

# Investment Decisions in Retirement: The Role of Subjective Expectations

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## Abstract

The rate of stock holding in the U.S. population is much below what theory suggests it should be. The leading explanations for this under-investing include excessive risk aversion, costs of entry, and misperceptions about possible returns in the stock market. We show that excessive risk aversion is not able to account for the low fraction of stock holding. However, a model with heterogeneous subjective expectations about stock market returns is able to account for low stock market participation, and tracks the share of risky assets conditional on participation reasonably well. Based on the model with subjective expectations, we estimate a welfare loss of up to 12% compared to investment under rational expectations, if actual returns follow the same distribution as in the past 50 years. However, the welfare loss is much smaller if individuals are not very risk averse or if actual returns follow the same distribution as in the past 10 years.

## 1 Introduction

Standard life-cycle models of consumption and labor supply have been successfully used to evaluate possible pension system and Social Security policy changes. These models assume that individuals are forward looking and seek to maximize lifetime utility. When appropriately calibrated to capture the economic incentives of public and private pensions, they provide realistic predictions of household savings and wealth accumulation (Scholz, Seshadri, & Khitatrakun, 2006) as well as of retirement decisions (French, 2005). On the other hand, they tend to generate investment patterns that are grossly at odds with observed portfolio choices, both inside and outside defined contribution (DC) pension plans (see for instance Ameriks and Zeldes (2004) for a description of how portfolio shares vary over the life cycle and Hung, Meijer, Mihaly, and Yoong (2009) for an analysis of retirement savings management). According to a typical life-cycle model of portfolio choice (and to financial experts), retirement-age individuals should

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hold a substantial fraction of their wealth in risky assets. Furthermore, they should gradually reduce the share of wealth held in stocks as the risk of facing high medical expenses increases with age and negative stock market realizations cannot be as easily compensated by labor income (e.g., Cocco, Gomes, & Maenhout, 2005). The empirical evidence is very different. In the U.S. less than 30% of retirement-age individuals hold stocks (Lee, Kapteyn, Meijer, & Yang, 2010). Moreover, conditional on stock holding, the mean (median) share of risky assets is around 25% (18%) and there is no evidence of a gradual reduction in portfolio shares with age (Hurd, 2002; Korniotis & Kumar, 2011).

Different reasons have been put forward to explain the observed patterns. For example, Heaton and Lucas (2000), Hochguertel (2003), Rosen and Wu (2004), and Curcuru, Heaton, Lucas, and Moore (2010) mention excessive risk aversion and background risks (e.g., medical costs), Alan (2006) points at startup costs of starting stock market participation, Haliassos and Michaelides (2003) study borrowing constraints, and Hong, Kubik, and Stein (2004) and Guiso, Sapienza, and Zingales (2008) propose social interaction and trust as explanations. While each of these have their merits, they do not explain empirical patterns satisfactorily and observed investment behaviors are still considered puzzling in the household finance literature (Campbell, 2006).

Recently, authors have focused on subjective equity return expectations to explain the observed low rates of stock holding (Hurd, Van Rooij, & Winter, 2011; Kézdi & Willis, 2008). Individual beliefs may diverge substantially from estimates of returns based on historical series. In particular, low average expectations, coupled with high uncertainty and large heterogeneity in beliefs may be the reason why so many individuals are reluctant to invest in stocks. Data eliciting subjective equity return expectations seem to support this hypothesis. Individuals on average have very pessimistic expectations about the stock market, but beliefs vary substantially.

This paper quantifies the importance of knowledge about stock market rates of return in reducing stock market participation and how economic preparedness for retirement would increase were individuals fully informed about the distribution of stock market returns. We estimate the costs of misperceptions in terms of foregone rates of return and in terms of loss of lifetime utility. In doing so, we study an important issue that has not been addressed in the literature, namely whether a standard lifecycle model of saving and portfolio choice incorporating subjective expectations can reproduce observed investment profiles. Clearly the model would predict that individuals holding negative equity return expectations should exclusively allocate their wealth to risk-free assets. We estimate subjective return distributions from the answers to survey questions (Dominitz & Manski, 2007; Hurd et al., 2011; Kézdi & Willis, 2008) and simulate consumption and investment behavior of individuals with these subjective beliefs. We compare the results with analogous simulations under beliefs based on historical returns. We assess to what extent observed subjective beliefs induce poor portfolio choices and what the consequences are for economic wellbeing.

We limit ourselves to individuals who are fully retired. The advantage of this is that retirees do not face the income risk (job loss, uncertainty about promotions, and generally future labor income) that workers face (Guiso, Jappelli, & Terlizzese, 1996), which simplifies the model and the estimation considerably, and avoids incorrectly attributing patterns due to income risk to misperceptions about stock market risk.

The outline of the paper is as follows. In section 2, we describe the lifecycle model we use. The data, the estimation of auxiliary processes, and the sources of the baseline versions of the parameter estimates are described in section 3. The predictions of the resulting baseline model are evaluated in section 4. This section also explores to what extent different values of the parameters, such as risk aversion or expected stock market returns, are able to match observed patterns in the data. Section 5 discusses the survey questions that elicit subjective beliefs on stock market returns, and computes the resulting subjective distributions of stock market returns. Subsequently, section 6 computes the patterns of stock holding, wealth, and consumption that are generated by the lifecycle model with the subjective expectations governing investment decisions, and compares these with the predictions of the model using historical returns and the patterns observed in the data. Section 7 concludes.

## 2 Model

In this section, we describe a model of life-cycle saving and portfolio decisions for individuals in retirement. In order to keep the computations tractable, the model contains a highly simplified representation of the economic environment that individuals face. Agents choose consumption and allocate their savings between risky and risk-free assets seeking to maximize expected utility over a finite time horizon. The stock of savings comprises private wealth and non-annuitized pension wealth held in Individual Retirement Accounts (IRAs).

We treat retirement income as a constant real income flow,  $P_t = P$  for all  $t$ , consisting of Social Security benefits, defined-benefit (DB) pensions, and annuities. In the current U.S. Social Security system, individuals can start receiving retirement benefits as early as age 62. Retirement need not be concurrent with claiming Social Security benefits and other factors, such as private pensions and health insurance, play an important role in determining the time of withdrawal from the labor force. Spikes in the pattern of retirement at ages 62 and 65 are well documented empirical regularities and are often linked to the incentives embedded in the Social Security system, employer pension plans, and social insurance programs like Medicare (Blau, 1994; Coile & Gruber, 2007; Hurd, 1990). When taking our model to the data, we will only select fully retired individuals age 60 and older. This approach greatly simplifies the model's specification since it allows us to abstract from labor supply decisions and labor income risk.

At each age  $t$ , the individual can be either in good health,  $H_t = \text{good}$ , or in bad health,  $H_t = \text{bad}$ . Survival and health status evolve according to a first-order Markov process with transition probabilities that depend on age and previous year's health status:

$$s_t^h \equiv \Pr(\text{alive}_t \mid \text{alive}_{t-1}, H_{t-1} = h) \quad (1)$$

$$\phi_t^h \equiv \Pr(H_t = \text{good} \mid H_{t-1} = h, \text{alive}_t), \quad (2)$$

where  $h \in \{\text{good}, \text{bad}\}$ .

Besides receiving health shocks, agents incur out-of-pocket medical expenses. The logarithm of real health costs is modeled as a function of the logarithm of retirement income, age, and health status,

$$\ln \text{HC}_t = \delta_0 + \delta_1 \ln P + f(t, H_t) + \eta_t, \quad \eta_t \sim \mathcal{N}(0, \sigma_\eta^2). \quad (3)$$

In order to save one state variable and keep the dynamic programming problem tractable, the innovations  $\eta_t$  are assumed to be i.i.d., except that we allow the variance  $\sigma_\eta^2$  to depend on health status. Hence, persistence in medical expenses is only attributable to persistence in health status. French and Jones (2004) show that the stochastic process for health care costs is well represented by the sum of a white noise term and a quite persistent AR(1) component. Our i.i.d. assumption may, therefore, imply an underevaluation of the duration of health cost shocks and, consequently, of the background risk associated with uncertain medical expenses.

