The Costs of Failure: Negative Externalities in High School Course Repetition

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Abstract This paper investigates the extent to which course repeaters in high school mathematics courses exert negative externalities on their course-mates. Using individual and school-specific course fixed effects to control for ability and course selection, it shows that doubling the number of repeaters in a given course (holding the number of course-takers constant) results in a 0.15 reduction in GPA scores for first-time course-takers. Evidence is presented that course repetition externalities widen the achievement gap between high- and low-performing students, and results suggest that effects are not just operating through course repeaters being low achievers. Further results suggest that the negative effect is only evident when the share of repeaters reaches a threshold of five to ten percent of the total number of course-takers.

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1 Introduction

The questions of whether low-achieving students should be retained in a grade or required to repeat a failed course are answered by the extent to which grade or course repetition affects the retained or repeating individual *and* the extent to which grade or course repeaters affect their classmates. An extensive literature has investigated the effect of grade retention on the individual, but there is less evidence on the potential effects of grade or course repeaters on their classmates. An important exception is Lavy, Paserman and Schlosser (2011) who find that the the share of students who are old for their grade (likely having been retained) negatively affects their same-grade schoolmates in Israeli middle schools. This paper provides new evidence of course-specific repeater externalities in US high schools. Using unique longitudinal transcript data in which individual and school-specific course fixed effects control for ability and course selection, it shows that doubling the number of repeaters in a given high school mathematics course (holding the number of course-takers constant) results in a 0.15 reduction in GPA scores for first-time course-takers.

There has been a widespread push to raise standards in American high school education.[1] In the past decade, many US states have both increased the number of mathematics credits required for high school graduation and specified particular mathematics courses that need to be passed (Reys et al, 2007).[2] Media reports indicate that this has generated considerable public contention and may have increased the likelihood of repetition for students who fail high school mathematics courses (Helfand, 2006; Hacker, 2012). This paper provides evidence of negative course repetition externalities in mathematics even before these waves of standards' increases. Seven percent of students in the data used in this paper repeat a failed mathematics course in high school. This increases to fifteen percent for students taking Algebra I. The effects of repetition in high school mathematics course are clearly important to understand. The negative externalities exerted by repeaters on their classmates found in this paper suggest a cost to course repetition ignored by much of the previous analyses, and, to the extent that the above policies and increases in standards increase course repetition, a cost to these policies that may have been overlooked by policy-makers.[3]

[1] Organizations such as Achieve (www.achieve.org) have been promoting college and career-ready high school curricula since the mid-2000s. More recently, they partnered with the National Governors Association and the Council of Chief State School Officers to release the Common Core State Standards, a set of overall guidelines for what students should learn throughout their schooling. These standards have been adopted by and are currently being implemented in a vast majority of states.

[2] The state-specific policies as of 2006 are summarized in Appendix Tables 8 and 9.

[3] There may, of course, be a benefit or cost experienced by the repeating individual. This is not the focus of this paper, but is clearly important for a complete policy analysis. Rose and Betts (2004) find that advanced high school mathematics courses have greater effects on students' earnings a decade after graduation than less advanced courses, and Goodman (2012a) uses state-specific policy reforms to find that

Understanding the externalities imposed by repeaters in high school mathematics courses may also inform the grade retention debate. This is because both grade retention and course repetition result in students being exposed to a set of low-achieving classmates who are likely to share similar characteristics.[4] To the extent that repeating and retained students exert similar externalities on their classmates, this paper suggests grade retention analyses should include effects exerted on classmates of the retained individual.

The paper uses a fixed effects strategy on longitudinal transcript data for multiple cohorts of US high school students to estimate the causal effect of course repeaters on their course-mates, students taking the same course but not necessarily in the same class. Essentially, the study compares the achievement of first-time course-takers in the same mathematics course (such as Algebra I) in the same high school in different years using year-to-year variation in the number of repeaters in the course to identify the effect. It is assumed that unobserved year-specific shocks to classroom education production in the previous year provide variation in the number of course repeaters in the current year. An example of this would be increased teacher absenteeism in a specific course causing a higher course failure rate.

Holding the number of students in a course constant (either parametrically or using course size fixed effects), the academic achievement of first-time course-takers is shown to be negatively correlated with the number of repeaters in the course that year. This relationship is robust to a variety of different specifications. The effect is concentrated in the lower and middle parts of the achievement distribution, indicating the effect widens the achievement gap between high- and low-performing students. Evidence that the negative externalities exerted by course repeaters are due to their both failing *and* repeating is provided, suggesting the effect is not just due to an increase in the number of low achievers in the course.

Course repeaters may exert externalities on their course-mates in a variety of ways. These course composition effects can be grouped into two categories: general effects arising from repeaters being low-achievers and specific repeater effects not exerted by other low-achievers. Low-achieving students are likely to disproportionately extract teacher inputs or redirect

increases in the mathematics requirements for high school graduation introduced in the mid and late-1980s increased subsequent earnings for black students. In an investigation of how increases in standards affect individual achievement, Goodman (2012b) shows little impact, which may be consistent with findings in this paper if potential gains to high-achieving students are offset by losses to low-achieving students who are both more likely to repeat failed courses and experience the negative externalities exerted by repeating course-mates.

[4] The effects of grade retention and course repetition on the individual, however, are likely to differ along several dimensions. This is primarily because retained and repeating students are likely to be of different ages and maturities (retention typically occurs in junior and middle schools while course repetition typically occurs in high school). In addition, retained and repeating students are exposed to a different peer group shock (retained students repeat all courses associated with a particular grade so are exposed to a completely new set of peers while repeating students are only exposed to new peers in the course they repeat).

teacher inputs away from first-time course-takers. They may need more time to understand concepts, slowing the pace of the class, and may also be more likely to misbehave in the classroom given that disruptive behavior is generally correlated with classroom ability, requiring teacher intervention. Low-achieving classmates may also be more likely to directly distract their classmates, lowering education production even without affecting teacher inputs.

In addition to these low-achiever effects, course repeaters may exert additional externalities specifically related to failing and retaking a course. They may be bored and inattentive when encountering course material for the second time, increasing the likelihood of disruptive behavior. Repeaters may also have a poor attitude or be uncooperative because they failed the course the previous year, and this may negatively affect both their classmates and the teacher.[5]

Course repeaters may also exert externalities through course size, and, for courses with more than one class, class assignment. Course size effects are fully controlled in the estimation procedure given the mechanical relationship between course repetition and course size.[6] Class assignment may matter if repeaters are assigned to classes non-randomly. For example, repeaters may be assigned to the best teacher for a particular course if failing for a second time is particularly costly (either from the perspective of the school or the student). This may increase the likelihood of first-time course-takers being assigned to another class with a worse teacher, leading to poorer performance for first-time course-takers. This effect has nothing to do with peer effects in the classroom, but is no less relevant from a policy perspective.

The primary focus of this study is an analysis of the combined low achiever and repeater effects that course repeaters exert on their course-mates. This is the appropriate level of analysis for an overall evaluation of course repetition effects. Secondary results attempt to separate the general low-achiever and specific repeater externalities, as well as probe whether effects are affected by class assignment. This has important policy implications. In an environment in which repeaters exert negative externalities only because they are low achievers, course re-assignment would only be effective if the low achiever effect did not persist in the new course. However, if the effect is specifically related to course repetition, the negative externality could be eliminated by students not repeating the course.

The results in this paper are best compared with those obtained by Lavy, Paserman

[5]Another potential repeater mechanism operates in the other direction; repeaters may provide examples of the consequences of failure, incentivizing more effort from first-time course-takers at risk of failing. This paper finds an overall negative effect, so this channel is at most a mitigating factor.

[6]Nonetheless, course size effects are unlikely to be a significant factor given the large changes in class sizes typically required to observe effects (Hanushek, 1999). The similar estimates for repeater effects with and without course size controls support this hypothesis.

and Schlosser (2011). Defining low-ability students as students who are old for their grade (most likely having repeated kindergarten or first grade), they find that the proportion of low-ability peers is negatively correlated with the academic achievement of regular students. Variation in the composition of seven adjacent cohorts of 10th grade students in Israeli high schools (from 1994 to 2000) is used to identify the effect. It is argued that the majority of students had little experience with their peers prior to entering high school, so results are not driven by common cohort-specific shocks.

