_c} \quad (6)$$

where b_1 captures the relative weight of the bequest and b_2 determines its curvature as in (DeNardi, 2004).

3.2 Solving the model

The timing of the model works as follows: At the beginning of each period leisure shocks, γ_t , and innovations in health status, S_t , are realized by the agent. He then simultaneously chooses consumption, labor supply, human capital investment, and social security application. After these decisions are made, ξ_t is drawn which determines the human capital level in the following period.

The recursive value function can be written as

$$V_t(X_t, \gamma_t) = \max_{c_t, \ell_t, I_t, sa_t} \{u_t(c_t, \ell_t, \gamma_t) + \delta E[V_{t+1}(X_{t+1}, \gamma_{t+1}) | X_t, c_t, \ell_t, I_t, sa_t]\} \quad (7)$$

where $X_t = \{A_t, S_t, H_t, AIME_t, ss_t\}$ is the vector of state variables. We assume there is no serial correlation in the stochastic shocks, $\{\gamma_t\}$ other than through health.

It is easiest to solve the model by dividing it into two stages. First solve for the optimal choices conditional on the labor supply decision and then second calculate the labor supply decision.

The optimal consumption $c_{t,0}(X_t)$, investment $I_{t,0}(X_t)$, and social security $sa_{t,0}(X_t)$ claiming decisions conditional on participating in the labor market ($\ell_t = 0$) depend only on X_t and can be obtained from

$$\{c_{t,0}(X_t), I_{t,0}(X_t), sa_{t,0}(X_t)\} \equiv \arg \max_{c_t, I_t, sa_t} \left\{ \frac{c_t^{1-\eta_c}}{1-\eta_c} + \delta E[V_{t+1}(X_{t+1}, \gamma_{t+1}) | X_t, c_t, \ell_t = 0, I_t, sa_t] \right\} \quad (8)$$

and the conditional value function is

$$V_{t,0}(X_t) \equiv \frac{(c_{t,0}(X_t))^{1-\eta_c}}{1-\eta_c} + \delta E[V_{t+1}(X_{t+1}, \gamma_{t+1}) | X_t, c_{t,0}(X_t), \ell_t = 0, I_{t,0}(X_t), sa_{t,0}(X_t)] \quad (9)$$

Similarly, conditional on not working ($\ell_t = 1$), we can calculate the optimal consumption and claiming decision from

$$\{c_{t,1}(X_t), sa_{t,1}(X_t)\} \equiv \arg \max_{c_t, sa_t} \left\{ \frac{c_t^{1-\eta_c}}{1-\eta_c} + \gamma_t + \delta E[V_{t+1}(X_{t+1}, \gamma_{t+1}) | X_t, c_t, \ell_t = 0, I_t = 0, sa_t] \right\} \quad (10)$$

and define the value function apart from γ_t to be

$$V_{t,1}(X_t, \gamma_t) \equiv \frac{c_{t,1}(X_t)^{1-\eta_c}}{1-\eta_c} + \delta E[V_{t+1}(X_{t+1}, \gamma_{t+1}) | X_t, c_{t,1}(X_t), \ell_t = 0, I_t = 0, sa_{t,1}(X_t)]. \quad (11)$$

Notice that since we assume there is no serial correlation in the stochastic shocks $\{\gamma_t\}$, the policy and value functions do not depend on γ_t . Therefore the individual works if

$$V_{t,0}(X_t) \geq V_{t,1}(X_t) + \gamma_t.$$

This means that there exists a threshold value $\gamma_t^*(X_t)$ such that

$$l_t = \begin{cases} 1, & \text{if } \gamma_t > \gamma_t^*(X_t) \\ 0, & \text{if } \gamma_t \leq \gamma_t^*(X_t) \end{cases} \quad (12)$$

where the threshold value $\gamma_t^*(X_t)$ is

$$\gamma_t^*(X_t) = V_{t,0}(X_t) - V_{t,1}(X_t) \quad (13)$$

We can also calculate the conditional expectation of the value function as

$$E[V_t(X_t, \gamma_t) | X_t] = \text{Prob}(\gamma_t \leq \gamma_t^*(X_t)) V_{t,0}(X_t) + \text{Prob}(\gamma_t > \gamma_t^*(X_t)) [V_{t,1}(X_t) + E(\gamma_t | \gamma_t > \gamma_t^*(X_t))] \quad (14)$$

Assume the parametric form for γ_t ,

$$\gamma_t = \exp(a_0 + a_s S_t + a_\varepsilon \varepsilon_t) \quad (15)$$

where ε_t follows an independent and identically-distributed (iid) standard normal distribution. Therefore γ_t follows a log-normal distribution, $\ln \gamma_t \sim \mathcal{N}(a_0 + a_s S_t, a_\varepsilon^2)$. Then we can calculate the threshold value of ε_t as

$$\varepsilon_t^*(X_t) \equiv \frac{1}{a_\varepsilon} \{ \ln [V_{t,0}(X_t) - V_{t,1}(X_t)] - a_0 - a_s S_t \} \quad (16)$$

We know that a property of log normal random variables is that

$$\begin{aligned} E(\gamma_t \mid \gamma_t > \gamma_t^*(X_t)) &= E(\gamma_t \mid \varepsilon_t > \varepsilon_t^*(X_t)) \\ &= \exp\left(a_0 + a_s S_t + \frac{a_\varepsilon^2}{2}\right) \frac{\Phi(a_\varepsilon - \varepsilon_t^*(X_t))}{\Phi(-\varepsilon_t^*(X_t))} \end{aligned}$$

So

$$\begin{aligned} E[V_t(X_t, \gamma_t) \mid X_t] &= \Phi(\varepsilon_t^*) V_{t,0}(X_t) \\ &\quad + (1 - \Phi(\varepsilon_t^*)) \left[V_{t,1}(X_t) + \exp\left(a_0 + a_s S_t + \frac{a_\varepsilon^2}{2}\right) \frac{\Phi(a_\varepsilon - \varepsilon_t^*(X_t))}{\Phi(-\varepsilon_t^*(X_t))} \right] \end{aligned}$$

Finally note that X_{t+1} is a known function of $X_t, c_t, \ell_t, I_t, sa_t$ and ζ_t , so to solve for

$$E[V_{t+1}(X_{t+1}, \gamma_{t+1}) \mid X_t, c_t, \ell_t, I_t, sa_t] = E[E(V_{t+1}(X_{t+1}, \gamma_{t+1}) \mid X_{t+1}) \mid X_t, c_t, \ell_t, I_t, sa_t]$$

we just need to integrate over the distribution of ζ_t .

4. Estimation

4.1 Pre-set Parameters

One period is defined as one year.² The model starts at age 18 and ends at age 90. The early retirement age is 62 and the normal retirement age is 65. The time endowment available for labor supply at each period is normalized as one.

The rent rate of human capital is normalized to be one. The risk free real interest rate is set as $r = 0.03$ and the time discount rate is set as $\delta = 0.97$. The coefficient of constant relative risk aversion in the utility of consumption is set as $\eta_c = 4.0$.

The consumption floor is set as $\underline{c} = 2.19$, following [French and Jones \(2011\)](#).³

The parameter which determines the curvature of the bequest function is set as $b_2 = 300$. This number is close to [French \(2005\)](#) where he sets $b_2 = 250$ or [French and Jones \(2011\)](#) where they estimate $b_2 = 222$.

We assume all individuals start off their adult life with no wealth and zero level of AIME.

These normalized or pre-set parameters are summarized in [Table 1](#).

²Mid-year retirement might be an issue. However, more than half of workers are never observed working half-time approaching retirement, so it would not be a big issue.

³ $\underline{c} = 4380/2000 = 2.19$ since we normalize the total time endowment for labor supply at one period as one.

Table 1: Parameters normalized or pre-set.

Parameters		Normalized/Pre-set Values
H rent	w	1.0
Interest rate	r	0.03
Discount	δ	0.97
CRRA	η_c	4.0
Initial wealth	A_{t_0}	0.0
Consumption floor	\underline{c}	2.19
Bequest shifter	b_2	300.0

4.2 Estimation Procedure

The remaining parameters are to be estimated using the method of simulated moments,

$$\Theta = \left\{ \underbrace{b_1}_{\text{bequest}}, \underbrace{a_0, a_s, a_\epsilon}_{\text{leisure}}, \underbrace{\sigma, \pi, \alpha, \beta, \sigma_\xi}_{H \text{ production}}, \underbrace{\bar{H}_{18}, \sigma_{H_{18}}}_{\text{initial value}} \right\}$$

We do not try to match moments in consumption or asset as those are not the focus of this paper. Of course the marginal utility of consumption plays an important role in determining the optimal human capital accumulation. Given total wealth level, the consumption allocation across periods are jointly determined by η_c , A_{t_0} , \underline{c} , b_1 , and b_2 . Separately identifying these parameters require matching moments in consumption or asset. For this reason we fix η_c , A_{t_0} , \underline{c} , and b_2 , and only estimate b_1 . As robustness check, later we will vary these parameters and see how they affect our results.