We assume the existence of a single saving habitat, which comprises conventional savings and investment accounts as well as Individual Retirement Accounts (IRAs). In the U.S. tax code, IRAs are treated differently, so our specification is a simplification. Our model assumes that returns are taxed at the source, but no taxes are paid when resources are used for consumption. Taxes are paid on nominal returns. The marginal tax rate  $\tau_r$  applies to nominal investment returns and the marginal tax rate  $\tau_i$  applies to non-investment (pension) income. Since we do not distinguish between capital gains and yield on risky assets, there is no differential taxation on dividend and capital gains.

The investment set is constant and consists of two financial assets: a risk-free one, that can be thought of as bonds, T-bills, or cash, and a risky one, representing the stock market. In each period, individuals choose the share of wealth to hold in risky assets, denoted by  $\alpha_t$ . There is no entry fee to participate in the stock market or cost associated with portfolio rebalancing.<sup>1</sup> Short sales are not allowed, hence  $\alpha_t \in [0, 1]$  for all  $t$ .

The risk-free asset yields a constant real return  $r$  and a real after-tax return

$$\tilde{r} = \frac{1 + [(1 + r)(1 + \pi) - 1](1 - \tau_r)}{1 + \pi} - 1, \quad (4)$$

where  $\pi$  is a constant inflation rate. The equity portfolio delivers a stochastic excess nominal return

$$r_t^e - r = \mu + \epsilon_t, \quad (5)$$

where  $\epsilon_t$  is i.i.d.  $\mathcal{N}(0, \sigma_\epsilon^2)$ , and a real after-tax return

$$\tilde{r}_t^e = \frac{1 + [(1 + r_t^e)(1 + \pi) - 1](1 - \tau_r)}{1 + \pi} - 1. \quad (6)$$

Denoting real wealth with  $W_t$ , its evolution over time is given by

$$W_{t+1} = [\alpha_t(1 + \tilde{r}_{t+1}^e) + (1 - \alpha_t)(1 + \tilde{r})] \{W_t + (1 - \tau_i)P - C_t - HC_t\}, \quad (7)$$

where  $C_t$  is real consumption.

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<sup>1</sup>Market frictions in the form of fixed entry costs are often mentioned as plausible, although partial, explanations for the observed limited stock market participation. Vissing-Jørgensen (2002), Alan (2006), and Paiella (2007) all find evidence of nonnegligible entry costs. While introducing a fixed entry cost in a conventional investment account, Gomes, Michaelides, and Polkovnichenko (2009) consider stock market participation as costless in a retirement account. The idea is that informational and set-up costs associated with direct stockholding are bypassed in the latter. As the present model focuses on relatively older investors, we assume that agents have already sustained such costs earlier in their life.

Preferences are described by a time-separable iso-elastic utility function with coefficient of relative risk aversion  $\gamma$  and discount factor  $\beta$ . Thus, at time  $t$  agents solve the following maximization problem:

$$\max_{C_t, \alpha_t} U_t = \frac{C_t^{1-\gamma}}{1-\gamma} + \sum_{j=t+1}^T \beta^{j-t} \sum_{g \in \{\text{good}, \text{bad}\}} S(j, g | t, H_t) \mathbb{E}_t \left[ \frac{C_j^{1-\gamma}}{1-\gamma} \mid \text{alive}_j, H_j = g \right]. \quad (8)$$

subject to the budget constraint (7) and a borrowing constraint

$$W_t \geq 0 \quad \forall t, \quad (9)$$

where  $\mathbb{E}_t$  is the expectation conditional on information available at time  $t$ , and  $S(j, g | t, h) \equiv \Pr(\text{alive}_j, H_j = g | \text{alive}_t, H_t = h)$ , which follows from the  $\{s_t^h\}$  and  $\{\phi_t^h\}$  defined in (1) and (2).

Since health costs are stochastic, the borrowing constraint (9) cannot be strictly enforced unless we allow for negative consumption or censor the distribution of health costs. In practice, individuals who incur large medical expenses that cannot be covered by the value of their assets may rely on Medicaid or other means-tested government programs. In the literature, this is typically modeled as a minimum consumption floor guaranteed by public transfers. Hubbard, Skinner, and Zeldes (1995) find that such social insurance programs discourage saving at the bottom of the wealth distribution, but have little effect on the wealth accumulation trajectory of more affluent individuals. De Nardi, French, and Jones (2010) show that the size of the consumption floor greatly influences saving decisions at all levels of income. In fact, since out-of-pocket medical expenses rise with income (as they do in our model), a consumption floor constitutes a valuable safeguard against catastrophic medical costs, even for the wealthier. In our model, we set the consumption floor at 1% of retirement income (i.e.,  $0.01P$ ). Since our focus is on portfolio choices, we intentionally choose a relatively low value for this parameter so as to minimize the “safety net” effect of the consumption floor on investment decisions. Indeed, the more generous the level of government transfers, the more shielded are model’s agents against the risk of catastrophic events and the more willing they become to take financial risks. In view of this, our analysis emphasizes the role of self-insurance against the risk of large medical expenses at older ages through not only a buffer stock accumulation, but also optimal portfolio rebalancing.

Very few papers have studied portfolio choices in retirement and how they are affected by the background risk of adverse health shocks resulting in high medical expenses. In the absence of unpredictable medical costs, the constraint in (9) is satisfied without introducing a minimum consumption floor (e.g., Campbell, Cocco, Gomes, & Maenhout, 2001, and Cocco et al., 2005, among others). Yogo (2009) develops a model where retirees choose the level of health expenditure and the allocation of wealth between bonds, stocks, and housing. In his setting, a consumption floor is not required since medical expenditures are endogenous. Hence, the focus is on how individuals can increase their lifetime horizon and self-insure against longevity risk by optimally allocating resources to different asset categories, including their health capital.

In addition to the path of consumption, lifetime utility may depend on bequests. Hurd (1989) estimates a life cycle model of consumption and finds that the marginal utility of bequests is small and, consequently, so are desired bequests. Similarly, using a model of saving for retired

individuals, De Nardi et al. (2010) show that a bequest motive is only important for the richest retirees and that the average wealth profile predicted by the model is virtually unaffected by its presence. Conversely, Lockwood (2012) finds that bequests may explain why few individuals buy annuities. Because we focus on individuals who are already retired and whose annuities we assume to be fixed, the omission of bequests from our model is relatively inconsequential.

## 2.1 Model's Solution

Adapting from Deaton (1991), define cash-on-hand as the sum of liquid assets and retirement income, net of out-of-pocket medical expenses:

$$X_t = W_t + (1 - \tau_i)P - HC_t.$$

The life-cycle maximization problem under consideration involves two continuous state variables—cash-on-hand and retirement income—and two discrete state variables—age and health status. In order to reduce the dimensionality of the problem, we follow Carroll (1992, 1997) and divide all variables by the constant flow of retirement income. After this normalization, the model's state space features the ratio of cash-on-hand to retirement income, which we indicate with  $x_t = X_t/P$ , age, and health status. We refer to the Appendix A for a more detailed discussion of how the solution algorithm is implemented.

A solution consists of a set of policy functions for consumption and portfolio composition:  $c_t^* = c(x_t, H_t, t)$  and  $\alpha_t^* = \alpha(x_t, H_t, t)$ . Optimal consumption is increasing and convex in normalized cash-on-hand (Gourinchas & Parker, 2002). The optimal portfolio share of the risky asset is decreasing in normalized cash-on-hand. This is because at lower levels of cash-on-hand, the risk-free asset position represented by the constant retirement income stream is relatively high and agents can tilt their portfolios more heavily toward risky investments. As cash-on-hand increases, the relative importance of retirement income on total wealth decreases and so does the optimal share of the risky asset.<sup>2</sup> The optimal share of risky assets is not defined for very low values of wealth at which saving is zero and no resources are available for investment.