This paper has three key distinctions from Lavy et al (2011). First, we observe course enrollment and achievement for all students in a set of high schools for multiple years allowing the inclusion of both individual and school-specific course fixed effects. This approach deals with potentially confounding individual effects (such as cohort-specific shocks and ability differences) and course effects (such as repeaters being more likely to repeat difficult courses) that cannot be dealt with using repeated cross-sectional data. Second, it isolates the effects of course-mates rather than grade-mates. Students in the same grade may have little interaction and may not take many of the same courses, which would attenuate effects for analyses performed at the grade level. And, third, it focuses on high school mathematics courses in the US, which is particularly relevant given policies stipulating minimum mathematics requirements for graduation in US high schools increasing the likelihood of mathematics course repetition.

Repeaters are low-achieving peers for first-time course-takers. Results can therefore be compared with the literature investigating ability peer effects in high school. These papers exploit a variety of identification strategies and typically find moderately-sized, negative achievement effects for individuals exposed to low-ability peers. Lavy et al (2012) find evidence of negative peer effects at the bottom of the ability distribution for English secondary school students, which is consistent with the finding that course repeaters widen the achievement gap. Burke and Sass (2013) use Florida public schools data and find significant peer effects in nonlinear models. They also find that ability peer effects are stronger at the class level than at the grade level. The course repeater externalities estimated in my paper are larger and more precise when the sample is restricted to courses likely consisting of one class, consistent with the idea that class peer effects dominate grade peer effects. Sacerdote (2011) provides a full discussion of the literature investigating ability peer effects.

The externalities exerted by course repeaters may also be placed in the context of the related literature investigating the effects of grade retention. This is because how grade retention affects retained students suggests how course repetition may affect repeating students. The literature investigating the causal effect of retention on the retained has exploited a variety of policies to overcome selection into retention. It provides evidence of both positive and negative effects. Positive achievement effects of retention for third grade students are found by Jacob and Lefgren (2004) and Greene and Winters (2007). These papers use Chicago and Florida accountability policies respectively to obtain exogenous variation in grade retention. Ding (2010) finds that holding children back in kindergarten has positive but diminishing effects on their academic performance up to third grade. Eide and Showalter (2001) use kindergarten entry dates as an instrument for retention and find that retention reduces the probability of dropping out of high school for white students.[7] These findings suggest generally positive effects of retention on students retained up to the third grade.

The effects for older students (more like those studied in this paper) appear to be more nuanced. Jacob and Lefgren (2004, 2009) find that retention in the sixth grade does not significantly affect achievement or high school graduation, while retention in the eighth grade reduces the probability of high school graduation. Using data from junior high schools in Uruguay and a policy of automatic grade failure for certain low-achieving students, Manacorda (2012) shows that grade failure increases dropout rates and lowers educational attainment. Fruehwirth, Navarro and Takahashi (2011) recognize that retention effects are likely to differ by the grade at which the student is retained and the unobservable behavioral and cognitive abilities of the student. They allow for heterogeneous effects in their econometric model and obtain generally negative effects from retention, suggesting grade retention is not an effective policy for raising the performance of low-ability students. To the extent that the effect of grade retention on the retained is similar to the effect of course repetition on the repeating, these findings suggest that high school course repetition may not benefit the repeating student. If this is the case, the course repetition externalities found in this paper aggravate the negative effects of repeating courses in high school.

This remainder of this paper is organized in the usual way: methodology, data, results and then interpretation.

2 Empirical Methodology

This paper considers course-specific rather than class-specific externalities. A course-level analysis has two favorable features. First, as discussed in the introduction, externalities may extend beyond the classroom if repeating students are systematically assigned to better (or worse) teachers. A class-level analysis would not capture these effects. And, second, we do not need to be concerned about sorting into classes within courses. For example, students may be assigned to classes precisely to mitigate potential negative externalities exerted by course repeaters. This would bias class-specific estimates of repeater effects. The peer

[7] Estimates for black students were uninformative.

composition literature typically uses grade and not class variation to identify composition effects precisely for this reason (Hoxby, 2000; Hanushek et al, 2009; Lavy and Schlosser, 2011).

The academic achievement of first-time course-taker i (who did not fail any math course the previous year) in course j, high school s, cohort c and year t is modeled as a linear function of the natural logarithm of one plus the number of repeaters (who did fail some math courses the previous year) in the course R_{jst}[8], the natural logarithm of the number of students in the course C_{jst} (course size), and a composite error term:

$$GPA_{ijsct} = \beta \ln(1 + R_{jst}) + \gamma \ln C_{jst} + \epsilon_i + \epsilon_{js} + \epsilon_{sc} + \epsilon_t + \epsilon_{jst} + \mu_{ijsct} \quad (1)$$

The coefficient β represents the level change in student GPA score for a percentage change in the number of course repeaters. The $\ln C_{jst}$ term controls for the potential negative effect of course size on achievement and is necessary because of the mechanical relationship between the number of course repeaters and course size. Without controlling for course size, estimates of the negative externalities exerted by repeaters may exaggerate the effect.

This parametrization of the education production function is chosen so estimated coefficients are easy to interpret. Results from a variety of other specifications in which course repeaters enter linearly, quadratically and as shares are reported in the appendix. The overall pattern of results is consistent across specifications.

The error term is modeled to consist of individual ability ϵ_i, school-specific course difficulty ϵ_{js}, a school-specific cohort effect ϵ_{sc}, a general time trend ϵ_t, a school-specific course time trend ϵ_{jst}, and a remaining idiosyncratic shock to achievement μ_{ijsct}.

Several components of this error term may be correlated with the number of course repeaters, which would bias estimates of the coefficient of interest β. A variety of fixed effects and time trends are included in the estimation to remove these potential biases.[9] Most of these rely on observing multiple years of student achievement in high school for multiple cohorts, representing an advantage over repeated cross-sectional analyses or longitudinal analyses of one cohort.

$$GPA_{ijsct} = \beta \ln(1 + R_{jst}) + \gamma \ln C_{jst} + \theta_{js} + \theta_t + \theta_{jst} + \theta_i + \mu_{ijsct} \quad (2)$$

School-specific course fixed effects θ_{js} control for course difficulty as well as any other

[8]The addition of one to the number of repeaters ensures that the natural logarithm of zero is avoided. Results are qualitatively similar when courses with zero repeaters are dropped from the sample.

[9]These are implemented using a two-stage procedure in which fixed effects are applied to demean dependent and explanatory variables in the first stage before the analysis is performed on the demeaned variables in the second stage. This is because the final estimation is only performed on first-time course-takers, but the year fixed effects and course-specific trends are more accurately computed using the whole sample.

course-specific factor affecting both the achievement of first-time course-takers and the number of course repeaters. A positive correlation between course difficulty and the number of course repeaters is expected if students are more likely to fail and repeat difficult courses. Alternatively, low-ability students who consider themselves more likely to repeat a course may select out of difficult courses (if the course is not required for graduation). This would generate a negative correlation between course difficulty and the number of repeaters. The net direction of the correlation between course difficulty and the number of course repeaters could be either positive or negative, which respectively would bias estimates of the effect upwards or downwards in the absence of these school-specific course fixed effects.

Year fixed effects θ_t and linear school-specific course trends θ_{jst} control for any correlated trends in student achievement and course repetition. Consider grade inflation. Every subsequent year, fewer students fail a given course, resulting in fewer course repeaters every subsequent year. At the same time, first-time course-takers perform better every year. This generates a pattern of increased achievement associated with fewer course repeaters that has nothing to do with course repetition externalities. In the absence of this set of controls, estimates of the effect of course repeaters would be upwardly-biased.

School-specific cohort fixed effects θ_{sc} may be included to control for cohort effects. An alternative approach to dealing with cohort effects is the inclusion of individual fixed effects θ_i, which nest cohort fixed effects. Individual fixed effects are preferred as they improve precision by controlling for individual ability, as well as any other forms of individual selection.

Finally, it is noted that grading to a curve would bias estimates of the effects. Repeaters are low-achieving students, so maintaining a constant course average in the presence of an increase in the number of repeaters would necessitate higher GPA scores for first-time course-takers. This would attenuate estimates of the externalities exerted by course repeaters on first-time course-takers, so results would be a lower bound of the true effect. Similarly, teachers endogenously lowering the level of the course in response to having a larger number of low-ability repeaters in the course would also attenuate rather than amplify effects. Significant variation in the unconditional means of school-specific course GPA scores in different years suggest that year-to-year grading to a curve may not be that pervasive.