We apply the Method of Simulated Moments (MSM) to estimate the parameters of interest, Θ , according to the following procedure.

1. Calculate the moments from the data.
2. Simulate individuals, generating initial conditions (the human capital level) as well as stochastic shocks (leisure, health, human capital innovation) at each period for each simulated individual.
3. Iterate on the following procedure for different sets of parameters of Θ until the minimum distance has been found.
 - (a) Given a set of parameters, solve value functions and policy functions for the entire state space grid.
 - (b) Generate the life-cycle profile for each simulated individual.

- (c) Calculate the simulated moments, and the distance between the simulated moments and the data moments.

4.3 Data and Moments

The main data we use in estimation is the Survey of Income and Program Participation (SIPP). The SIPP is comprised of a number of short panels of respondents and we use all of the panels starting with the 1984 panel and ending with the 2008 panel. To focus on as homogeneous a group as possible, the sample includes white male high school graduates only. For the difference of labor force participation rates between workers with good health and those with bad health, we use the Health and Retirement Survey (HRS) data.

Our measure of labor force participation is whether the individual worked during the survey month. Clearly the aggregation is imperfect. We could use participation in a year, but this would miss much of the extensive labor supply decisions of men. Ideally we would estimate the model at the monthly level, but this is not computationally feasible. One could think of our model as operating at the monthly level but for computational reasons we only solve it at the annual basis.

For workers, we construct the hourly wage as the earnings in the survey month divided by the total number of hours worked in the survey month.

Four sets of moment conditions at each age from 22 to 65 (except the second set) are chosen to represent the life-cycle profiles.

- (1). The labor force participation rates;
- (2). The difference of labor force participation rates between workers with good health and workers with bad health, from age 55 to 65.
- (3). The first moments of the logarithm of observed wages;
- (4). The first moments of the logarithm of wages after controlling for individual fixed effects.

Figure 1 a-c presents these four profiles. Figure 1b plots two profiles of the difference in LFPR, one from the HRS data and the other one is the smoothed profile by regression on age polynomials. We match the smoothed profile.

The observed wage in the model is defined as

$$W_t = wH_t(1 - I_t) \tag{17}$$

We assume this is the wage actually observed by econometrician and used to match the data moments.

We match both age-wage profiles, with and without controlling for individual fixed effect, for the following reason.

The discrepancy between the age-wage profile with or without controlling for individual fixed effects has been documented in various data sets, including the National Longitudinal Survey of Older Men (NLSOM) data (Johnson and Neumark, 1996), the Panel Study of Income Dynamics (PSID) data (Rupert and Zanella, 2012), and the Health and Retirement Survey (HRS) data (Casanova, 2013). All of this work find that after controlling for individual fixed effects the age-wage profile is much more flat than the hump-shaped age-wage profile estimated using pooling observations, and it does not decline until 60s or late 60s. All three of these papers argue that this evidence is not consistent with the traditional human capital model, since the traditional human capital model would predict a hump-shaped age. When the human capital depreciation outweighs the investment, wage starts to decline and therefore generates a hump-shaped profile.

To verify this result we compare our SIPP results with the Current Population Survey (CPS) data. From the CPS Merged Outgoing Rotation Groups (MORG) data, we match the same respondent in two consecutive surveys⁴ using the method proposed in Madrian and Lefgren (2000), and we have a short panel with each individual interviewed twice, one year apart. We construct a similar short panel from the CPS March Annual Social and Economic Supplement files (March). The difference is that the wage information is collected from the reference week in the CPS MORG data and from the previous year in the CPS March data.

Figure 2d presents the age-wage profiles with or without controlling for individual fixed effects for male high school graduates from the CPS MORG data and the CPS March data. We find very similar discrepancy in the age-wage profiles. The age-wage profiles for male high school graduates in the SIPP data present similar pattern, as shown in Figure 1c.

Our model is able to reconcile such discrepancy in the age-wage profiles, as we show in the next section.

5. Estimation Results

The estimates of parameters are listed in Table 2. The model fits the data fairly well, as shown in Figures 3a-3d.

The simulated labor force participation rates fit the data generally well (Figure 3a), even though they are a little bit off at old ages, suggesting human capital depreciation might not be the only factor inducing massive retirement at those ages. We do not view this as surprising. We have tried to focus on a very simple model that focuses on the main idea. Adding many other realistic components to the model presumably would allow us

⁴For MORG data, they are the fourth and eighth interview.

to fit the places where we miss. The main point here is that our simple model can reconcile the main facts: a small increase in labor supply/large increase in wages at the beginning of the lifecycle along with the large decrease in labor supply/small decrease in wages at the end of the lifecycle. One should keep in mind that this might be a limitation on our policy couterfactus as adding other features to obtain a better fit might impact those simulations.

The age-wage profile from the model after controlling for individual fixed effects fits data very well (Figure 3b). The fit with the mean wages is reasonable (Figure 4d). This model is able to generate the discrepancy between the age-wage profiles with or without controlling for fixed effects, as shown in Figure 4a. The fit with labor supply hits the main features but is not perfect, so does the fit with the difference of labor supply between workers with good health and bad health (Figure 3d). Please notice that the difference is quite small in the data.

The separation of observed labor ($1 - \ell_t$) and effective labor ($1 - \ell_t - I_t$), as in Equation (17), has several advantages. First of all, it helps generate the pattern that the working hours profile peaks earlier than the wage profile (Weiss, 1986), as shown in Figure 4b. The working hours increases slightly with age when the worker is young, with a large portion devoted to human capital investment. The working hours profile peaks around age 35 (actually it is pretty flat between age 30 and 40) and starts declining at age 40. However, with proportionally less time devoted to human capital investment and most time to effective labor supply (Figure 4c), the observed wage keeps increasing from young to age 50 and does not decline as much as the labor supply between age 50 and 65.

More importantly, such separation helps generate retirement at old ages. As shown in Figure 4d, at old ages the actual human capital level has already depreciated to a relatively low level (lower than the initial level at age 18), but the observed wage level is still quite high due to the ability of adjusting time allocation between investment and effective working. However, over age 60, each individual has already allocated most time in the effective working, there is no further room for such adjustment. This implies that the observed wage declines almost at the same speed as human capital depreciates, which leads to massive retirement at old ages.

This also explains why the depreciation rate of human capital is quite high in our estimation, $\sigma = 10.4\%$, comparing with 2.4% in Manuelli et al. (2012).

6. Elasticity

In this subsection, we calculate elasticities of labor supply from the model. Since we assume discrete labor supply choice, the elasticity of labor supply on the intensive margin

Table 2: Estimates in the baseline model.

		Estimates
Leisure: constant	a_0	-5.920
Leisure: health	a_s	0.552
Leisure: stochastic	a_ϵ	0.552
H depreciation	σ	0.104
H innovation coef	π	1.322
I factor	α	0.497
H factor	β	0.391
H innovation shock, sd	σ_ξ	0.492
initial H , mean		14.738
initial H , sd		2.635

is zero by assumption. Therefore the relevant one is the elasticity of labor supply on the extensive margin.

We increase the human capital rental rate at different ages by 10% (from 1 to 1.1), and then compare the labor force participation rate with the baseline model to calculate two different types of labor supply elasticities.

The first type is our counterpart to the Marshallian (uncompensated) elasticity. Let h_t^b be the labor force participation rate at age t in the baseline model and h_t^t be the labor force participation rate at age t in the simulation in which we increase the rental rate at age t by 10%. Then our version of the Marshallian is calculated as

$$\epsilon_t^u = \frac{\log(h_t^t) - \log(h_t^b)}{\log 1.1}.$$

We also calculate our version of the Intertemporal Elasticity of Substitution (IES) as,

$$ies_t = \frac{\log(h_t^t/h_{t-1}^t) - \log(h_t^b/h_{t-1}^b)}{\log 1.1}.$$

Please note in both calculations, we assume that when the human capital rental rate increases by 10% the wage also increases by the same proportion, which is just an approximation. The whole life-cycle age-wage profile will be different in this model even when the rental rate only changes at age t .

The calculated Marshallian elasticity and IES at each age are plotted in Figure 5a. Table 3 also lists elasticities and IES at selected ages.

Figure 5b presents the LFPR profiles for cases where the 10% increase of the human capital rental rate happens at different ages, specifically at ages 25, 40, and 60. This shows the response in LFPR at different ages for the positive shock at one specific age.

Table 3: Elasticities at selected ages.