At older ages, the optimal share of risky assets conditional on normalized cash-on-hand is lower. Since retirement benefits will be received over a shorter time horizon, the net present value of the risk-free income stream is lower and so is the willingness to bear financial risks in the form of risky investments. This behavior is reinforced by the risk of large out of pocket medical expenses increasing with age.

## 3 Data and parameters

We use the Health and Retirement Study (HRS; Juster & Suzman, 1995). The HRS is a longitudinal biennial survey of individuals over the age of 50 and their spouses, which represents

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<sup>2</sup>As cash-on-hand increases, the optimal share of the risky asset asymptotes to the value of  $\mu/\gamma\sigma_\epsilon^2$ . This is the constant, optimal share of risky assets generated by a portfolio model with CRRA preferences when markets are frictionless and complete and/or agents face no uncertainty about available future resources (Samuelson, 1969; Merton, 1969).

the primary source of information about the elderly and future elderly in the U.S. The data contain extensive information on household economic condition and demographics, individual employment history, retirement planning, pensions, and health status. We use data from 2000 to 2010. Because assets are measured at the household level, the household is our unit of analysis. For individual-level variables, such as age, we use the values for the financial respondent. An exception is health, which plays an important role in determining medical expenses. The financial risk of medical expenses is primarily caused by individuals in bad health, and therefore, we define a household-level health as being “bad” if at least one of the spouses reports being in fair or poor health, and as “good” otherwise.

In our baseline analysis, we adopt a very comprehensive measure of household wealth, which includes housing, vehicles, and financial wealth, but in section 6.1, we explore the sensitivity of our results to the definition of wealth by repeating the analysis with only financial assets. Appendix B provides more details about the data and construction of relevant variables.

We consider stocks as the risky asset. Figure 1 shows the fraction of individuals in our sample who hold stocks, and among those who hold stocks, what the average fraction of their wealth in stocks is. Stock holding is relatively constant across age at slightly below 40%, whereas the share of risky assets slopes upward a little with increasing age.<sup>3</sup> In the following sections, we will present analogous patterns based on the model’s predictions and compare them to their empirical counterparts.

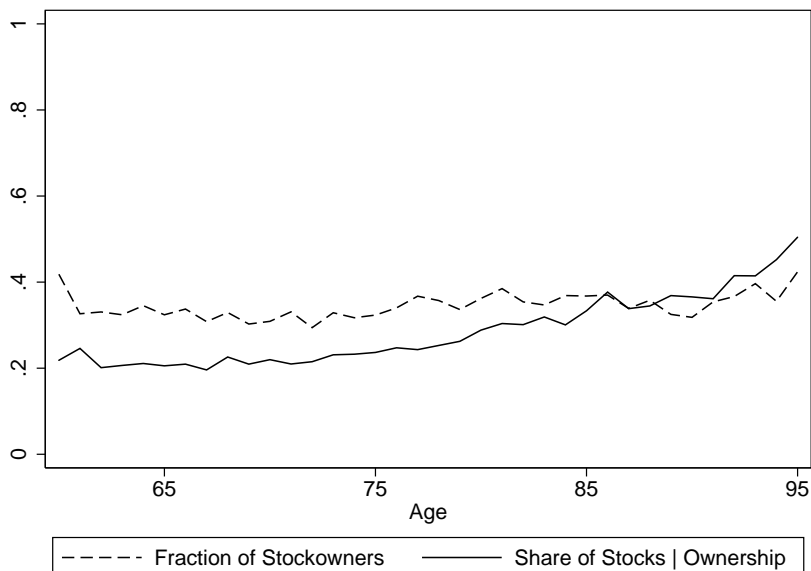
Two auxiliary models inform the life-cycle model presented above. The first is used to estimate conditional survival and health transition probabilities. Following De Nardi et al. (2010), we take the unconditional survival probabilities from life tables and estimate the probabilities of being in a given health status conditional on being currently alive, alive next year, in good and bad health last year, from the data. These four conditional probabilities are specified as a function of a cubic polynomial in age and jointly estimated by maximum likelihood. Observed two-year transitions are written as functions of these four conditional probabilities and life tables survival probabilities and inferred accordingly. One-year survival and transition probabilities are then derived and used as inputs in the dynamic programming problem. The second auxiliary model deals with the process of out-of-pocket medical expenses. Equation (3) is expressed in terms of medical costs normalized to retirement income and estimated as a panel data regression model with individual and year fixed effects, a cubic polynomial in age, and an interaction of health and age. The specification and estimation of the auxiliary models is discussed in more details in Appendix C.

The remaining parameters of the model—tax rates, the inflation rate, the distribution of asset returns, the discount factor, and relative risk aversion—are taken from the literature or external data sources. Table 1 summarizes the values for the baseline model’s parameters. See Appendix D for further details.

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<sup>3</sup>Several reasons may explain the observed increase in the share of risky assets at very old ages. For example, retirees may spend down their non-equity wealth before tapping their stock market investments. Also, mortality may play a role, as richer and more educated individuals are more likely to hold stocks and live longer.

**Figure 1:** Stock holding and average share of risky asset conditional on stock holding among retired individuals age 60 and older in the HRS 2000–2010



**Table 1:** Baseline model’s parameters

Initial age	60
Final age ( $T$ )	100
Discount factor ( $\beta$ )	0.96
Relative risk aversion ( $\gamma$ )	4
Real risk-free return ( $r$ )	1.57%
Mean real equity premium ( $\mu$ )	5.0%
Standard deviation of real risky asset returns ( $\sigma_\epsilon$ )	17.0%
Inflation ( $\pi$ )	4.0%
Pension income tax rate ( $\tau_i$ )	20%
Investment income tax rate ( $\tau_r$ )	15%

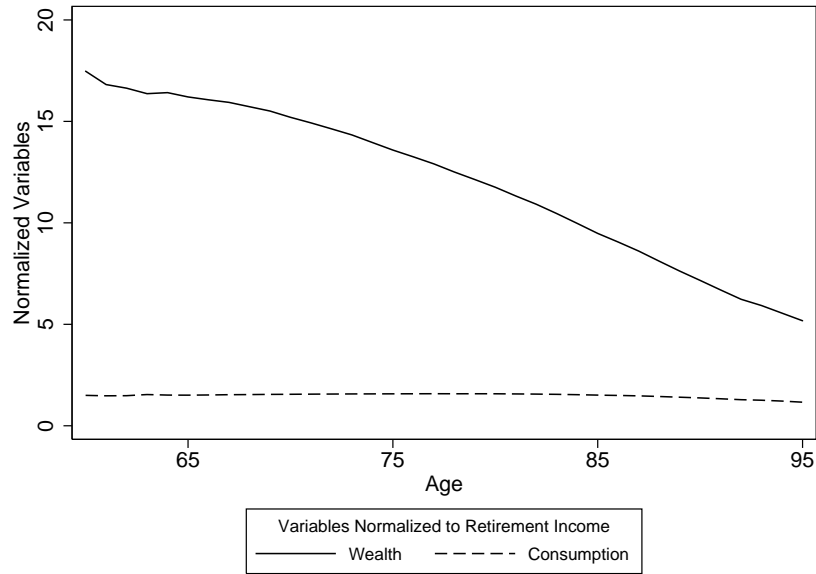
## 4 Model predictions with historical expectations

In this section we describe the wealth accumulation and investment patterns obtained when agents hold rational expectations and base their estimates of future market gains on historical returns. We thus solve and simulate the model under the assumptions that (1) individuals expect that returns in the future are characterized by the parameters in Table 1 and (2) that their expectations are fulfilled, that is, future realizations will be in line with the historical trend.