Descriptive statistics include results from ordinary least squares regressions of current achievement on an individual's past mathematics course achievement (such as failing and repeating the course). Estimates from this equation do not have a causal interpretation as we cannot control for non-random selection into course repetition, but are included to describe what happens to the performance of individual students when they fail a course.

Placebo tests in which achievement depends on the number of repeaters in the same course and same school but in different years are conducted using the following equation where $p \in \{t-1, t, t+1, t+2\}$:

$$GPA_{ijsct} = \beta_p \ln(1 + R_{jsp}) + \gamma_p \ln C_{jsp} + \theta_{js} + \theta_t + \theta_{jst} + \theta_i + \mu_{ijsct} \tag{3}$$

The number of repeaters at time $t-1$ should be uncorrelated with the achievement of first-time course-takers at time t, so it is expected that $\beta_{t-1} = 0$. This is the primary placebo test, and evidence that $\beta_{t-1} \neq 0$ would raise concerns. When first-time course-takers perform poorly, more of them may repeat the course. This suggests that β_{t+1}, the correlation between the number of repeaters in the course at $t+1$ and the performance of first-time course-takers at t, may be negative. Furthermore, in an environment in which negative course repetition externalities are sufficiently large to result in persistent effects over time (repeaters causing first-time course-takers to fail and repeat the course), it would also be possible that $\beta_{t+2} \leq 0$. Estimates of β_{t+1} and β_{t+2} are reported to provide comparisons with the primary parameter of interest β_t.

Separating the general low achiever and specific repeater effects is investigated by introducing a separate measure of the number of low achievers in the course. Several students progress to a more difficult mathematics course after failing some mathematics the previous year. Denote the number of students in course j who failed some mathematics course the previous year but still progressed by F_{jst}. These students are low achievers, and potentially exert low achiever externalities on other students taking the course for the first time who did not fail any mathematics course the previous year (the primary sample of interest). A more accurate measure of the number of low achievers in a course is therefore provided by the sum of these students and the repeaters in a particular course, $F_{jst} + R_{jst}$. Both of these measures can be included in the model.

$$GPA_{ijsct} = \beta_R \ln(1 + R_{jst}) + \beta_F \ln(1 + F_{jst} + R_{jst}) + \gamma \ln C_{jst} + \epsilon_{ijsct} \tag{4}$$

The specific repeater effects on first-time course-takers are captured by the parameter β_R, while general low achiever externalities are reflected in the coefficient on the sum of repeaters and first-time course-takers who failed some previous course β_F. The extent to which β_R is attenuated by the inclusion of the measure of low achievers in the model reflects the extent to which course repetition externalities operate through low achiever effects. Course repetition externalities operating solely through low achiever effects would result in $\beta_R = 0$.

Results from this specification are presented with a caveat. Students who progressed after failing some mathematics course the previous year may have already exerted negative externalities on current course-mates as they may have been course-mates the previous year (when they failed some course). This would bias the magnitude of the low achiever effect upwards. If the primary objective was to estimate these low achiever peer effects (β_F), this would be a problem, but in the context of examining the persistence and checking the robustness of specific repeater effects (β_R), it provides a more powerful test given more variation will already be explained.

Finally, a suggestive test of whether mechanisms operate within classes or across courses is performed. Without course assignment data, the only method to ensure course-mates are in the same class is considering courses in which there is only one class. If repeaters are uniformly distributed across classes, the estimates of course repetition externalities would be no different in courses with one class and courses with more than one class. However, if repeaters were not uniformly distributed across classes, estimates of course repetition externalities would be more precisely estimated in courses with one class. For example, in a course with two classes in which all the repeaters are assigned to one of the two classes, about half of the first-time course-takers would experience no repeater effect and about half the first-time course-takers would experience an intensified effect. This would result in a less precise estimate of the overall effect than in a course with one class in which all first-time course-takers were exposed to the repeaters. The analysis can therefore be performed on courses likely consisting of one class and courses likely consisting of more than one class, which is done by considering courses with less than and more than 25 students, respectively.

The subsequent section provides a full description of the data used in the analysis.

3 Data and Descriptive Statistics

This paper uses data from the National Longitudinal Study of Adolescent Health (Add Health). The Add Health is a school-based longitudinal study of a nationally representative sample of US adolescents who were in grades 7 to 12 during the 1994-1995 school year. A core sample was selected to participate in a series of detailed surveys. Complete high school transcript data (grades 9 to 12) are available for individuals selected for the core sample.[10] For all of these individuals, the transcript data include a categorization of the mathematics course taken in every year of high school (or an indication that no mathematics courses were taken that year), the GPA score obtained in each of these courses, and a failure index variable describing whether the student passed or failed these courses.[11] This information is required

[10]The extent of these data is beyond many of the National Center for Education Statistics' transcript surveys which typically focus on single cohorts, allowing for a more specified model.

[11]A subset of students may have taken more than one mathematics course in a given year. For these students, the provided course categorization is for the highest level mathematics course taken that year, the reported GPA score is the mean GPA score over all mathematics courses taken, and the failure index describes the share of mathematics courses failed.

for all students in a school in order to accurately compute the course composition measures used in the analysis. If transcript information is only available for a subset of students in the school, information is only available for a subset of a student's course-mates. The study is therefore restricted to fifteen schools in which all students in the school were selected for the core sample. This is known as the saturated sample. Figure 1 plots the number of students who enrolled in at least one mathematics course in each of these schools, showing that there are two large schools and thirteen smaller schools.

The analysis is restricted to the years between 1992 and 1996 to ensure that courses are mostly populated by students included in the above sample. The pooled sample includes 6341 student-years. Appendix Table 10 reports the demographics of the sample. There are 2270 unique students in the sample, so course achievement data is observed an average of 2.8 times per student. There are 3191 student-year observations describing the achievement of male students and 3150 student-year observations describing the achievement of female students. Descriptive statistics are provided in Table 1. Female students consistently outperform male students across all measures of academic achievement.

Past achievement for each student in each year is described by a set of six variables: failing all mathematics courses the previous year, failing any mathematics course the previous year, failing any mathematics course the previous year and repeating it (which identifies the repeaters studied in this paper), failing all mathematics courses the previous year and progressing[12], failing any mathematics course the previous year and progressing, and repeating the mathematics course from the previous year (not necessarily having failed it). Note that failing any and failing all mathematics courses perfectly correspond for the majority of students who take only one mathematics course.

The first column of Table 1 indicates that six percent of students in the sample fail all their mathematics courses the previous year.[13] This share increases to 17 percent for students failing any mathematics course the previous year. Seven percent of students repeat a failed math course, which corresponds to about two repeaters in a class of 25 who failed the previous year. Students who fail may take no mathematics course, an easier mathematics, the same mathematics course or progress to a higher mathematics course the subsequent year. One percent of students progress to a higher-level mathematics course after failing all their mathematics courses the previous year. This anomalous behavior serves as a reminder that school progression may be somewhat fuzzy. Six percent of students fail some mathematics course, but still progress to a higher course. This could happen, for example, if a student

[12] Progressing is defined as taking a course of a higher level. Table 2 orders courses by level.

[13] Technically, these are student-years, so 6 percent of student-year observations describe students who failed all their mathematics courses the previous year.

takes Algebra I in the first semester, fails it, but then repeats and passes it in the second semester. The set of students who fail some mathematics course and progress are used to identify the set of low achievers the subsequent year. Finally, 14 percent of student repeat their highest mathematics course from the previous year, indicating that several students repeat a course even after passing it.

Course composition measures are obtained by averaging the individual achievement indicators of students in the same course in the same school in the same year. Course-mates may not be classmates if courses are divided into multiple classes within a school. The mean number of students per mathematics course is 112.[14] On average, first-time course-takers are exposed to five students who are repeating the course after failing it the previous year. Course composition is also described in terms of shares rather than counts. The distribution of course sizes for all of the course-years included in the analysis is plotted in Figure 3. The identification relies on variation in the number of repeaters in the larger courses; effects are imprecisely estimated if these courses are excluded. Figure 4 plots the variation in the number of students repeating a failed course per school-course-year. Thirty percent of student-course observations correspond to school-course-years in which no students are repeating a failed course, and the median and mean number of students repeating a failed course per school-course-year are 3 and 5.5, respectively.