Age	Marshallian ϵ_t''	IES
25	0.151	0.119
30	0.144	0.105
35	0.147	0.111
40	0.156	0.127
45	0.227	0.193
50	0.310	0.254
55	0.515	0.434
60	0.744	0.589
65	1.060	0.859

Figure 5c plots the total changes of LFPR for such positive shocks at different ages. Assume the human capital rental rate only increases at age t . For this case, the “Overall” represents the overall change of LFPR over the entire life-cycle (from age 18 to 90); the “Before t ” represents the total change of LFPR before age t ; the “After t ” is the total change after age t and the “At t ” is the spot change at age t . If the human capital rental rate increases at age t , the spot LFPR increases responding to this positive shock. Furthermore, before age t , the expected return of working and investing also increases. This leads to the increase of the LFPR before age t . This shows that a rational individual responds to the predicted shock at later age before it occurs in this dynamic model. On the other hand, if the positive shock occurs during early career, the wealth effect causes decline of the LFPR at later life. However, a positive shock at old ages would encourage higher LFPR afterwards. This is because one individual allocates more time in effective working at old ages than at young ages. Thus the substitution effect is more prominent at old ages, when the wage is around the peak.

7. Counterfactuals

We conduct five counterfactual policy experiments given the estimation fitting SIPP data.

(1) Remove the Social Security earnings test, which is effective between age 62 and 70 in the baseline model.

(2) Delay Normal Retirement Age (NRA) two years: the new NRA is age 67 in this counterfactual experiments, while it is age 65 in the baseline model.

(3) Remove the Social Security system. In this case, there is neither Social Security benefit, no Social Security tax (including the medicare tax).

(4) Extend the life expectancy from 90 to 100, keeping ERA and NRA unchanged.

(5) Extend the life expectancy from 90 to 100, and remove the Social Security system.

The first two policy changes intend to encourage labor supply, especially at old ages. The third experiment is to investigate the impact of the current Social Security system on individual's human capital accumulation pattern. In the last two, we try to estimate the effect of increased life expectancy, with or without the presence of the present Social Security system. The comparison between the baseline model and the five counterfactuals is plotted in Figure (6a)-(6d).

Removing the Social Security earnings test for age between 62 and 70 has small effect on all variables. This is because in the baseline model the benefits withheld due to earnings test is transferred to the delayed retirement credit and it is roughly actuarially fair. Therefore removing the Social Security earnings test does not change incentive much.

Delaying the normal retirement age (NRA), on the other hand, has larger impact. Individuals participate in the labor market more, especially around the NRA. They also invest more and therefore have higher human capital level, which leads to higher wages at old ages (around 2% higher). Over the life-cycle, they work more to offset the reduced Social Security benefit, and this response happens before and after the effective NRA.

Removing the entire Social Security benefits and taxes induce higher LFPR, especially at old ages. The investment and human capital level for an average individual is also higher at all ages in this experiment. This indicates that the presence of the Social Security system has some level of distortion—it increases the incentive to work at young ages disproportionately.

The distortion effect partly explains that in the fourth experiment where each worker lives ten extra years. In this case, each individual supplies more labor while young but enjoys more leisure after 40s. The average LFPR and human capital level over the life-cycle are actually lower when the life span is larger.

As comparison, the fifth experiment removes the Social Security system when each individual lives ten extra years. Comparing with the third experiment (No SS), the LFPR, investment and human capital level, and wages, are universally higher at all ages when the life span is larger, in the absence of the Social Security system. This implies that the current Social Security system has negative effect on growth in the context of improved mortality and increased life-expectancy, which most countries are experiencing. Even though the current U.S. Social Security system is largely a Pay-As-You-Go program, [Echevarría and Iza \(2006\)](#) have similar findings for a funded Social Security system.

Another point worth noticing is that, in all five experiments except the first one, the responses in the endogenously determined wages are non-trivial, especially at old ages. The changes in the wages vary from -4% (extending life expectancy by ten years) to 2%

(delaying NRA by two years), and to over 20% (removing Social Security system). For this reason, it is likely that ignoring human capital investment channel will generate bias in terms of predicting LFPR at old ages in similar experiments.

8. Concluding Remarks

This paper develops and estimates a Ben-Porath human capital model with endogenous labor supply and retirement, combining the standard Ben-Porath human capital model with the standard retirement model. In the model each individual makes decisions on consumption, human capital investment, labor supply and retirement. The investment in the human capital generates the wage growth over the life-cycle, while the depreciation of the human capital is the main driving force for retirement. We show that the simple model is able to fit the main features of lifecycle labor supply and wages. Given that this is still work in progress, it is premature for conclusions beyond this.

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Figure 1a: Labor Force Participation Rate-SIPP Data