We select fully retired individuals 60–64 years of age observed between 2000 and 2010, who



**Figure 2:** Wealth and consumption in the baseline model



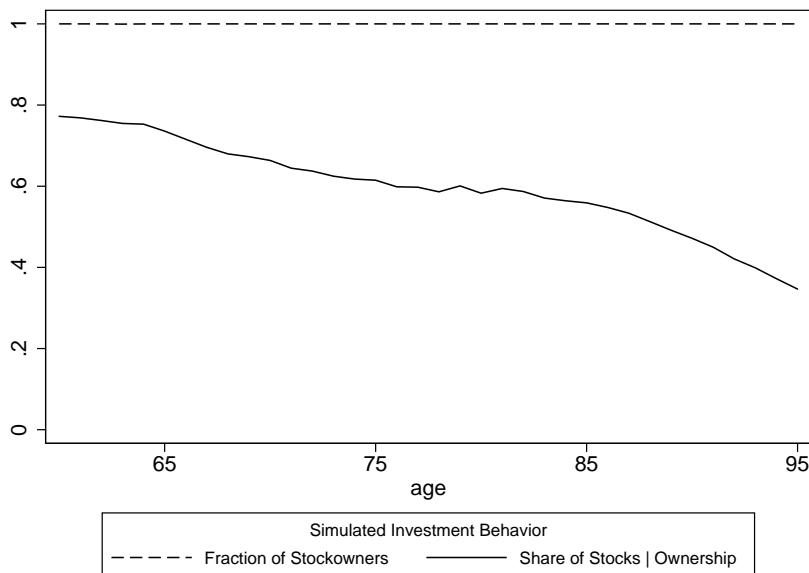
satisfy certain restriction on income and wealth (see Appendix B), so that their initial portfolio composition is a meaningful problem. This leaves us with a pool of 3,607 retirees, whose characteristics are input in the model as initial conditions. We sample from this pool 10,000 times with replacement to obtain our simulation sample. The distributions of the three state variables—age, health status, and normalized cash-on-hand—in the simulation sample can be found in Appendix B.

We simulate an outcome path for each individual in the sample, using the optimal decision rules (described above) that solve the model. Below we report cross-sectional means of relevant variables from age 60 to age 95. Wealth and consumption are both normalized to retirement income. Investment behavior is illustrated by the fraction of individuals who hold any wealth in stocks, and by the average share of wealth held in stocks conditional on holding any.

Figure 2 shows the evolution of (normalized) consumption and wealth over the life cycle. Consumption is perfectly smooth at about 1.5 times retirement income. The initial stock of household wealth in the simulation sample amounts to about 17 times retirement income. Total wealth declines rather modestly early into retirement and starts to be largely run down in the late 70s. This is consistent with the data, where the elderly are observed to reduce their asset holdings only slightly until very late in life. As in De Nardi et al. (2010), the increased risk of unexpected large out-of-pocket medical expenses induces older households to save more for precautionary reasons and significantly slows down asset decumulation.

Figure 3 shows the fraction of retirees holding stocks and the average share of wealth held in stocks conditional on stockholding. With the historical equity premium at 5% and return volatility at 17%, the model predicts that everyone should invest in stocks. This is greatly at odds with the

**Figure 3:** Stock holding and average share of risky assets conditional on stock holding in the baseline model

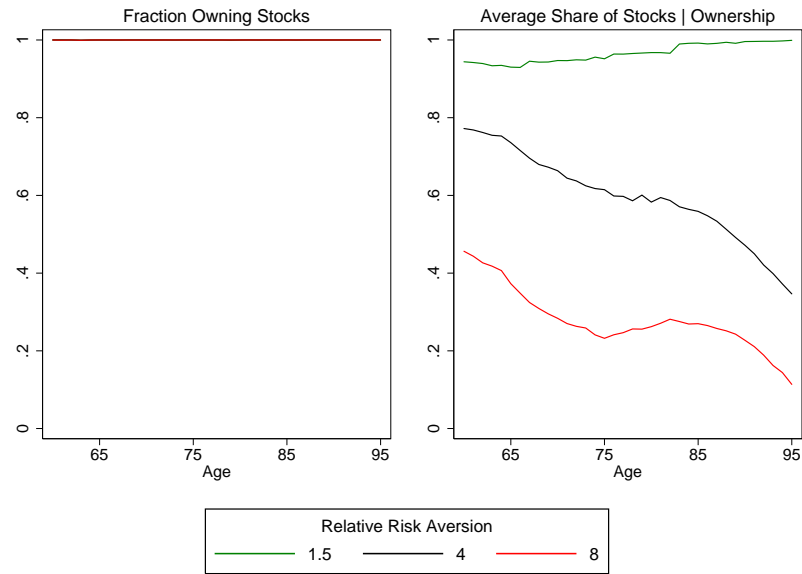


stock market participation rate of less than 40% that we observed in the HRS (Figure 1). This finding, however, is not surprising and confirms the well-documented “stock-market participation puzzle” (Haliassos & Bertaut, 1995): under the assumption of rational expectations based on historical returns, a standard life-cycle portfolio model cannot replicate the observed low rate of household stock market participation unless it features extremely high values of risk aversion or substantial market frictions in the form of large entry and transaction costs (Alan, 2006).

The model also predicts that individuals keep most of their wealth (75%) in risky assets at age 60 and gradually move towards more conservative portfolios as they age. This is in line with the typical financial planners’ advice of shifting investments away from stocks in retirement (e.g., Cocco et al., 2005). The reason for this is that younger individuals are better able to overcome negative portfolio returns using retirement income over a longer horizon. Because the optimal share of risky assets decreases in normalized wealth, asset decumulation towards the end of the life cycle could induce a higher share of the risky asset. This mechanism is, however, counteracted by the need for hedging against the risk of large medical expenses at old ages. The net effect is a sharp decline of the fraction of wealth held in risky assets from age 85 to 95.

The model approximates the portfolio profile observed in the HRS better with alternative parameter values. For instance, Figure 4 shows that if individuals are twice as risk averse as in the baseline model, the predicted share of risky assets is about 25% on average, which is much closer to the empirical pattern in the data. The need for precautionary savings increases with the degree of prudence (Kimball, 1990), which, with a CRRA utility function, is directly linked to the degree of risk aversion. For higher values of risk aversion, the preference for a buffer stock of

**Figure 4:** Effect of relative risk aversion ( $\gamma$ ) on investment behavior

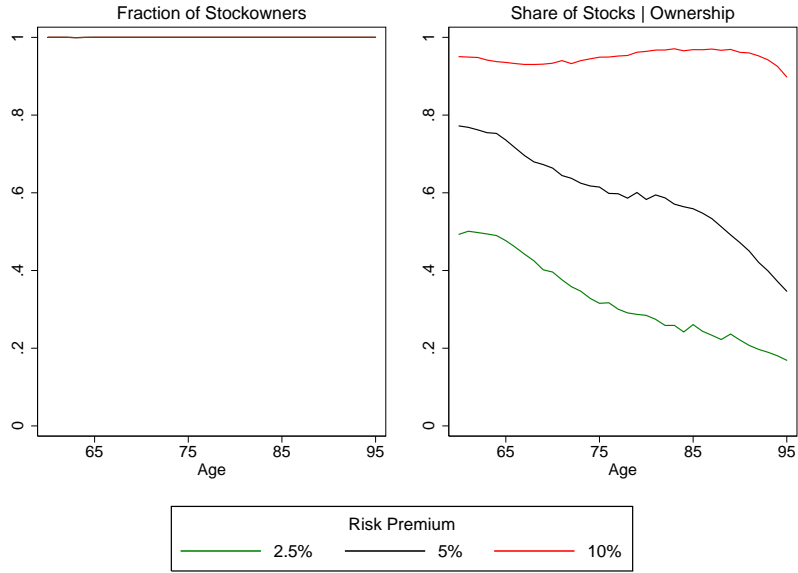


wealth increases, inducing individuals to run down their assets more slowly and bear less financial risk by holding a more conservative portfolio. Such a high value of risk aversion, however, appears to be in contrast with empirical estimates. Hurd (1989) and Attanasio and Weber (1993, 1995), among others, find evidence of a relative risk aversion as low as 1.