Course-specific descriptive statistics are provided in Table 2. Mathematics courses are categorized into nine different groupings by survey administrators.[15] These are loosely ordered by difficulty from Basic/Remedial Mathematics to Calculus. The three most popular high school mathematics courses (by enrollment) are Algebra I, Geometry and Algebra II. Results are largely driven by variation in the number of repeaters across these three courses.

Fifteen percent of students in Algebra I are repeating the course after failing it the previous year. The shares of students repeating the more advanced courses of Geometry and Algebra II are smaller. This indicates that low-ability students who are most likely to fail and repeat select out of mathematics courses after taking Algebra I. This may be by choice or because they are not allowed to progress given their achievement in Algebra I.

The transition of students between mathematics courses is described in the two panels of Table 3. The first panel is based on 3741 student-year observations and describes the course transition of students who passed their previous mathematics course. The second row of this panel indicates that of the 450 students who passed General/Applied Mathematics, 66

[14]Note that these means are computed by equally weighting student-year observations and not by equally weighting course-year observations, so large course-years with many students receive a greater weight. This also explains why the mean shares are not simply the ratios of the mean counts.

[15]The actual categorization process is not important for this paper given that students in the same course in the same school in the same year are necessarily categorized as taking the same course.

percent take Algebra I the following year. Seventy-one percent of students follow Algebra I with Geometry while seven percent follow with Algebra II. Somewhat surprisingly, 16 percent of students repeat Algebra I after passing it. The primary source of this irregularity appears to be one large school. This school is excluded from the analysis in a sensitivity check to confirm that this anomaly is not affecting results.[16] Ninety percent of students who pass Geometry follow it with Algebra II and 74 percent of students who pass Algebra II follow it with Calculus. A typical progression for passing students is a subset of the path General/Applied Mathematics to Algebra I to Geometry to Algebra II to Calculus, although several other course paths are also observed.

The second panel of Table 3 describes the course transitions of students who failed any of their previous year's mathematics course. It indicates that, for most courses, repetition is the modal behavior of students who failed. Interestingly, a nontrivial number of students still progress. Twenty-three percent of students who fail General/Applied Mathematics take Algebra I the next year, 29 percent of students who fail Algebra I take Geometry, and 37 percent of students who fail Geometry take Algebra II. These students are defined as low achievers and are useful for separating general low achiever and specific repeater effects in a secondary set of results.

The final set of descriptive statistics is provided in Table 4. This table describes how current student achievement is associated with past achievement in a series of OLS regressions. The negative coefficients in the first three columns reflect that students repeating a failed math course are lower achievers (and likely of lower ability) than first-time course-takers. The coefficient drops from -0.81 to -0.39 in the third column when school-specific course fixed effects control for course difficulty. This suggests that part of the reduced achievement of repeaters is because they are repeating more difficult courses than those taken by other students.

The remaining three columns in Table 4 include individual fixed effects to control for individual ability. The correlations between repeating a failed course and current achievement are imprecise but positive in the fourth and fifth columns, suggesting an increase in achievement for students repeating a failed course relative to when they took it for the first time. The sixth column includes an indicator for progressing to a higher level mathematics course after failing some mathematics course the previous year. This results in a positive and sizable increase in achievement. The gain in achievement repeating a failed mathematics course and progressing after failing some mathematics are shown to be statistically equivalent in column (7), although the precision of the estimates differs. It does not appear to be the case that students who progress after failing outperform students who repeat. It is

[16] One possible hypothesis is that two different courses at this school were categorized as Algebra I.

emphasized that these associations are descriptive and non-causal.

4 Results

Table 5 reports the primary set of results. Controls are added sequentially across columns. The coefficient on the log number of students failed and repeating of -0.25 in the first column is estimated without controlling for course size and excluding school-course and individual fixed effects. The coefficient falls in magnitude to -0.20 when course size is controlled in the second column, confirming that the previous estimate exaggerated the effect. The log of course size and GPA are negatively correlated. The specification in the third column includes school-course fixed effects to control for course difficulty, and the magnitude of the effect falls further to -0.17. This indicates that the net effect of course difficulty biased the estimates downwards; first-time course-takers systematically perform worse in more difficult courses with more repeaters.

The fourth column reports results from the preferred specification, controlling for course size and including the full set of fixed effects. The coefficient of -0.15 is the level change in GPA scores for first-time course-takers caused by a doubling of the number of repeaters in a course (a 100 percentage point increase in the number of course repeaters or a 1 unit increase in the natural logarithm of the number of course repeaters). For the average mathematics course, this is an increase from five to ten repeaters in a course of around 100 students. The relationship between course size and GPA is no longer statistically different from zero, although the estimate is much less precise.

Results from placebo tests in Table 6 support the robustness of empirical strategy. The first column indicates that doubling the number of repeaters in the course the year before it was taken is associated with a -0.02 (no) change in GPA scores for first-time course-takers. The second column is the original specification, while the third column reveals that the achievement of first-time course-takers is negatively correlated with the number of repeaters the next year, although the estimate is not significant. This is expected as course repeaters are course-takers from the previous year that performed poorly.

Distributional effects are investigated in Figure 5. This graph plots estimates from a series of linear probability models in which binary indicators for attaining at least the specified GPA score are the dependent variables. Results in this figure partially inform our understanding of the negative externalities exerted by repeaters. Negative effects at the top of the distribution may indicate teachers transferring inputs from high achievers to low achievers (such as slowing the pace of the class), negative effects throughout the ability distribution may indicate repeaters being generally disruptive, while negative effects concentrated at the bottom of the distribution may indicate repeaters specifically distracting other low achievers. The negative externalities exerted by repeaters are evident in the middle and lower parts of the distribution, consistent with the findings of Lavy et al (2012). This is evidence against the hypothesis that teachers transfer inputs away from high achievers when there are more repeaters in a course, and suggests repeaters may specifically distract other students in similar parts of the achievement distribution. Course repetition externalities appear to widen the achievement gap between high and low performing students.

Course repeaters may exert negative externalities on first-time course-takers only when they reach a threshold share of the course. This form of nonlinearity cannot be captured by the above specifications. Figure 6 investigates threshold effects by plotting coefficients from a series of regressions taking the form of Equation 2, but with the explanatory variable being a binary indicator of whether the share of repeaters exceeds the specified level. The estimated effect is the difference in GPA scores between first-time course-takers exposed to a share of repeaters above the specified level and first-time course-takers exposed to a share of repeaters below the specified level. The plot suggests the negative effect is already evident when the share of repeaters reaches five percent of course-takers, although it is only statistically significant when the share reaches nine percent of course-takers. The negative effect on first-time course-takers remains relatively flat until the share of repeaters reaches fifteen percent after which it becomes very imprecise.

Results in Table 7 attempt to distinguish the externalities exerted by course repeaters because they are low achievers and the externalities exerted specifically because they are repeating. Recall that low achievers are defined as course-mates who are either repeating a failed course or failed some mathematics course the previous year but still progressed to a higher level course. The estimate in the second column captures both the general low achiever effect and specific repeater effect. Results in column (3) separate these effects. The negative coefficient on the log number of students failed and repeating (-0.09^{**}) indicates the presence of specific repeater externalities. Low achiever effects are negative, but imprecisely estimated. These results suggest that course repetition externalities may operate through both channels, although the evidence supporting repeater effects is stronger. One implication of this is that encouraging low-achieving students to progress to higher-level mathematics course rather than repeat may not fully address the issue as the low achiever negative externalities exerted by these students would persist in the higher-level courses. A more appropriate policy for negating these externalities may be to direct failing students away from mathematics courses or towards less mathematically-demanding numeracy courses.

The final two columns of Table 7 investigate whether effects operate within classes or across courses. The empirical methodology section argues that more precise estimates in courses with one class support repeater externalities operating at the class level. The estimate for courses likely consisting of one class (courses of less than or equal to 25 students) is, indeed, larger in magnitude and more precisely estimated, while the estimate for courses likely consisting of more than one class is very similar in magnitude, but less precisely estimated, in comparison to the initial estimate. This indicates that repeater effects appear to operate inside the classroom either through direct peer interaction or teachers responding to the composition of the class rather than through class assignment within courses. This is consistent with Burke and Sass (2013) who find that ability peer effects operate at the class level rather than grade level.