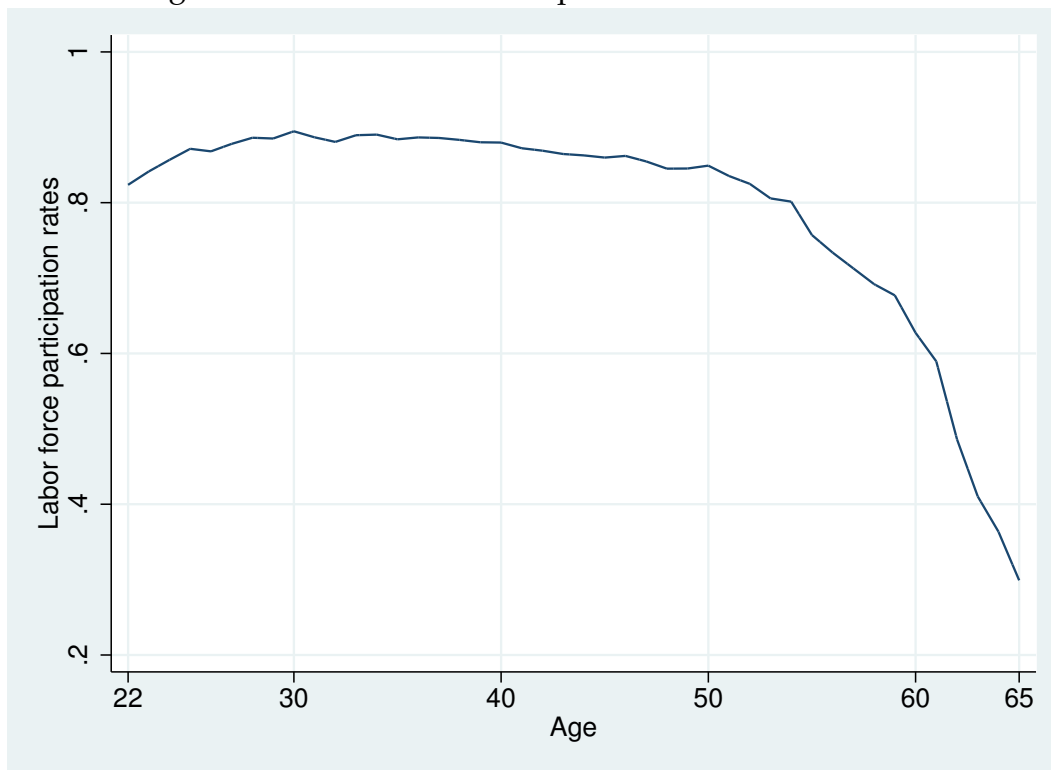


Figure 1b: Difference of LFPR between good health and bad health-HRS
Data

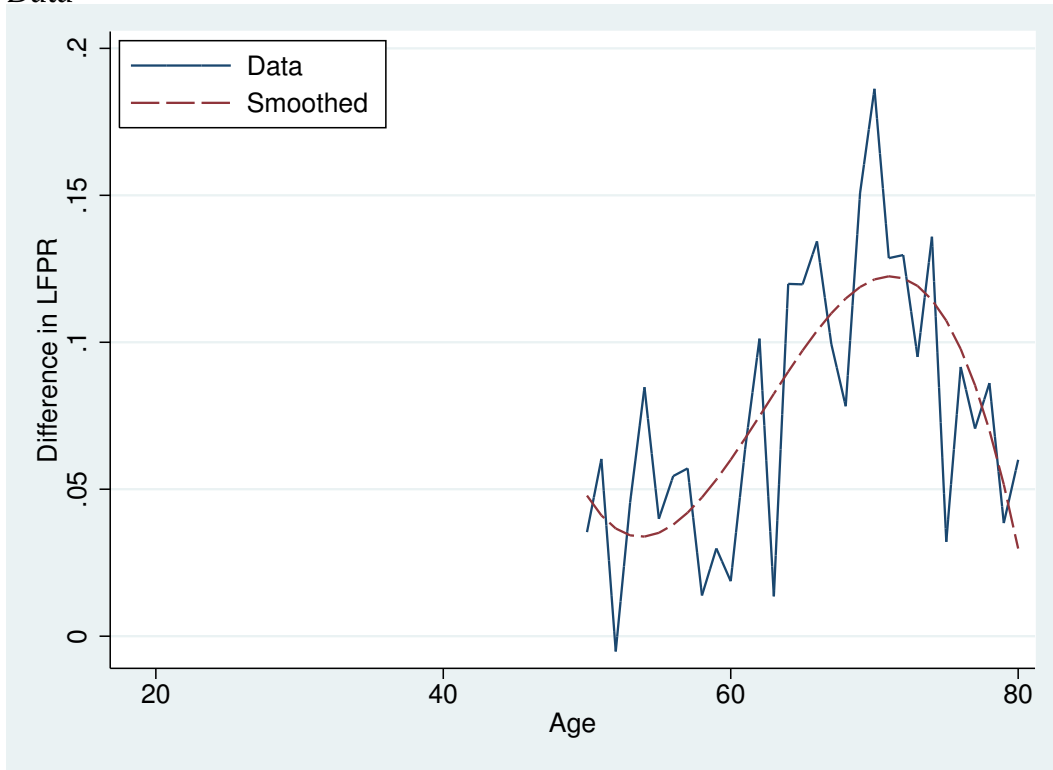


Figure 1c: Log Wages-SIPP Data

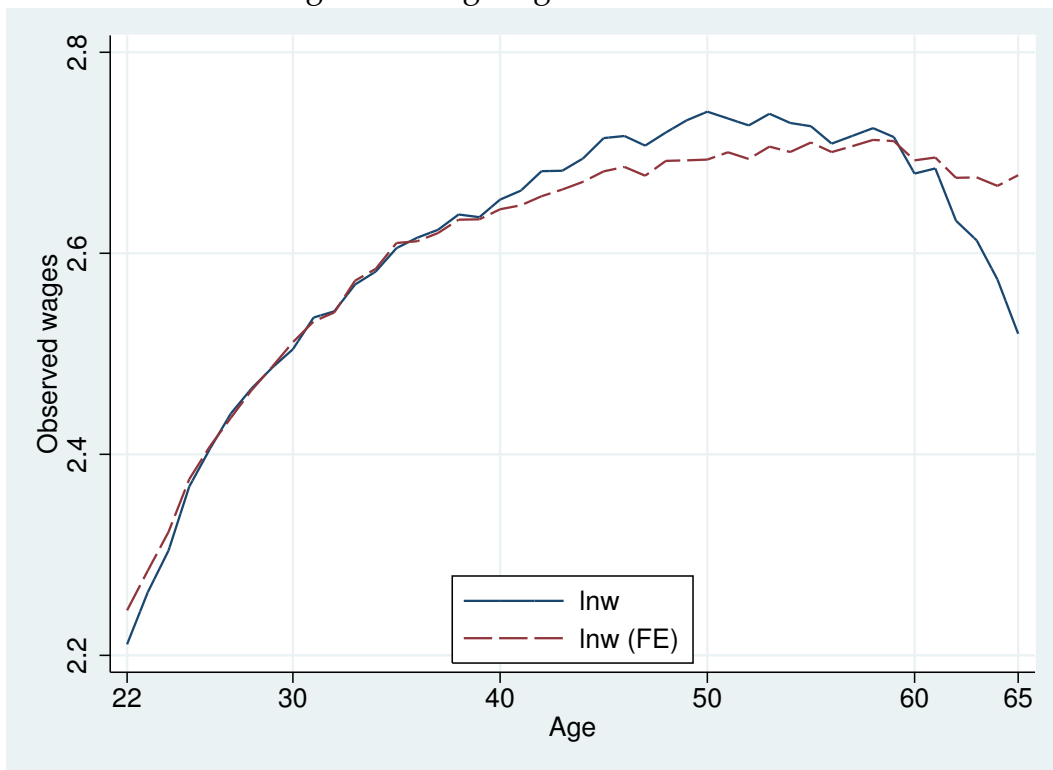


Figure 2d: Log wage profiles, CPS MORG and March, high school graduates

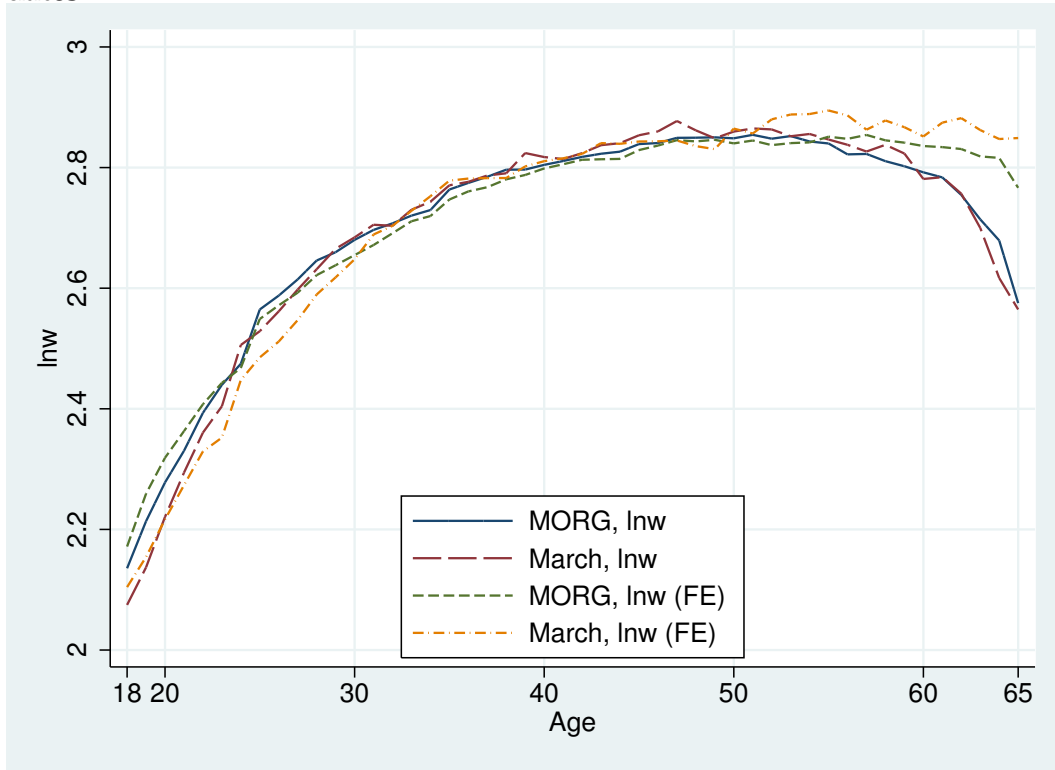


Figure 3a: Fit of Model: Labor Force Participation Rate

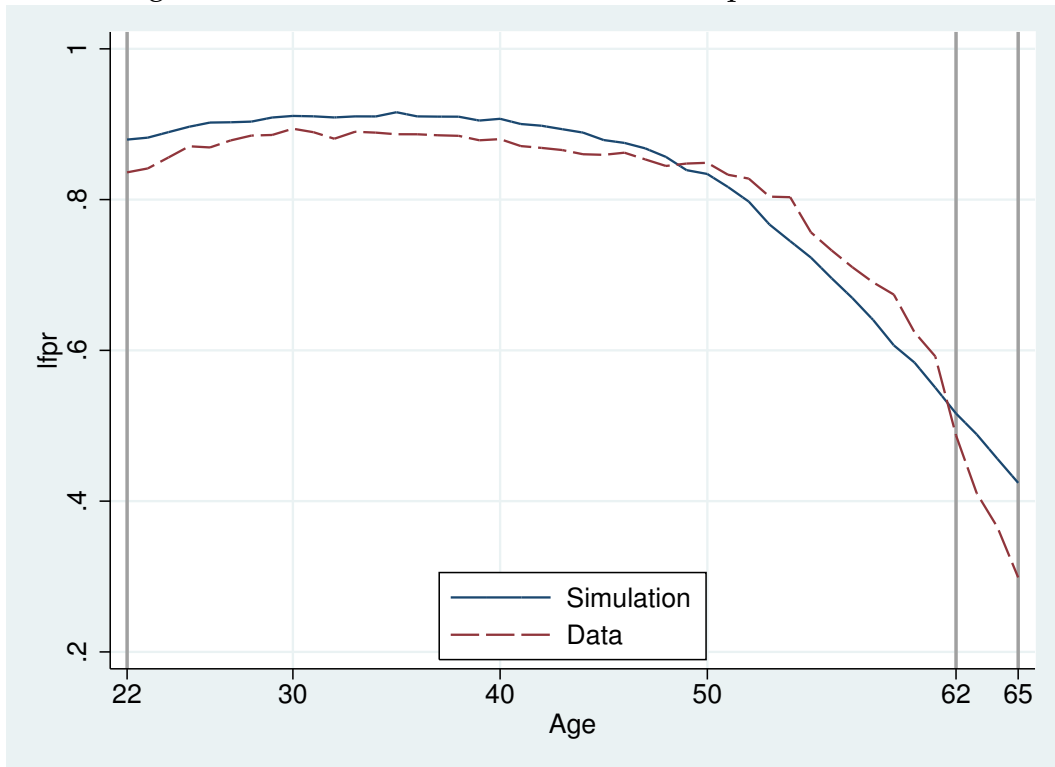


Figure 3b: Fit of Model: Log Wages after controlling for individual fixed effects