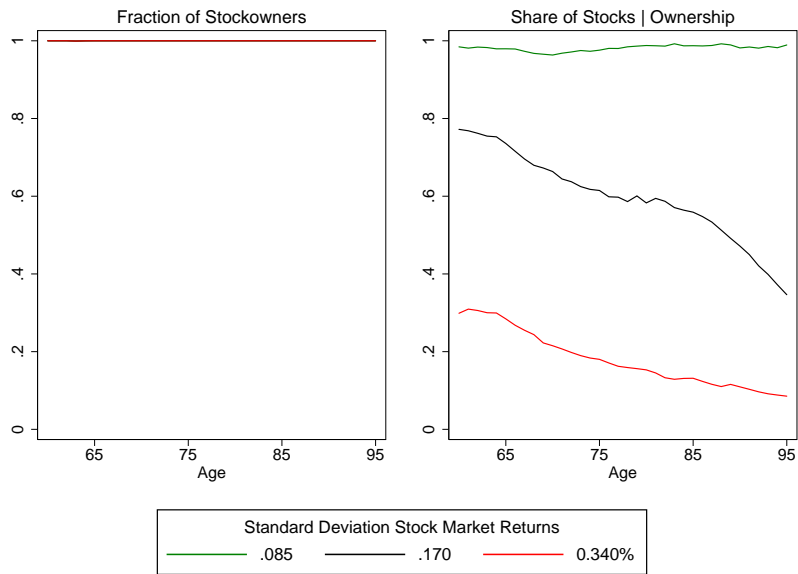
The investment patterns observed in the data can also be better approximated should the risk premium or its volatility differ from their historical values. Figures 5 and 6 show that if the risk premium is half the historical one or if return volatility is double the historical one, then the predicted share of risky assets is also about 25% on average. This exercise indicates the potential of using return expectations not based on historical returns to bring the model's predictions closer to observed investment patterns.

However, even with different parameter values, the model is not able to reproduce the low fraction of stock holding that we see in the data, but predicts that all individuals should participate in the stock market. In what follows, we will document that a substantial fraction of individuals with very pessimistic beliefs about the stock market, combined with fractions of individuals with moderate or optimistic beliefs, leads to predictions that resemble the data more closely on both the intensive and extensive margins.

**Figure 5:** Effect of the risk premium ( $\mu$ ) on investment behavior



**Figure 6:** Effect of the return volatility ( $\sigma_\epsilon$ ) on investment behavior



## 5 Subjective beliefs about the distribution of stock market returns

Since 2002, the HRS has elicited respondents' beliefs about stock market returns. The basic question, asked in each wave, is:

*By next year at this time, what is the percent chance that mutual fund shares invested in blue chip stocks like those in the Dow Jones Industrial Average will be worth more than they are today?*

We call this question P0. From 2006 onward, a follow-up question has been added to understand whether a 50% answer to P0 expresses epistemic uncertainty. In 2008, follow-up questions have been introduced after 0% and 100% answers to P0 so as to gain insight on whether these are rounded (in which case respondents are probed again to give a continuous answer) or actual beliefs of certainty. We take all this information into account when using answers to P0.

In 2002, one additional question, which we call P10, similarly asked the percent chance that they have grown by 10 percent or more, with the order of P0 and P10 randomized between two groups.

In 2010, two similar questions were asked. P20 asks about 20 percent gain and P[-20] asks the percent chance that stocks will *fall* by more than 20 percent.

In 2008, respondents were randomized into one of 8 follow-up questions after answering P0 and asked about the probability of stock market gains of 10, 20, 30, or 40 percent (P10–P40) or losses of 10, 20, 30, or 40 percent (P[-10]–P[-40]). Each respondent answered at most two questions, P0 and one follow-up.

Summarizing, for the distribution of the one-year ahead stock market return, HRS respondents gave one probability in 2004 and 2006, up to two probabilities in 2002 and 2008, and up to three probabilities in 2010. Under suitable assumptions, we can use these self-reported probabilities to compute individual-specific distributions of stock market returns. Throughout, we assume that these questions elicit beliefs about nominal before-tax returns.

Dominitz and Manski (2007) study the 2004 data, which only have P0. They assume that respondents' subjective distributions of stock market returns are normal with a constant standard deviation equal to the historical standard deviation of nominal returns (for which they use 0.206). Under these assumptions, the 2004 data can be exploited to infer individual-specific expected stock market returns. Let  $m_i$  be respondent  $i$ 's subjective mean,  $s_i = s$ ,  $\forall i$ , be respondent  $i$ 's subjective standard deviation, and  $p_{0i}$  be the answer to P0 divided by 100. Then  $m_i = -s\Phi^{-1}(1 - p_{0i})$ , where  $\Phi(\cdot)$  is the standard normal cumulative distribution function. This method can be used for the 2004 and 2006 data. This method can be generalized if more information is available. For instance, whenever respondents give two probabilistic answers, the assumption that  $s_i$  is constant across individuals can be relaxed and both the subjective mean and standard deviation of the distributions of returns can be inferred. Specifically, denoting with  $p_{xi}$  and  $p_{yi}$  the probabilities of a stock market gain of at least  $x$  and  $y$ , respectively (e.g.,  $x = 0$  and  $y = 0.10$  in 2002), the

following system of two equations in two unknowns can be solved:

$$s_i = \frac{y - x}{\Phi^{-1}(1 - p_{yi}) - \Phi^{-1}(1 - p_{xi})} \quad (10a)$$

$$m_i = x - s_i \Phi^{-1}(1 - p_{xi}) = \frac{x \Phi^{-1}(1 - p_{yi}) - y \Phi^{-1}(1 - p_{xi})}{\Phi^{-1}(1 - p_{yi}) - \Phi^{-1}(1 - p_{xi})}. \quad (10b)$$

When, as in 2010, three probabilistic questions are asked, the two parameters  $m_i$  and  $s_i$  would be over-identified (a system of three equations in two unknowns). In this case a minimum distance estimation procedure could be applied to obtain subjective means and standard deviations of individual-specific return distributions.

Hurd and Rohwedder (2011) take a different approach. They observe that respondents' answers to  $k$  questions reveal subjective probabilities that stock market returns take values within  $k + 1$  mutually exclusive intervals spanning the entire real line. They proceed to assume that the subjective distribution *within* each interval coincides with the historical distribution within that interval. For example, suppose the respondent answers only P0. From this we can deduce  $\Pr_i(r^e < 0)$  and  $\Pr_i(r^e \geq 0)$ , where  $r^e$  is the nominal stock market return (as in the model of section 2) and the subscript  $i$  denotes respondent  $i$ 's subjective distribution. Then

$$\begin{aligned} \mathbb{E}_i(r^e) &= \Pr_i(r^e < 0) \mathbb{E}(r^e \mid r^e < 0) + \Pr_i(r^e \geq 0) \mathbb{E}(r^e \mid r^e \geq 0) \\ \mathbb{E}_i[(r^e)^2] &= \Pr_i(r^e < 0) \mathbb{E}[(r^e)^2 \mid r^e < 0] + \Pr_i(r^e \geq 0) \mathbb{E}[(r^e)^2 \mid r^e \geq 0]. \end{aligned}$$

The conditional expectations on the right-hand side are taken from historical distributions, and the subjective mean and standard deviations are obtained as  $m_i = \mathbb{E}_i(r^e)$  and  $s_i = \{\mathbb{E}_i[(r^e)^2] - m_i^2\}^{1/2}$ . This method immediately generalizes to any number of intervals.