5 Conclusion

Mathematics is difficult for many students, and course repetition in high school mathematics courses is a common occurrence. This repetition is promoted by policies in several US states that stipulate a minimum level of mathematics to graduate high school, and may become an increasing concern as the Common Core State Standards Initiative is implemented in individual states. Mathematics is also generally considered important for future job market success, acting as further encouragement for students to repeat failed mathematics courses. Previous discussions around the benefits and costs of course repetition have focused on the potentially-repeating individual student.

This paper takes a new step by considering the externalities exerted by course repeaters on other students taking the course for the first time. A doubling of the number of repeaters in a mathematics course leads to a 0.15 reduction (approximately equal to the mean female-male achievement gap) in GPA scores for first-time course-takers. The effect appears to dominate course size effects, and, given the relationship between course size and class size and the extensive literature on class size, warrants more attention.

Using Israeli data, Lavy et al (2011) finds that higher proportions of low-ability students in a grade are associated with reductions in the general quality of the classroom environment. This provides a candidate mechanism through which the negative externalities reported in this paper may operate. The estimated distributional effects indicate that course repeaters negatively affect students at the middle and lower parts of the achievement distribution. This suggests that course repeaters may be more likely to distract classmates who are located in similarly-low parts of the achievement distribution rather than high achievers, which is particularly concerning given these students are already at risk. The effect does not appear to operate through teachers redirecting resources to low-ability students from high-ability students, so policies that promote maintaining a constant level of teacher inputs irrespective of the classroom distribution of repeaters may not be effective in alleviating the negative externalities.

Results also suggest that the negative externalities exerted by course repeaters arise because these students are both low-achieving *and* repeating. This is important because policies that reduce course repetition may not deal with the low-achiever effects. If the negative externalities exerted by course repeaters outweigh the potential benefits of repetition for the repeating student, a more fitting solution may be promoting numeracy courses rather than Algebra and Geometry for high school students who do not display an aptitude for mathematics.

Finally, suggestive evidence indicates that the negative effect is mitigated if the share of repeaters remains below five percent. This presents a possible policy response of stipulating a maximum share of repeaters permitted in a course. The overall finding of negative externalities emphasizes the need to include the effect of repeaters on their classmates when considering optimal grade retention, course repetition and high school graduation policies.

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6 Figures

Figure 1: Number of students enrolled in math courses per school

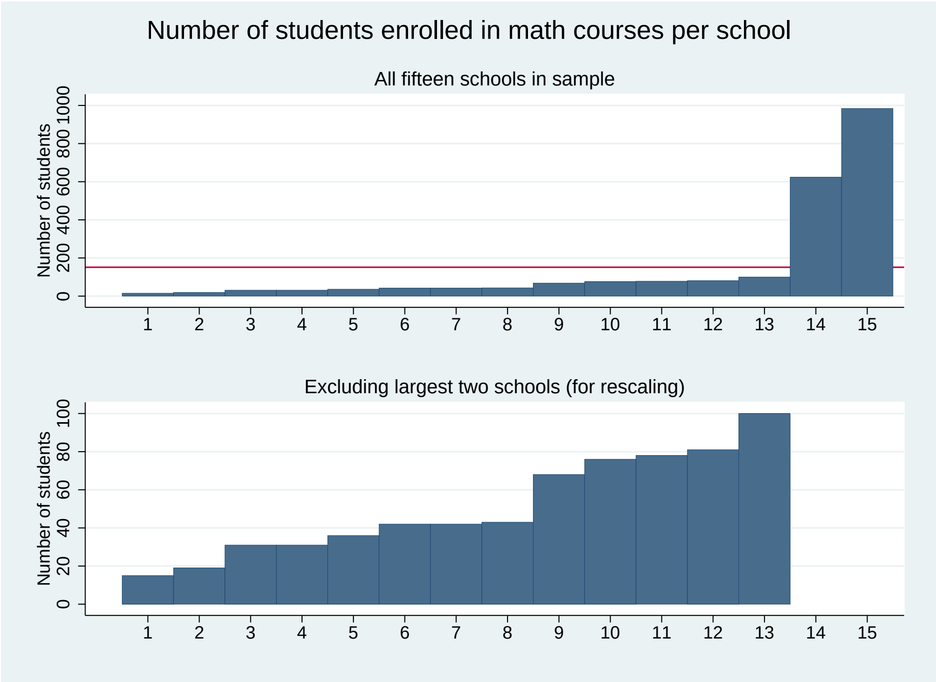


Figure 2: Distribution of math GPA scores by past achievement

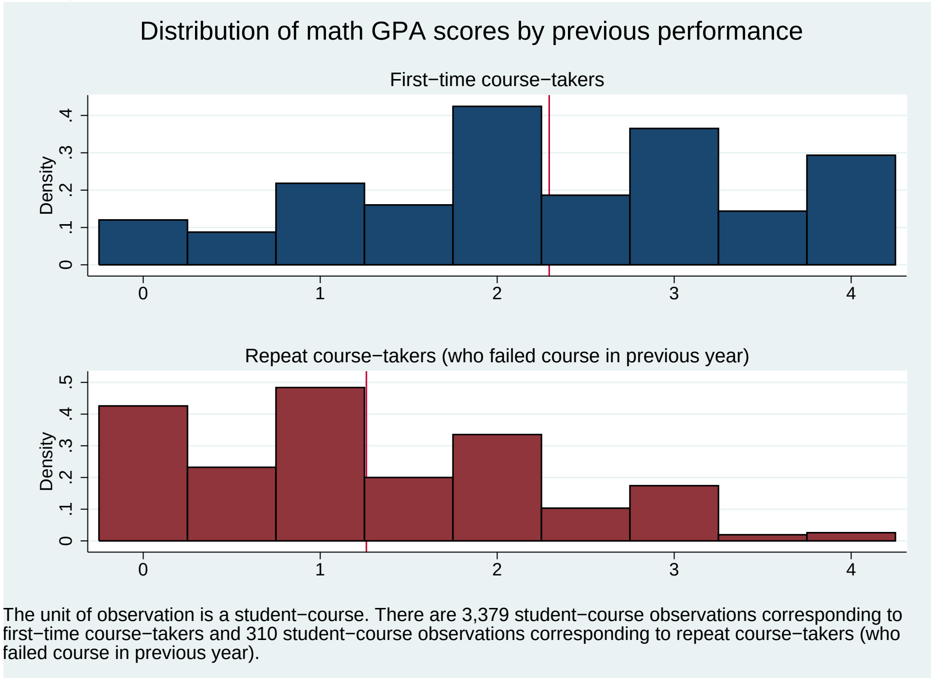


Figure 3: Number of students per school-course-year (class)