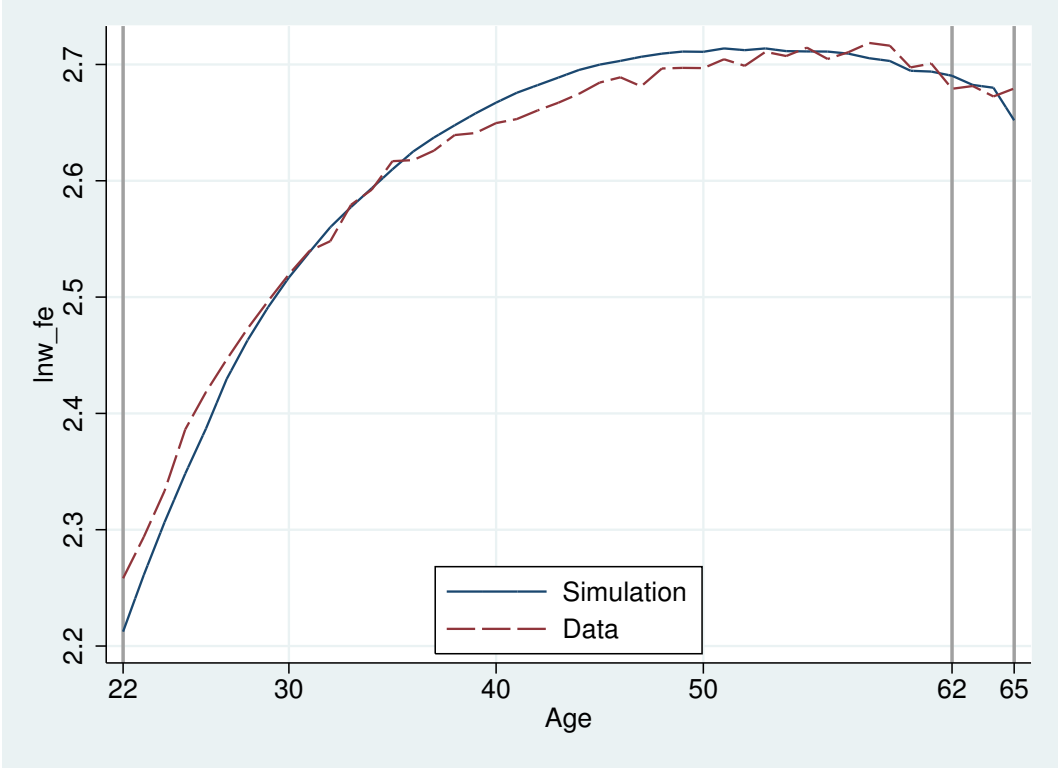


Figure 3c: Fit of Model: Log Wages

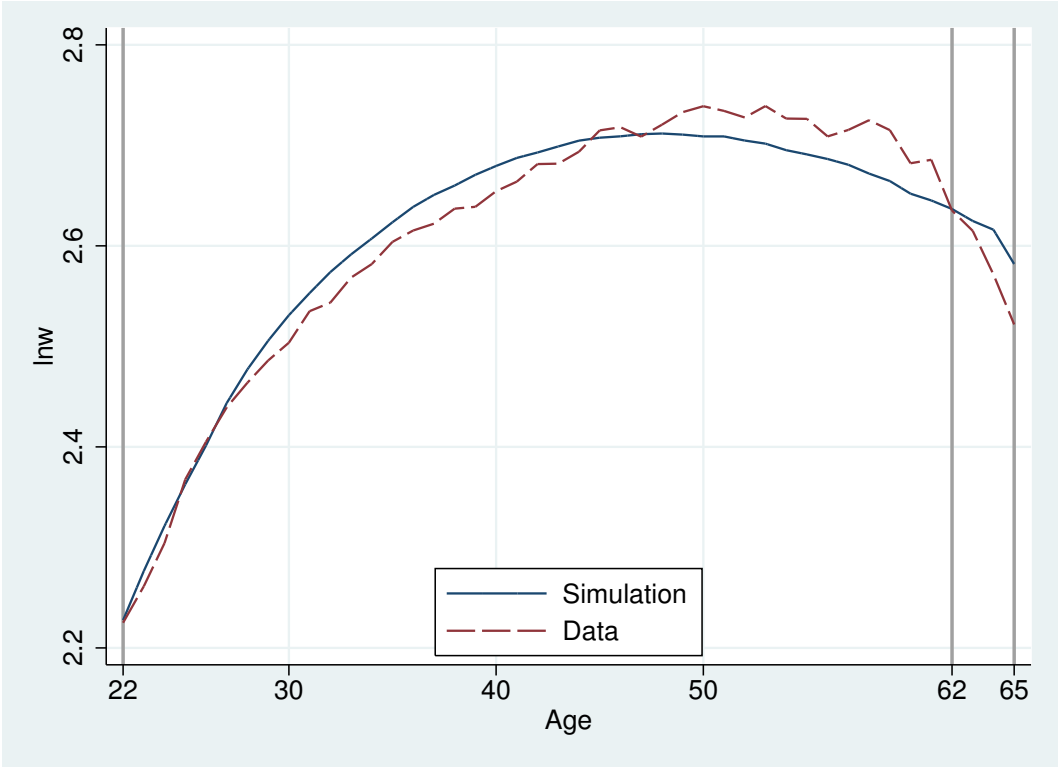


Figure 3d: Fit of Model: Difference of LFPR between workers with good health and bad health