We have experimented with both approaches and found that they give very similar results. Here, we only present results using the Hurd-Rohwedder approach.<sup>4</sup>

After computing subjective means and standard deviations of nominal returns, we transform them by correcting for inflation and the risk-free return so as to obtain the subjective real risk premium and subjective standard deviation of real excess returns. For consistency with the subjective expectations about stock returns, we should also use subjective expectations about inflation and the risk-free return. The HRS does not elicit subjective expectations about prices and the risk-free interest rate, but the Survey of Consumers (<http://www.sca.isr.umich.edu/>) does. The average subjective inflation rate reported by individuals in the Survey of Consumers

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<sup>4</sup>We use basic probability rules to identify inconsistent answers. First, we drop self-reported probabilities that are outside the  $[0, 1]$  interval. The procedure suggested by Dominitz and Manski (2007) implies that only values strictly greater than 0 and strictly less than 1 can be used. Given that a significant fraction of the respondents clusters at 0 or 1, we approximate a "zero probability" with 0.001 and a "one probability" with 0.999. This allows us to retain most of the selected individuals in the sample when estimating subjective distributions of returns. The Hurd-Rohwedder approach does not require this approximation as it can be implemented for probabilities taking value 0 or 1. Second, when two or more probabilities are elicited, we define as inconsistent all answers that do not conform to the probabilities of the single events and their union. If, for instance P0 and P10 are asked, we define as inconsistent all answers for which  $P0 \leq P10$ . In this case, we only use P0 and ignore P10. The inequality above is required to be strict under the Dominitz-Manski approach, but not under the Hurd-Rohwedder approach.

over the period 2002–2010 is 3.75%. We use this value to compute the subjective real expected stock market returns from the subjective nominal stock market returns. The Survey of Consumers asks its respondents if they expect any change in interest rates during the next 12 months, but not what the expected interest rate will be. However, most respondents expect the interest rate in the coming year to be about the same as the interest rate in the past year. Therefore, in order to construct a proxy of the subjective real interest rate, we take the average nominal interest rate from 2002 to 2010 and correct it by the aforementioned expected inflation rate of 3.75%. This returns a real interest rate of 0.3%, which we use in the computation.

We discretize the distribution of risk premium into five intervals using the following rule

$$\text{interval} = \begin{cases} 1, & \text{if } \text{risk premium} \leq 0; \\ 2, & \text{if } 0 < \text{risk premium} \leq 75^{\text{th}} \text{ percentile}; \\ 3, & \text{if } 75^{\text{th}} \text{ percentile} < \text{risk premium} \leq 85^{\text{th}} \text{ percentile}; \\ 4, & \text{if } 85^{\text{th}} \text{ percentile} < \text{risk premium} \leq 95^{\text{th}} \text{ percentile}; \\ 5, & \text{if } 95^{\text{th}} \text{ percentile} < \text{risk premium} \leq 99^{\text{th}} \text{ percentile}; \end{cases}$$

and collapsing the distribution to the interval midpoints. Within each of these intervals, we then take terciles of the standard deviation and again collapse distributions to the interval midpoints. The advantage of having a discrete distribution with a small number of distinct values is that we do not have to add a state variable to the model. Instead, we run the model for each group separately and aggregate the results afterwards, which is much less burdensome computationally. We pool individuals with zero or negative subjective risk premia in the same group since their optimal decision is to never invest in stocks. Also, because of ties, the grouping of standard deviation values does not always result in exactly one third of the sample, and we do not always have three different groups within each risk premium interval.

The resulting distributions are presented in Table 2. A large fraction of respondents (more than 60%) is very pessimistic about stock market returns and expects them to earn less than the risk-free rate. The fraction varies between waves, though, and appears to be related to the business cycle, with the highest fraction of pessimistic individuals observed during the recent financial crisis (2008).<sup>5</sup> The distributions are also skewed to the right, with noticeable fractions of respondents expecting risk premia of up to 20%. Our findings—more pessimistic expectations about stock market gains than historical averages, high degree of heterogeneity in subjective expectations, and reported beliefs linked to the recent stock market performance—are in line with those documented by Hurd (2009).

## 6 Model predictions with subjective expectations

We now analyze the extent to which replacing expectations based on historical returns with subjective expectations elicited through survey questions affects the model’s predictions about

<sup>5</sup>Cross-wave comparisons are subject to the caveat that expectations are elicited by different questions. For instance, HRS respondents are only asked about P0 in 2004 and 2006, while they report P0, P20, and P[–20] in 2010.

**Table 2:** Estimated distributions of subjective risk premium and volatility

2002			2004–2006			2008			2010		
RP	SD	Fraction	RP	SD	Fraction	RP	SD	Fraction	RP	SD	Fraction
≤ 0	14.4	60.0	≤ 0	16.8	68.4	≤ 0	17.9	68.7	≤ 0	16.5	63.2
.09	11.2	15.0	1.6	18.9	6.6	1.8	14.5	3.1	2.6	13.8	4.0
				19.2	0.4		20.2	2.0		21.1	4.0
							23.7	2.6		28.7	4.0
3.7	10.4	3.5	4.0	17.5	5.8	5.1	11.8	3.3	7.0	12.8	3.7
	17.4	3.5		18.1	4.2		18.8	3.2		21.9	3.1
	19.3	3.2		18.6	0.6		23.7	3.2		27.9	3.3
9.4	12.4	3.5	8.7	15.6	9.9	10.0	12.5	5.6	11.9	12.8	3.3
	16.5	3.5					17.2	1.1		18.9	3.3
	18.2	3.1					22.9	3.3		24.7	3.3
15.6	8.0	2.6	12.6	10.7	3.7	17.2	10.5	1.3	19.2	5.6	1.7
	9.6	0.9		11.8	0.4		17.2	1.3		10.7	1.6
	12.2	1.2					23.9	1.3		21.4	1.6

*Notes.* RP = subjective risk premium ( $\mu$ ), SD = subjective volatility ( $\sigma_e$ ), Fraction = fraction of the sample (weighted). Computations use P0 and P10 for 2002; only P0 for the two combined waves 2004–2006; P0 and one question among P10–P40 or P[–10]–P[–40] for 2008; P0, P20, and P[–20] for 2010.

household stock holding at both the intensive and extensive margins. We also gauge the robustness of the predictions to different values of key preference parameters (e.g., degree of risk aversion) and quantify the welfare implications of portfolio choices induced by subjective beliefs about future market returns that diverge from expectations based on historical returns.

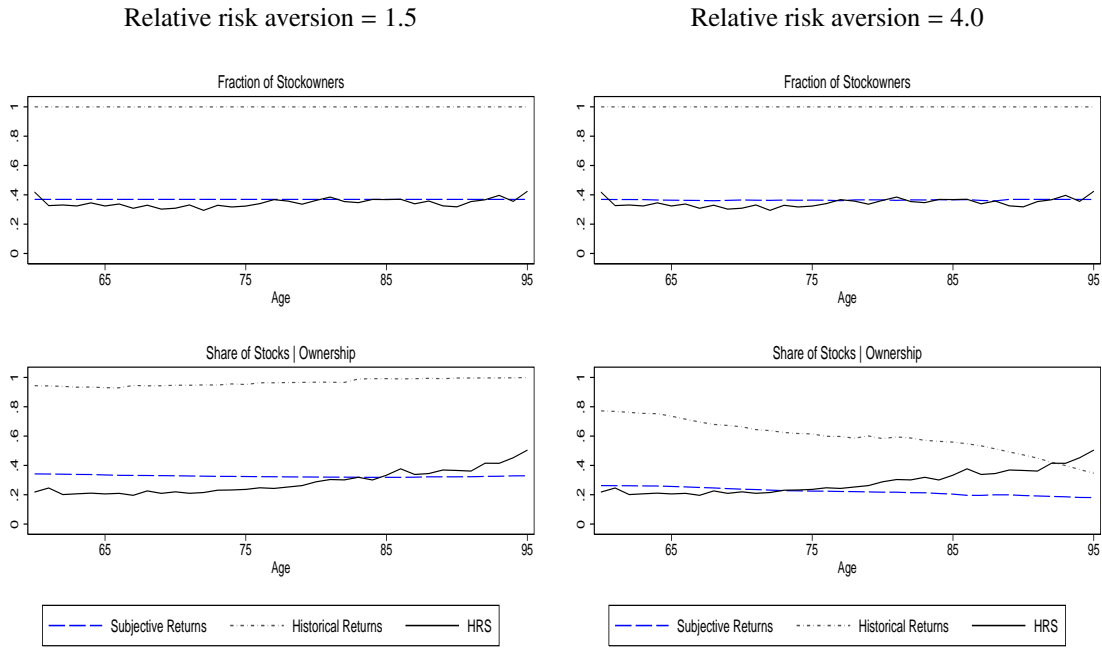
Throughout this section, we perform simulations using decision rules based on individual subjective expectations, but realized stock market returns following the historical trend. In other words, while agents make their financial decisions according to their beliefs, the actual returns on their portfolios are drawn from a distribution with historical mean and standard deviation. In these exercises, subjective beliefs are assumed to be constant over time. We leave inclusion of an updating process for further research.