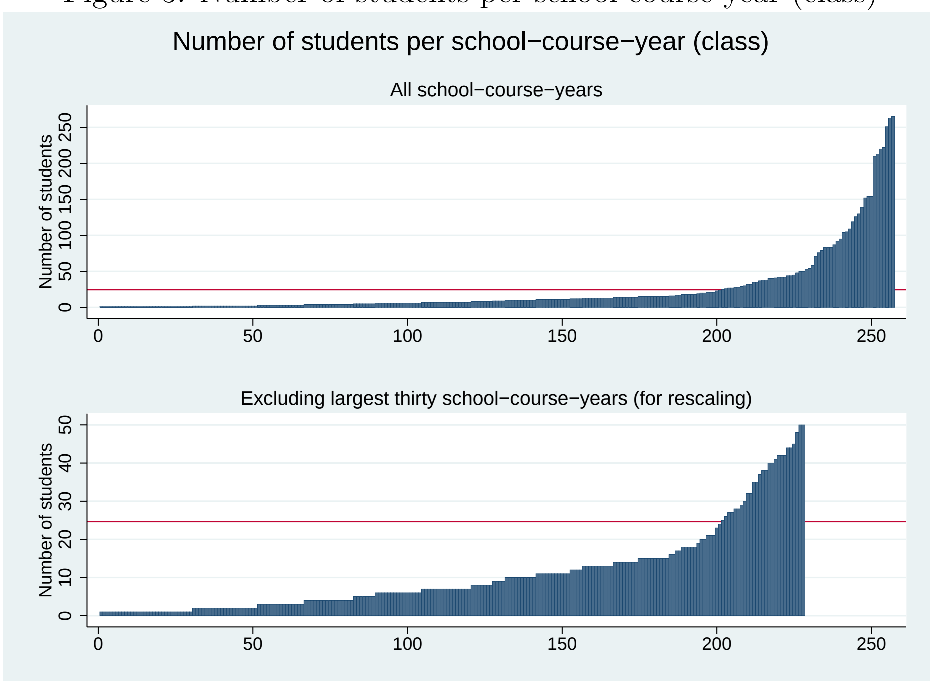
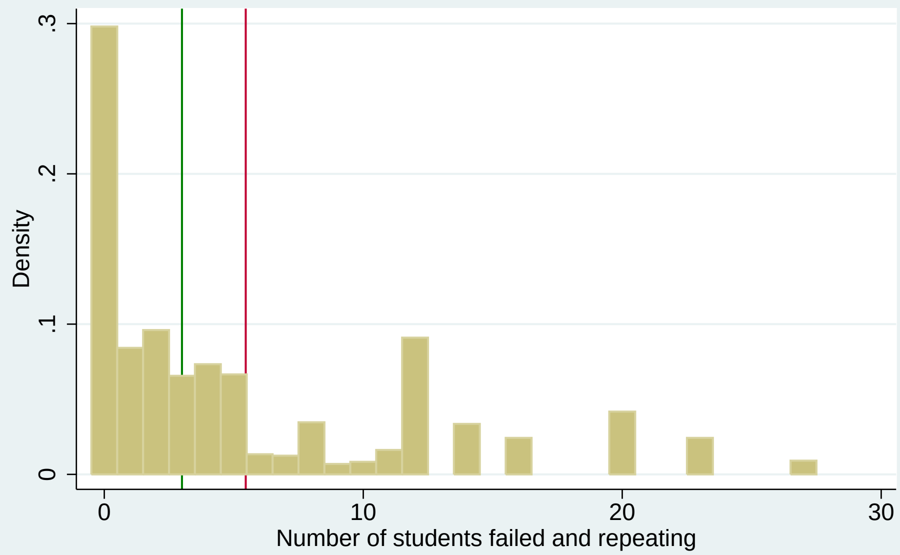
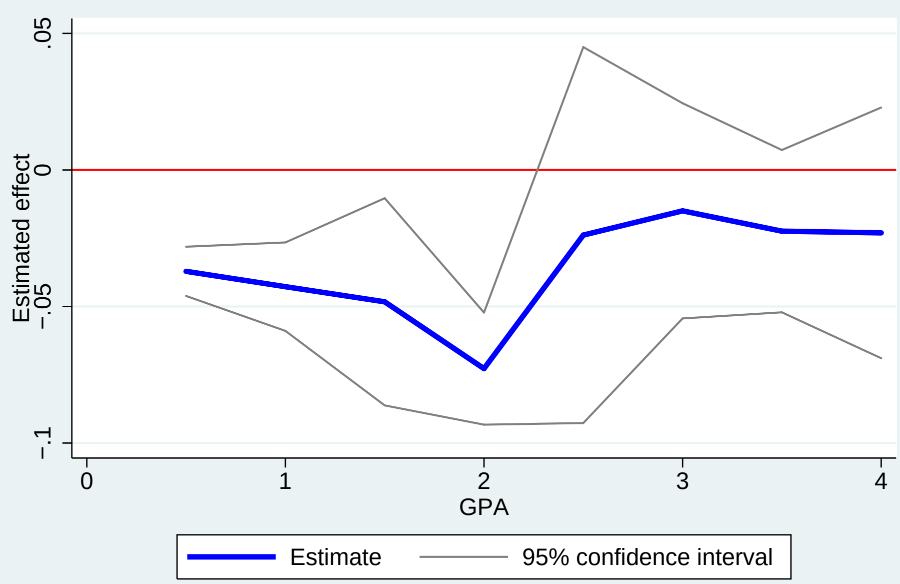


Figure 4: Distribution of number of students repeating a failed course per school-course-year



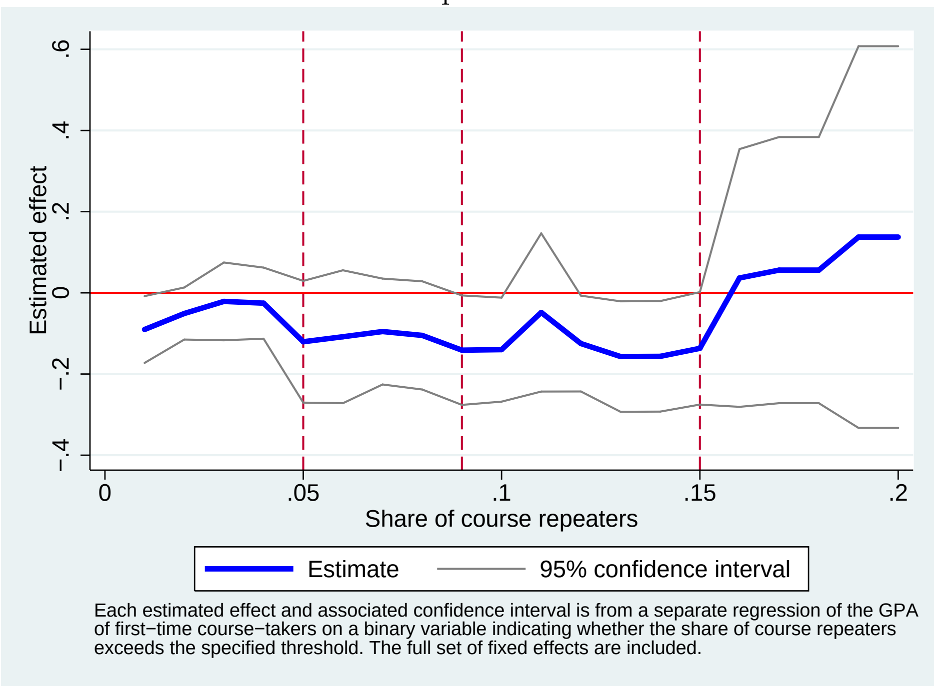
The unit of observation is a student−course. Thirty percent of student−course observations correspond to school−course−years (classes) in which no students are repeating a failed course. The median and mean number of students repeating a failed course per school−course−year are 3 and 5.5, respectively indicated by the green and red vertical lines.

Figure 5: Distributional effects



Each estimated effect and associated confidence interval is from a separate linear probability model. The dependent variable is an indicator for attaining the specified GPA and the independent variable is the natural logarithm of the number of course repeaters. The full set of fixed effects are included.

Figure 6: Threshold effects of share repeaters on GPA of first-time course-takers



7 Tables

Table 1: Descriptive statistics - Pooled (Units: student-years)

	Mean (standard deviation)		
	All	Males	Females
Academic outcomes:			
Math GPA score (transcript)[a]	2.17	2.05	2.28
	(1.17)	(1.16)	(1.17)
Individual past achievement - binary indicators:[b]			
Failed all math courses in previous year	0.06	0.07	0.05
Failed any math course in previous year	0.17	0.19	0.15
Failed[c] and repeating math course	0.07	0.08	0.06
Failed all math courses and progressed	0.01	0.01	0.01
Failed any math course and progressed	0.06	0.06	0.05
Repeating math course from previous year	0.14	0.16	0.13
Course-mates:[d]			
Course size (number of students)	111.63	113.11	110.12
	(88.10)	(87.47)	(88.72)
Number of students failed and repeating	5.46	5.55	5.37
	(6.55)	(6.57)	(6.52)
Share of students failed and repeating	0.08	0.08	0.08
	(0.10)	(0.10)	(0.11)
Observations	6341	3191	3150
Share	1	0.50	0.50

[a]The math GPA score is the mean GPA over all math courses taken in a given year if more than one course is taken in the year. [b]Means for these binary indicators based on smaller samples due to missing past achievement for some individuals. [c]*Failed* is defined as any failure in previous year's math courses for this variable. [d]Course-mates are students in the same school, taking the same course, in the same year.