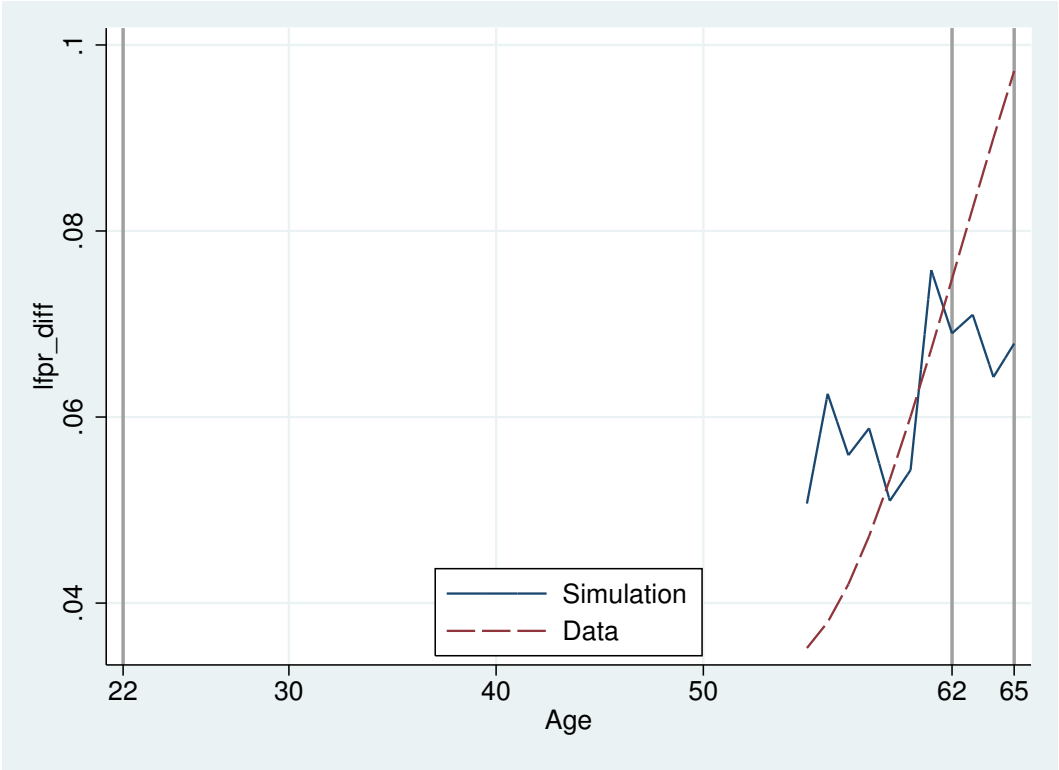


Figure 4a: log wages, with and without controlling for individual fixed effects

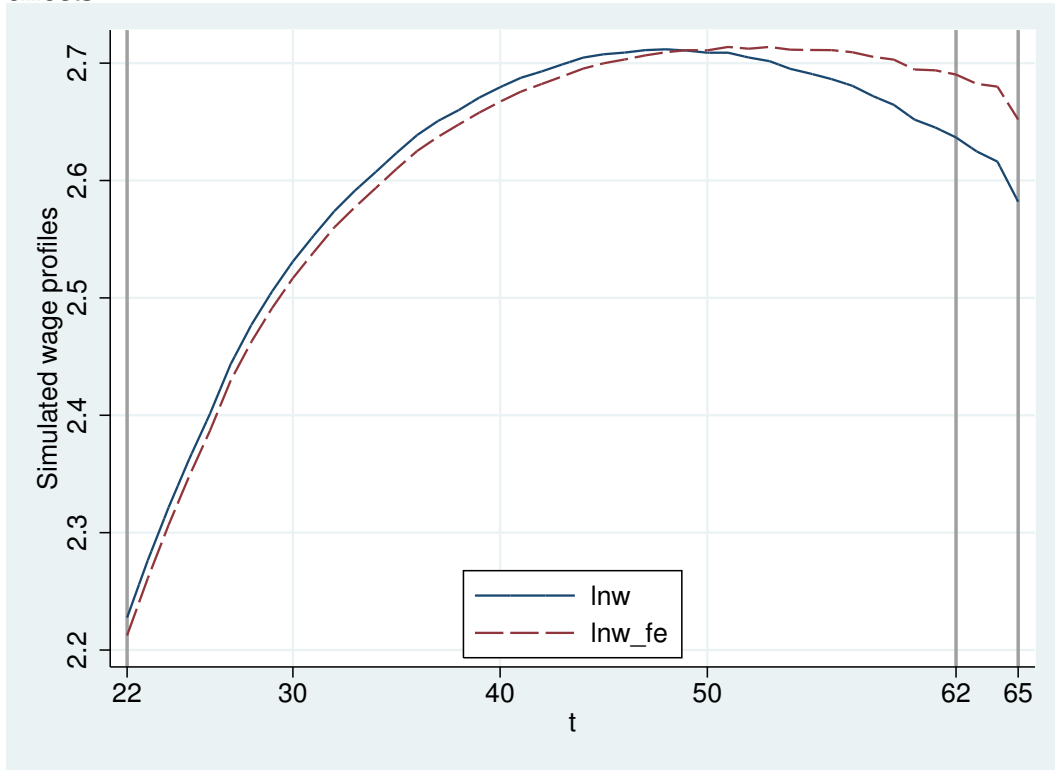


Figure 4b: Labor supply, log wages, and income

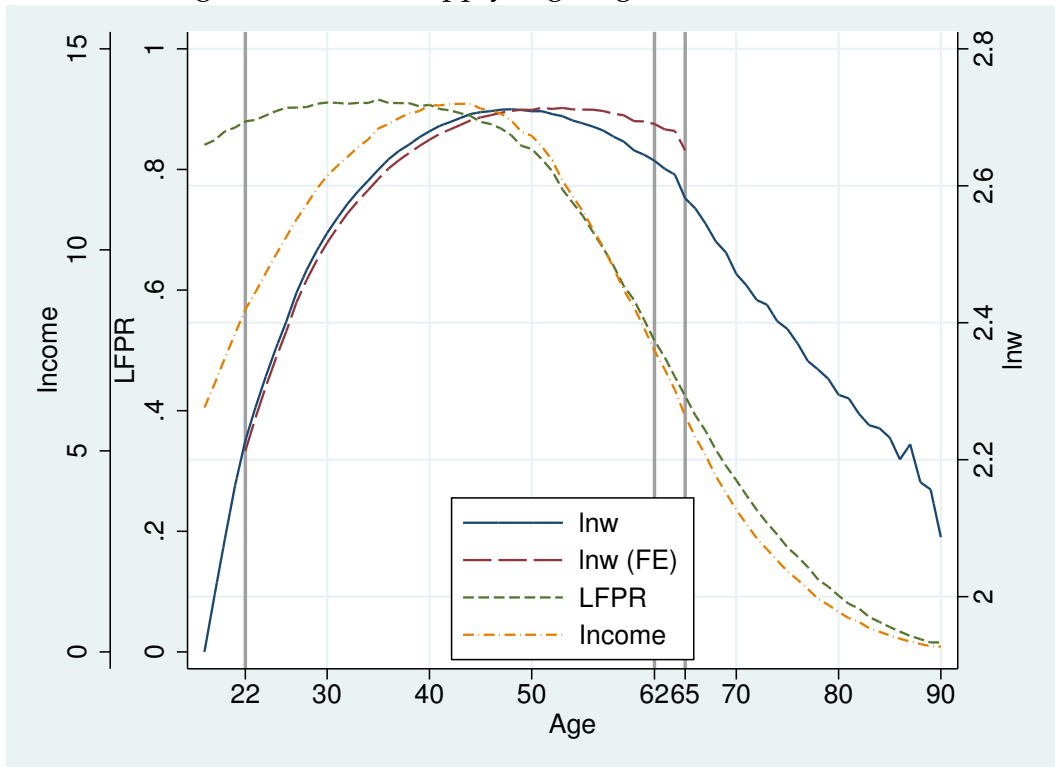


Figure 4c: Leisure, investment, and human capital

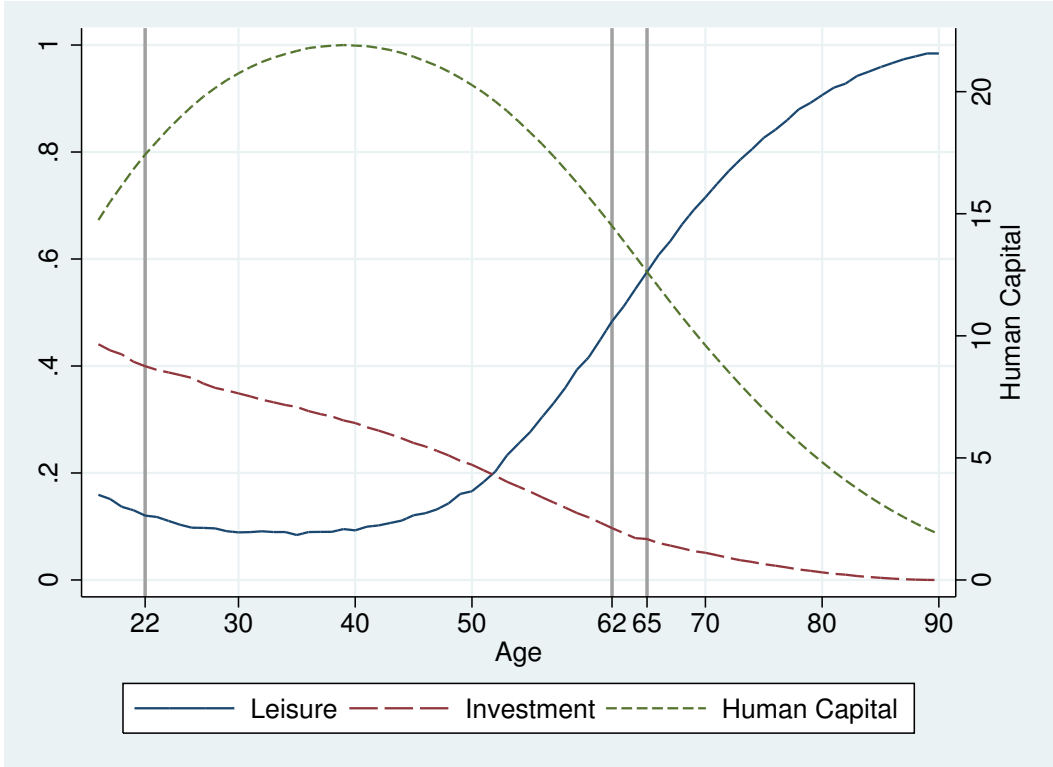


Figure 4d: log wages and human capital