We present results using the subjective expectations from 2010. We have also simulated the model taking the subjective expectations from the earlier years. The simulated investment patterns are qualitatively and quantitatively similar to the ones shown here and will be made available upon request.

Figure 7 shows stock market participation and average share of stocks conditional on stock holding. Stock market participation is now substantially below 1 and in line with the empirical pattern observed in the data. The model predicts that all those who expect a negative risk premium (about 65% of the sample) should not hold any stock. This leaves about 35% of stock owners, which is roughly what we see in the data. The average fraction of wealth held in stocks is also



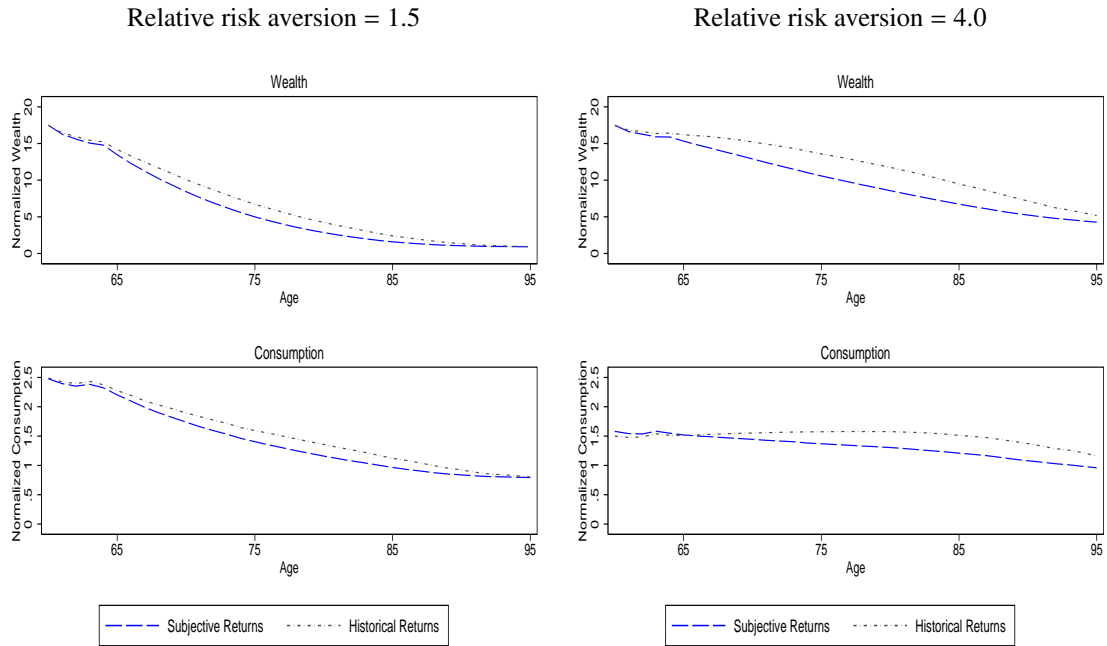
**Figure 7: Simulated Investment Behavior under Subjective and Historical Returns**



matched more closely than when expectations are based on historical returns. Interestingly, the simulated profiles with subjective expectations are only marginally affected by the hypothesized degree of risk aversion. With subjective expectations, changing the coefficient of relative risk aversion from 4 to 1.5 implies an increase of 10 percentage points in the average share of risky assets over the simulation period. The same change causes the average share of risky assets to increase by 40 percentage points when it is assumed that individuals base their expectations on historical returns. However, the increase with age of the average fraction of wealth held in stocks that we see in the HRS is not reproduced by the model.

Figure 8 compares wealth and consumption profiles generated by the model under subjective and historical returns. With a coefficient of relative risk aversion of 4, portfolio strategies generated by subjective return expectations produce wealth levels that are significantly below the level implied by investment choices driven by historical return expectations. Over the simulation period, retirement wealth and consumption are respectively 20% and 13% lower on average when portfolio choices are based on subjective rather than historical returns. As a result, we compute that if future stock market realizations follow the historical trend but households portfolios are determined by subjective beliefs the welfare loss (in utility terms) is 12%. With a degree of risk aversion of 1.5, individuals are substantially less prudent and tend to run down their assets relatively quickly. In this scenario, subjective beliefs about future risk premia that diverge from the historical trend induce wealth and consumption losses of 15% and 8%, respectively, and a welfare loss of about 2%. In general, the welfare loss associated with “inaccurate”

**Figure 8:** Simulated Wealth and Consumption Profiles under Subjective and Historical Returns



return expectations is higher, the higher the degree of risk aversion. This is because higher risk aversion implies more prudent behaviors and a higher preference for accumulating a buffer stock to hedge against background risks. As a consequence, any missed investment opportunity stemming from overpessimistic expectations and unrealized investment returns associated with overoptimistic beliefs translate into a higher welfare cost due to the need of reducing consumption and maintaining a sufficiently high level of wealth.

### 6.1 Robustness Checks

So far we have adopted a very comprehensive measure of household total wealth. In doing so, we have implicitly treated housing as a liquid and risk-free form of wealth which individuals may tap into to finance their consumption during retirement. Although there is some evidence that Americans homeowners move out of their owned home in later life and tend to downsize their housing consumption during retirement (Banks, Blundell, Oldfield, & Smith, 2010), the vast majority of individuals may be reluctant to spend down their housing equity due to large transaction costs, price volatility and emotional attachment to their home. We therefore repeat our simulation exercises using only financial wealth—including the net value of stocks, bonds, checking and savings accounts, treasury securities and IRAs—as a measure of household wealth. In our simulation sample, the ratio of financial wealth to retirement income is about 7 as opposed to a value of 17 for the ratio of total wealth to retirement income. This indicates that households hold most of their wealth in the form of housing and other non-perfectly liquid assets such as vehicles and businesses.