Table 2: Descriptive statistics by math course - Pooled (student-years)

	Basic/ Remedial	General/ Applied	Pre-algebra	Algebra I	Geometry	Algebra II	Advanced	Pre-calculus	Calculus
Academic outcomes:									
Math GPA score (transcript)[a]	1.66	2.07	1.96	1.92	2.26	2.32	3.00	2.65	3.04
Individual past achievement									
- binary indicators:[b]									
Failed all math courses in previous year	0.24	0.12	0.21	0.08	0.03	0.01	0.00	0.01	0.00
Failed any math course in previous year	0.54	0.36	0.51	0.21	0.13	0.10	0.00	0.05	0.00
Failed[c] and repeating math course	0.11	0.08	0.12	0.15	0.05	0.04	0.00	0.01	0.00
Failed all math courses and progressed	0.00	0.02	0.04	0.01	0.00	0.00	0.00	0.00	0.00
Failed any math course and progressed	0.00	0.02	0.16	0.05	0.08	0.06	0.00	0.04	0.00
Repeating math course from previous year	0.22	0.28	0.20	0.36	0.07	0.05	0.04	0.04	0.00
Course-mates:[d]									
Course size (number of students)	51.12	89.09	41.61	159.69	130.24	100.96	8.07	58.28	24.85
Number of students failed and repeating	3.70	1.92	2.18	10.94	4.48	4.34	0.00	0.79	0.00
Share of students failed and repeating	0.12	0.12	0.13	0.14	0.04	0.04	0.00	0.01	0.00
Observations	340	571	313	1790	1469	1160	72	483	143
Share	0.05	0.09	0.05	0.28	0.23	0.18	0.01	0.08	0.02

[a]The math GPA score is the mean GPA over all math courses taken in a given year if more than one course is taken in the year. [b]Means for these binary indicators based on smaller samples due to missing past achievement for some individuals. [c]*Failed* is defined as any failure in previous year's math courses for this variable. [d]Course-mates are students in the same school, taking the same course, in the same year.

Table 3: Transition matrices - shares: Mathematics (student-years)

Panel A: No math course failure in previous year

	Current course									
Previous course	1	2	3	4	5	6	7	8	9	**Total**
1 - Basic/Remedial	0.19	0.10	0.21	0.47	0.02	0	0	0	0	**163**
2 - General/Applied	0.12	0.11	0.06	0.66	0.03	0.01	0	0.00	0	**450**
3 - Pre-algebra	0.06	0.05	0.08	0.78	0.02	0.00	0	0	0	**202**
4 - Algebra I	0.01	0.02	0.00	0.16	0.71	0.07	0	0.01	0	**1,264**
5 - Geometry	0.00	0.01	0.00	0.02	0.03	0.90	0.01	0.02	0	**947**
6 - Algebra II	0.00	0.04	0.00	0.01	0.12	0.03	0.05	0.74	0.01	**539**
7 - Advanced	0	0	0	0	0.11	0	0.33	0.44	0.11	**9**
8 - Pre-calculus	0	0	0	0	0	0.01	0.09	0.07	0.83	**167**
9 - Calculus	0	0	0	0	0	0	0	0	0	**0**
Total	**115**	**139**	**89**	**758**	**1,012**	**972**	**64**	**449**	**143**	**3,741**

Panel B: Any math course failure in previous year

	Current course									
Previous course	1	2	3	4	5	6	7	8	9	**Total**
1 - Basic/Remedial	0.41	0.07	0.28	0.24	0	0	0	0	0	**68**
2 - General/Applied	0.46	0.18	0.12	0.23	0.01	0	0	0	0	**95**
3 - Pre-algebra	0.30	0.11	0.39	0.18	0	0.02	0	0	0	**56**
4 - Algebra I	0.10	0.08	0.07	0.44	0.29	0.02	0	0.00	0	**323**
5 - Geometry	0.06	0.09	0.09	0.05	0.34	0.37	0	0.01	0	**163**
6 - Algebra II	0.04	0.11	0.04	0.03	0.08	0.53	0	0.18	0	**79**
7 - Advanced	0	0	0	0	0	0	0	0	0	**0**
8 - Pre-calculus	0	0.13	0	0	0.13	0.13	0	0.63		**8**
9 - Calculus	0	0	0	0	0	0	0	0	0	**0**
Total	**134**	**78**	**93**	**199**	**156**	**110**	**0**	**22**	**0**	**792**

Table 4: Correlation between previous and current mathematics achievement

Dependent variable: GPA score	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Previous year academic achievement:							
Failed and repeating course	-0.89***	-0.81***	-0.39***	0.36	0.30		0.33
	(0.04)	(0.02)	(0.08)	(0.25)	(0.21)		(0.21)
Failed course and progressed						0.24***	0.29***
						(0.06)	(0.09)
Fixed effects:							
Year (5)		x	x	x	x	x	x
School-cohort (56)		x	x	x	x	x	x
School-course (84)			x		x	x	x
Individual (2047)				x	x	x	x
Observations (student-years)	4533	4533	4533	4533	4533	4533	4533
Number of students	2047	2047	2047	2047	2047	2047	2047

Robust standard errors clustered by school in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Table 5: Effect of course repeaters on academic performance of first-time course-takers

	Sample: First-time course-takers			
Dependent variable: Math GPA score	(1)	(2)	(3)	(4)
Course-mates:				
Log number of students failed	-0.25***	-0.20***	-0.17***	-0.15**
and repeating	(0.04)	(0.03)	(0.02)	(0.04)
Log number of students in course		-0.18***	-0.11***	-0.03
		(0.03)	(0.04)	(0.15)
Fixed effects:				
Year (5) and school-cohort (53)	x	x	x	x
School-course (78)			x	x
School-course trends (78)			x	x
Individual (1810)				x
Observations (student-years)	3379	3379	3379	3379
Number of students	1810	1810	1810	1810

Robust standard errors clustered by school in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Table 6: Placebo tests: Pseudo course-mate achievement at time $t-1$ to $t+2$

	Sample: First-time course-takers at time t			
Dependent variable: Math GPA score	(1)	(2)	(3)	(4)
Pseudo course-mate achievement at time:	$t-1$	t	$t+1$	$t+2$
Pseudo course-mates:				
Log number of students failed	-0.02	-0.15**	-0.09	0.10
and repeating	(0.04)	(0.04)	(0.08)	(0.09)
Fixed effects[a]	x	x	x	x
Observations (student-years)	3160	3379	3324	3337
Number of students	1739	1810	1790	1783

[a]Year, school-cohort, school-course, school-course trends and individual fixed effects, as well as log number of students in course included. Robust standard errors clustered by school in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Table 7: Mechanisms of course repeater externalities

	Sample: First-time course-takers				
Dependent variable: Math GPA score	(1)	(2)	(3)	(4)	(5)
				Course size:	
				≤ 25	> 25
Course-mates:					
Log number of students failed	-0.15**		-0.09**	-0.36***	-0.14
and repeating	(0.04)		(0.03)	(0.11)	(0.07)
Log number of low achievers		-0.17**	-0.12		
		(0.07)	(0.08)		
Fixed effects[a]	x	x	x	x	x
Observations (student-years)	3379	3379	3379	760	2619
Number of students	1810	1810	1810	534	1408

[a]Year, school-cohort, school-course, school-course trends and individual fixed effects, as well as log number of students in course included. Robust standard errors clustered by school in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

APPENDIX

The appendix describes results from a variety of robustness and sensitivity checks. Appendix Table 10 shows the composition of the sample and is discussed in the text. Appendix Table 11 reports regression results in which course repeaters enter the education production function of first-time course-takers in a variety of functional forms. The first column replaces the log course size control with fully flexible course size fixed effects. The parameter of interest remains negative and significant (-0.58^{**}). The second to fifth columns consider the number of repeaters entering either linearly or quadratically with alternating parametric and non-parametric controls for course size. The estimated coefficients are all negative and precisely estimated except the terms on the higher orders of the quadratic, which are not significantly different from zero. These results are all consistent with course repeaters exerting negative externalities, and it is likely that with considerably more data, nonlinearities in the effects could be more precisely identified. The final two columns report effects of the shares of course repeaters on achievement. The estimated coefficients are negative, but imprecisely estimated. This is not surprising given the evidence in Figure 6, which does not suggest a linear trend in the share.