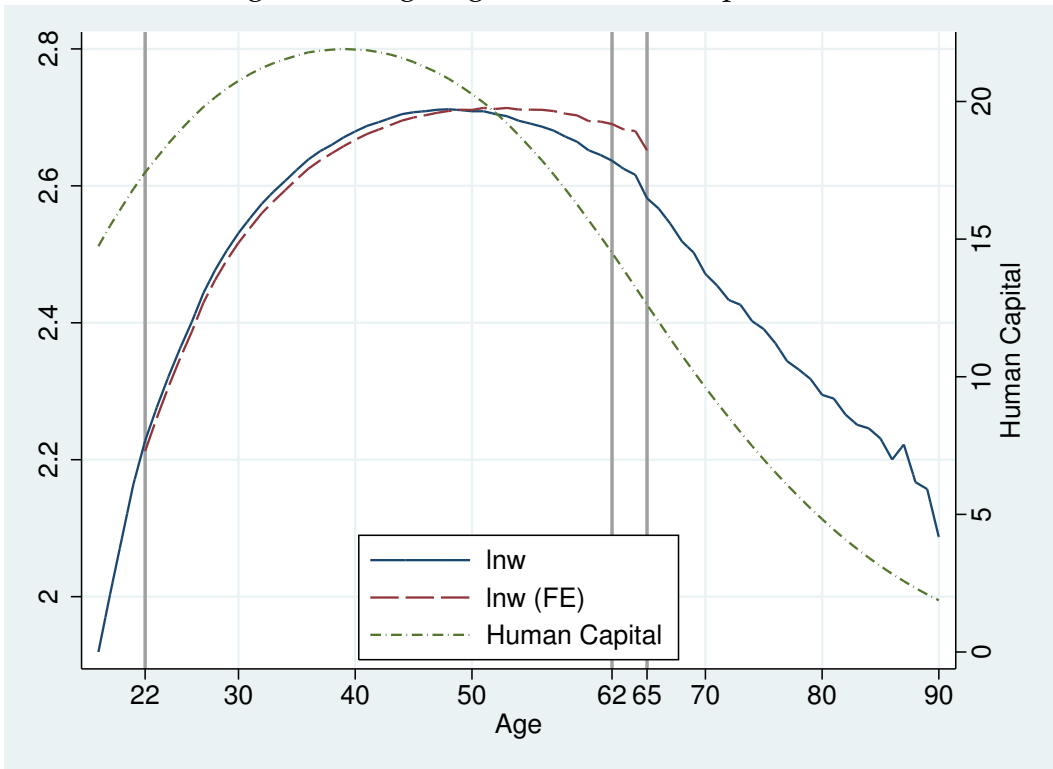


Figure 5a: Calculated elasticities

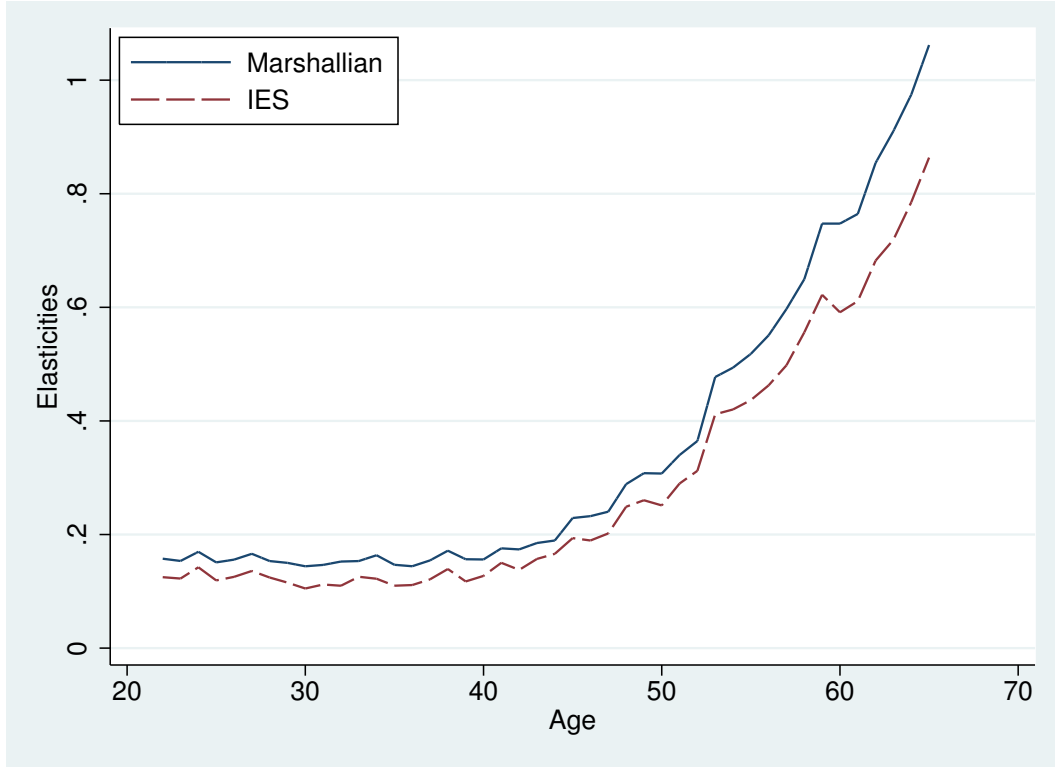


Figure 5b: LFPR profiles for positive shocks at different ages

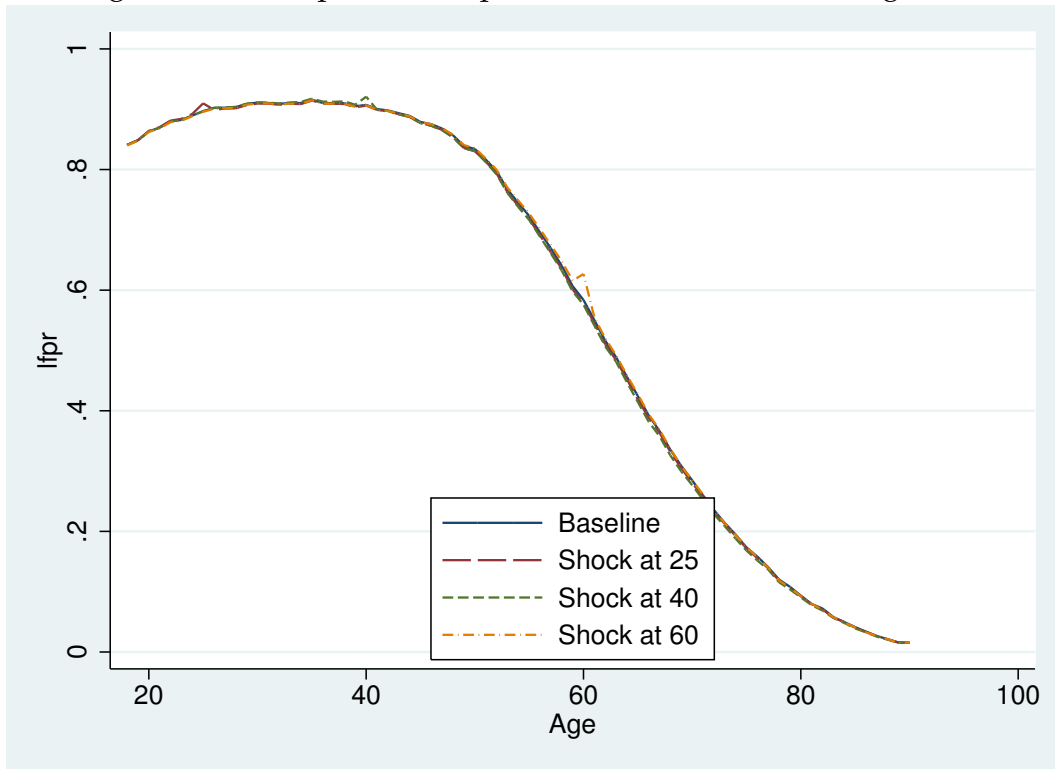


Figure 5c: Total changes in LFPR for positive shocks at different ages

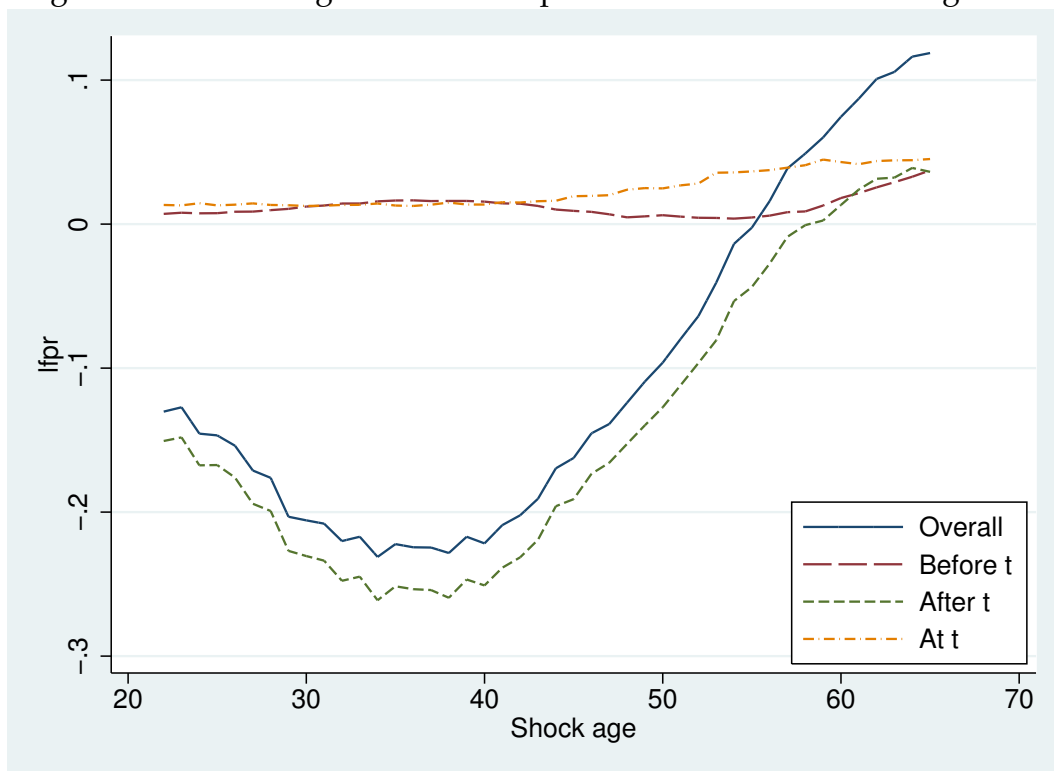


Figure 6a: Counterfactual experiments: difference in LFPR

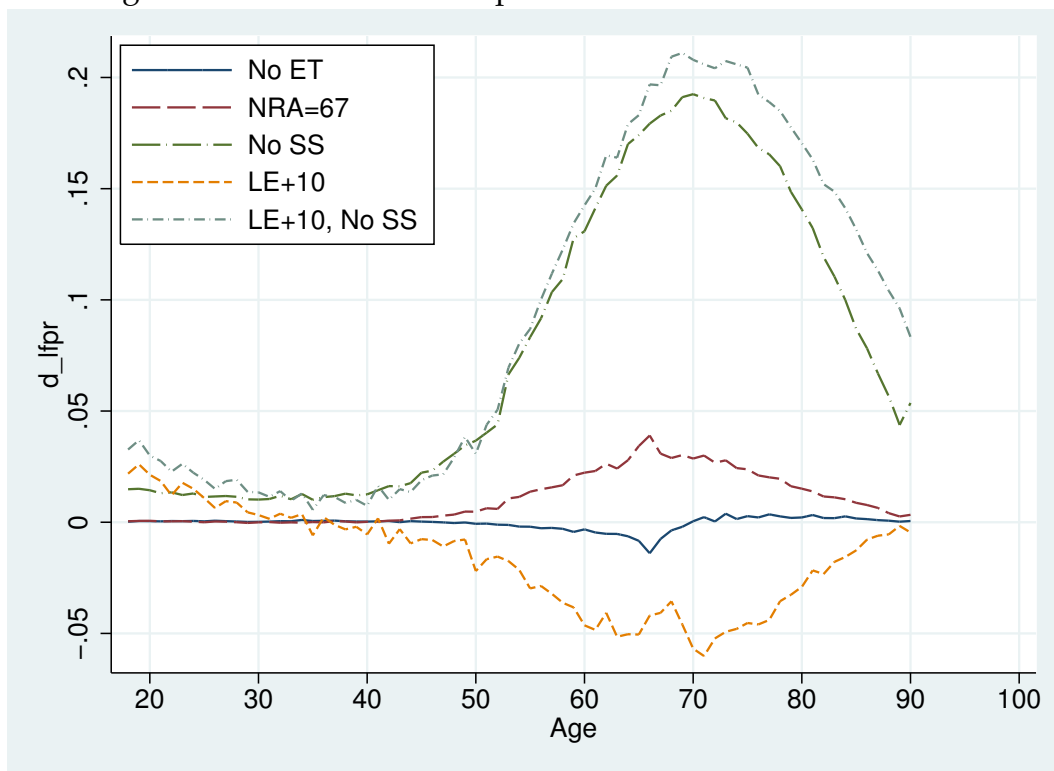


Figure 6b: Counterfactual experiments: difference in lnw

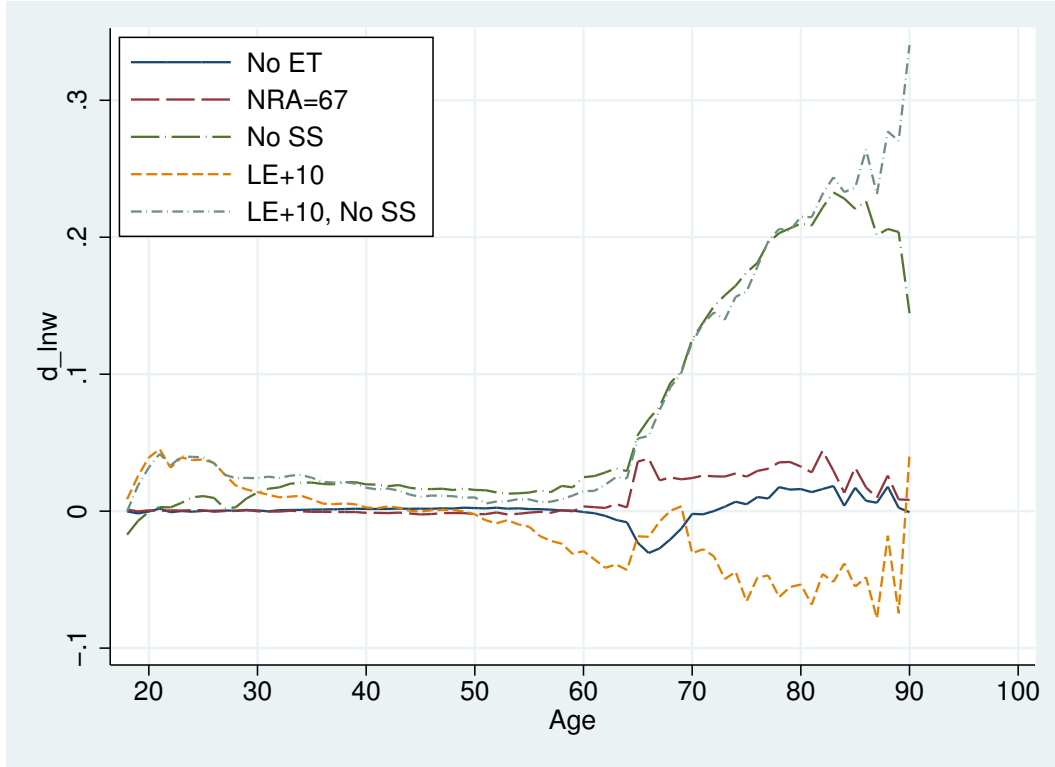


Figure 6c: Counterfactual experiments: difference in human capital levels

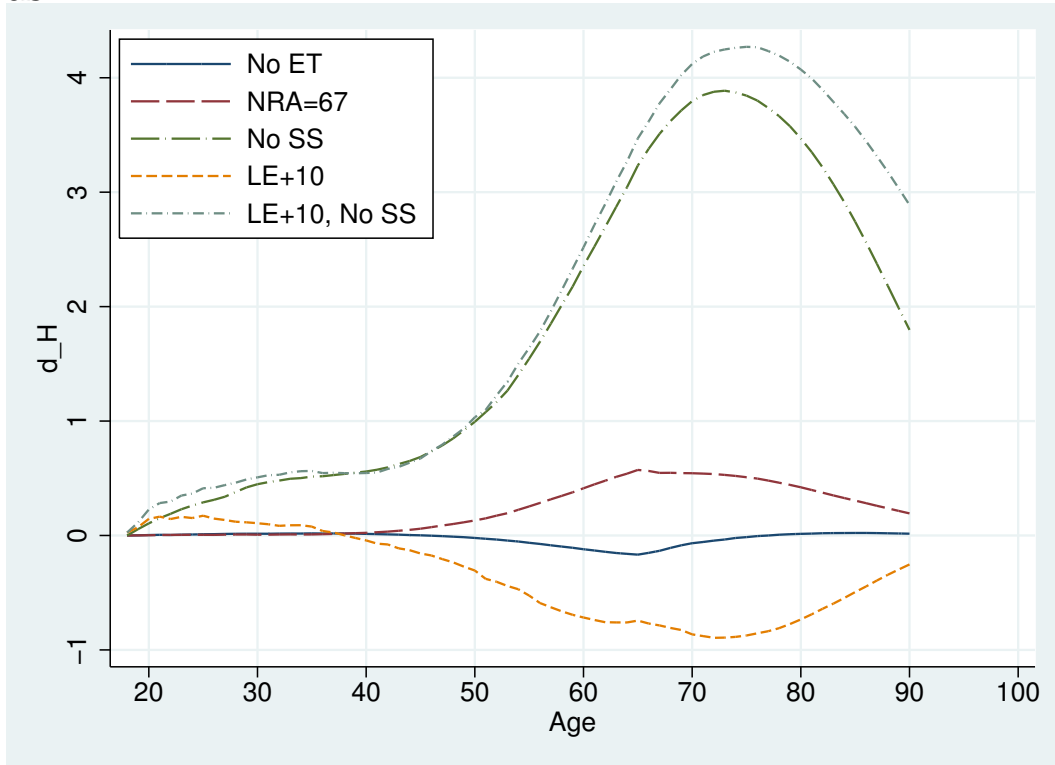


Figure 6d: Counterfactual experiments: difference in investment