Gender and race heterogeneity in the effect is investigated by interacting the number of repeaters with gender and race indicators. These results are reported in Appendix Table 12. The third column includes both gender and race interactions. Doubling the number of repeaters in a course reduces the GPA scores of white males (the omitted category) by 0.27. Females are slightly less affected than males, but the gender difference is not statistically different. The negative externalities exerted by repeaters on black first-time course-takers are significantly smaller than those exerted on white students, while other differences by race are imprecisely estimated. Descriptive statistics in Appendix Table 10 indicate that black students are more likely to fail and repeat mathematics courses. The smaller effect for black students suggests smaller effects in schools with more black students, and, given that black students are more likely to repeat, may indicate a declining effect for each additional percentage point increase in the number of repeaters.[17]

Appendix Table 13 reports results that show the phenomenon of students seemingly repeating passed courses does not generate the effects. The first column excludes Algebra I, the main course in which this behavior is observed, and the estimated coefficient remains negative, but is imprecisely estimated. The second column excludes one large school to

[17]The logarithmic functional form captures some nonlinearity in the effect, but actual nonlinearities may be more pronounced or take a different form. The small sample and the related absence of statistical power do not allow a fuller investigation of this; a nonparametric analysis in which a series of bins for the number of repeaters were included as explanatory variables was uninformative.

which the majority of students repeating a passed course attend. The parameter of interest is negative and precisely estimated (-0.13^{**}). The final column excludes both Algebra I and the selected school; the pattern of negative externalities is preserved, but results are imprecise.

Finally, Appendix Table 14 investigates whether course-takers who pass subsequently benefit if a larger number of their previous course-mates failed, thereby reducing their future exposure to low-achieving course-mates. This potential effect would be mitigated if deviations from course-specific failure rates are caused by achievement shocks experienced throughout the distribution, such as teacher absenteeism, as even students who passed would have a poorer academic base from which to perform in the subsequent course. There is no evidence of a positive effect from reducing exposure to low-achieving course-mates. In fact, results indicate potential persistence in mathematics achievement. Passing students from courses in which an above-average number of students failed perform below average in their subsequent mathematics course, suggesting that the negative shock that caused the above-average number of students to fail still adversely affects their achievement. These results are presented with the caveat that the majority of estimates in this table are imprecisely measured.

A Appendix Tables

Table 8: Number of years of high school mathematics courses/credits required for graduation

Years	States	Total
Specified at local level	CO, IA, ME, MA, NE	5
1 year		0
2 years	AK, AZ, CA, ID, MT, ND, WI	7
3 years	CT, DC, DoDEA, HI, IL, KS, KY, LA, MD, MN, MO, NH, NM, NJ, NV, NY, OH, OK, OR, PA, TN, UT, VT, WY	24
4 years	AL, AR, DE, FL, MI, MS, RI, SC, TX, WA, WV	11
Varies by diploma	IN (2-4 yrs), GA (3-4 yrs), NC (3-4 yrs), SD (3-4 yrs), VA (3-4 yrs)	5

Source: Reys et al, 2007

Table 9: Courses required for high school graduation/diploma

Course	States	Total
Algebra I	AL, AR, CA, DoDEA, DC*, FL*, GA*, IL, KY, MD, MI, MS, ND, NH, NM**, OK**, SD, TX, UT*	19
Algebra I Integrated Mathematics I	IN, LA*, NC, TN*	4
Geometry	AL, AR, DoDEA, IL, KY, MD, MI, TX, UT*	9
Geometry or Integrated Mathematics II		0
Algebra II	AR, MI	2
Algebra II Integrated Mathematics III	DE*	1
Algebra I, Geometry, Algebra II OR Integrated Mathematics I-III	LA, TN*, VA	3

* Or an equivalent course, ** Minimum requirement.
Source: Reys et al, 2007

Table 10: Descriptive demographic statistics - Pooled (student-years)

	All	First-time course-takers	Failed and repeating course-takers	Course-takers who pass
Math GPA score (transcript)	2.17	2.29	1.26	2.61
Gender and race:				
Female	0.50	0.51	0.41	0.52
White	0.45	0.45	0.32	0.51
Black	0.13	0.12	0.22	0.11
Hispanic	0.21	0.21	0.22	0.18
Asian	0.18	0.21	0.19	0.18
Other	0.02	0.02	0.04	0.02
Age (years and months)	16.80	17.03	17.12	16.65
Immigrant status and home language:				
Not born in US	0.13	0.15	0.11	0.13
Home language: English	0.84	0.83	0.85	0.85
Home language: Spanish	0.10	0.10	0.13	0.09
Home language: Other	0.06	0.07	0.02	0.06
Parent characteristics:				
Mother ed: Less than high school	0.19	0.18	0.21	0.16
Mother ed: High school	0.32	0.31	0.26	0.33
Mother ed: Some college	0.18	0.18	0.19	0.19
Mother ed: College	0.24	0.26	0.25	0.25
Father ed: Less than high school	0.16	0.15	0.18	0.14
Father ed: High school	0.24	0.23	0.21	0.24
Father ed: Some college	0.16	0.17	0.15	0.17
Father ed: College	0.22	0.25	0.14	0.25
Parent not born in US	0.23	0.26	0.26	0.22
Household income:				
Household income: <$20k	0.09	0.08	0.11	0.09
Household income: $20k-$40k	0.23	0.23	0.24	0.23
Household income: $40k-$60k	0.20	0.21	0.17	0.21
Household income: >$60k	0.18	0.19	0.16	0.19
Observations	6341	3379	310	3937
Share	1	0.53	0.05	0.62

Table 11: Effect of course repeaters on academic performance of first-time course-takers

Dependent variable: Math GPA score	Sample: First-time course-takers (1)	(2)	(3)	(4)	(5)	(6)	(7)
Course-mates:							
Number of students failed and repeating:							
Natural log	-0.58**						
	(0.23)						
Linear		-0.02***	-0.09**	-0.04***	-0.13***		
		(0.004)	(0.03)	(0.01)	(0.04)		
Quadratic				0.001	0.004		
				(0.001)	(0.003)		
Share of students failed and repeating						-0.20	-0.11
						(0.82)	(1.46)
Course size (number of students):							
Linear		-0.001***		0.003			
		(0.0003)		(0.003)			
Quadratic				0.000			
				(0.000)			
Non-parametric	x		x		x		x
Fixed effects[a]	x	x	x	x	x	x	x
Observations (student-years)	3379	3379	3379	3379	3379	3379	3379
Number of students	1810	1810	1810	1810	1810	1810	1810

[a]Year, school-cohort, school-course, school-course trends and individual fixed effects included. Robust standard errors clustered by school in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Table 12: Gender and race heterogeneity in effect of course repeaters

	Sample: First-time course-takers		
Dependent variable: Math GPA score	(1)	(2)	(3)
Course-mates:			
Log number of students failed	-0.21*	-0.21***	-0.27***
and repeating	(0.11)	(0.05)	(0.08)
x Female	0.11		0.12
	(0.14)		(0.13)
x Black		0.21*	0.22*
		(0.11)	(0.10)
x Hispanic		-0.01	-0.01
		(0.11)	(0.11)
x Asian		0.19	0.19
		(0.06)	(0.07)
x Other		0.25	0.25
		(0.41)	(0.40)
Fixed effects[a]	x	x	x
Observations (student-years)	3377	3377	3377
Number of students	1808	1808	1808

[a]Year, school-cohort, school-course, school-course trends and individual fixed effects, as well as log number of students in course included. Robust standard errors clustered by school in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Table 13: Robustness check - excluding selected subjects and schools

	Sample: First-time course-takers		
Dependent variable: Math GPA score	(1)	(2)	(3)
Exclusions:			
Algebra I	x		x
Selected schools		x	x
Course-mates:			
Log number of students failed	-0.06	-0.13**	-0.04
and repeating	(0.04)	(0.06)	(0.06)
Fixed effects[a]	x	x	x
Observations (student-years)	2828	2414	2023
Number of students	1565	1291	1130

[a]Year, school-cohort, school-course, school-course trends and individual fixed effects, as well as log number of students in course included. Robust standard errors clustered by school in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Table 14: Correlation between course failure rate and subsequent GPA

Dependent variable:	Sample: Course-takers who pass			
Subsequent year math GPA score	(1)	(2)	(3)	(4)
Course-mates:				
Log number of students who	-0.09			-0.13
fail and repeat current course	(0.11)			(0.09)
Log number of students who		-0.10*		-0.03
fail current course		(0.05)		(0.07)
Log number of students who			0.02	0.09
repeat current course			(0.07)	(0.06)
Fixed effects[a]	x	x	x	x
Observations (student-years)	3276	3276	3276	3276
Number of students	1860	1860	1860	1860

[a]Year, school-cohort, school-course, leading school-course and individual fixed effects, as well as log number of students in course included. Robust standard errors clustered by school in parentheses. *** p<0.01, ** p<0.05, * p<0.1.