**Figure 9:** Simulated Investment Behavior under Subjective and Historical Returns, using only financial wealth

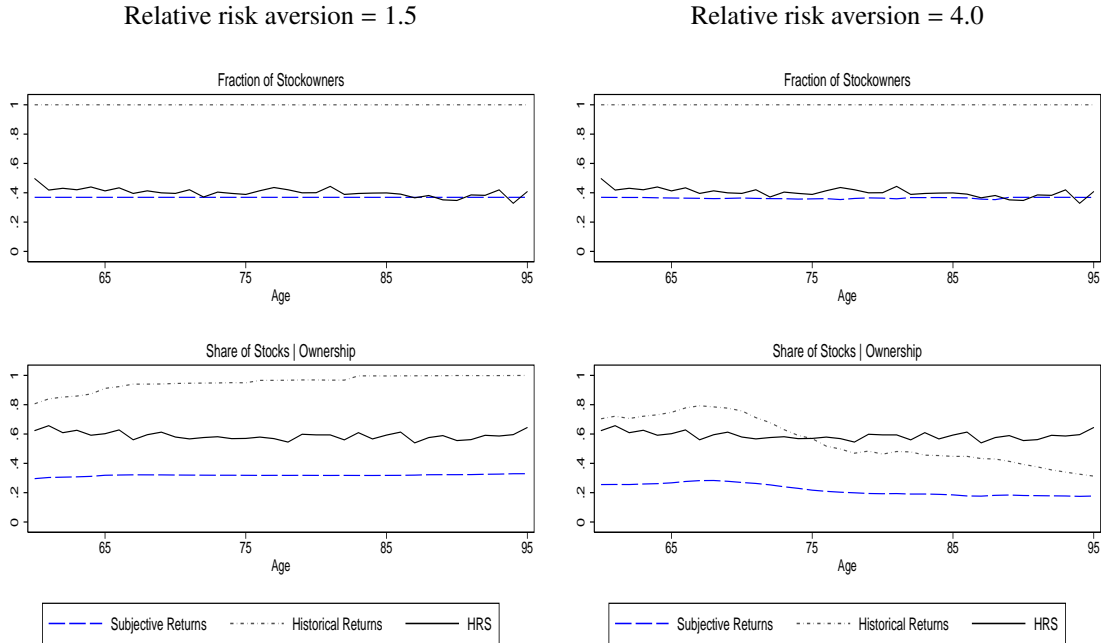
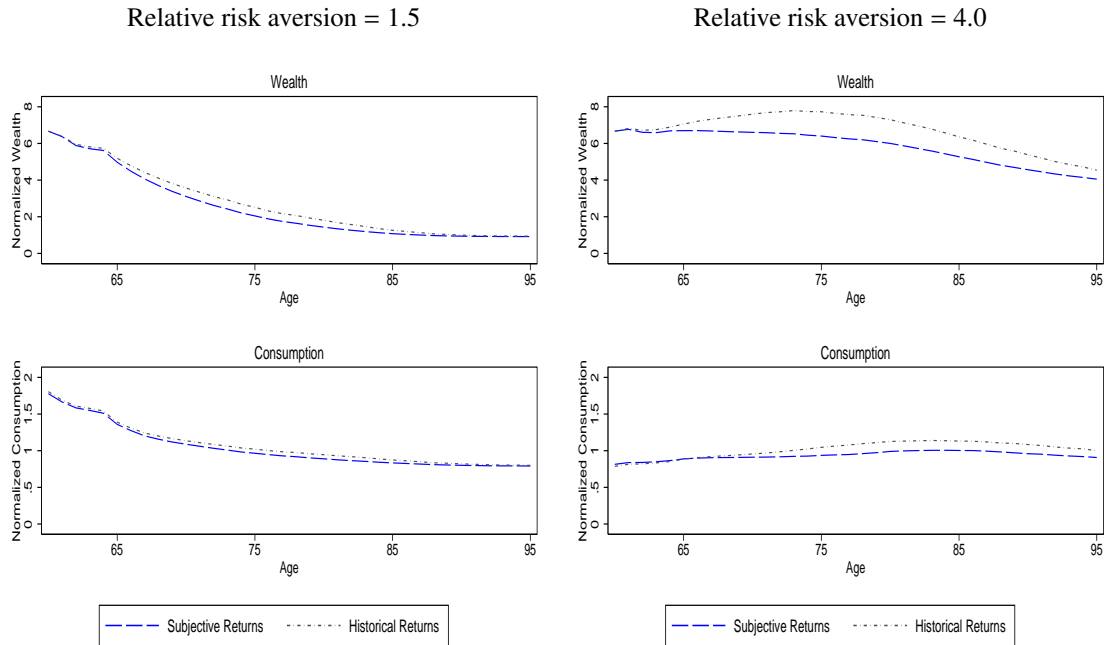


Figure 9 shows that the model using subjective expectations again reproduces observed stock market participation very closely, and the model using expectations based on historical returns again predicts a 100% stock market participation rate. In the HRS data, conditional on stockholding, the average fraction of financial wealth invested in stocks is constant over time at about 60%. This pattern is qualitatively well replicated by the model with subjective expectations. Quantitatively, however, the match is less satisfactory. The model predicts that on average 32% and 22% of household financial wealth should be held in stocks when relative risk aversion is 1.5 and 4, respectively. With historical expectations the simulated profile for the average share of risky assets is increasing and asymptotes at about 95% when relative risk aversion is 1.5, while it is decreasing and asymptotes at 55% when relative risk aversion is 4.

Figure 10 presents the corresponding simulated wealth and consumption profiles. The welfare implications of having subjective return expectations that diverge from the historical trend are similar to those discussed in the previous section, although welfare losses are now more modest. When portfolio choices are informed by observed subjective beliefs rather than by expectations based on historical returns the average welfare loss is 6% with relative risk aversion of 4 and 1% with relative risk aversion of 1.5.

In Figures 11 and 12 we present the simulation results when historical returns are computed using data over the last 10 years as opposed to the last 50 years. As for household wealth, we return to the baseline measure of total wealth including housing, financial wealth and other assets. Over

**Figure 10:** Simulated Wealth and Consumption Profiles under Subjective and Historical Returns, using only financial wealth



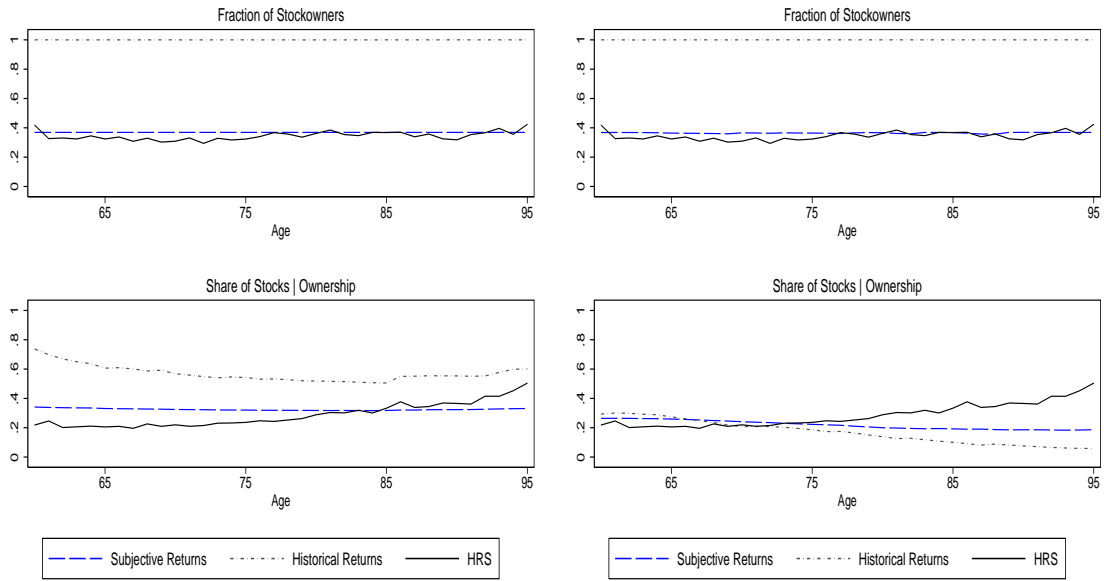
the period 2001–2011, risk-free assets have given a real return of  $-0.23\%$ , with inflation at  $2.5\%$ . The average real risk premium has been  $1.3\%$  and the standard deviation of stock market returns has been  $0.19$ . If one believes that this trend will continue in the future and the stock market will not revert to its average performance over the last 50 years, investing in stocks becomes clearly less appealing. Indeed, this is what we observe when we solve and simulate the model assuming that agents hold expectations based on returns over the last 10 years. Specifically, while the model still predicts a  $100\%$  stock market participation rate, the average optimal share of risky assets is much lower than before. With relative risk aversion of  $1.5$ , the average risky share is about  $50\%$  versus the  $95\%$  obtained when returns are computed over the last 50 years. With relative risk aversion of  $4$ , the low risk premium coupled with high volatility significantly inhibits stock market investments. In this case, the average optimal share of risky assets is less than  $20\%$ , as opposed to an average of  $57\%$  obtained when returns are computed over the last 50 years.

As in the previous section, the model using subjective expectations replicates empirical stock market participation patterns very well, while failing in reproducing the increase with age in the fraction of household wealth held in stocks. More interesting are the simulated wealth and consumption profiles in Figure 12. When historical returns refer to the last 10 years, the effect of implementing portfolio strategies informed by subjective beliefs on wealth and consumption is rather small and so is the overall impact on individual well-being. We find a welfare loss of  $3\%$  when relative risk aversion is  $4$  and a virtually zero welfare loss ( $0.04\%$ ) when relative risk

**Figure 11:** Simulated Investment Behavior under Subjective and Historical Returns, with historical returns from 2001–2011

Relative risk aversion = 1.5

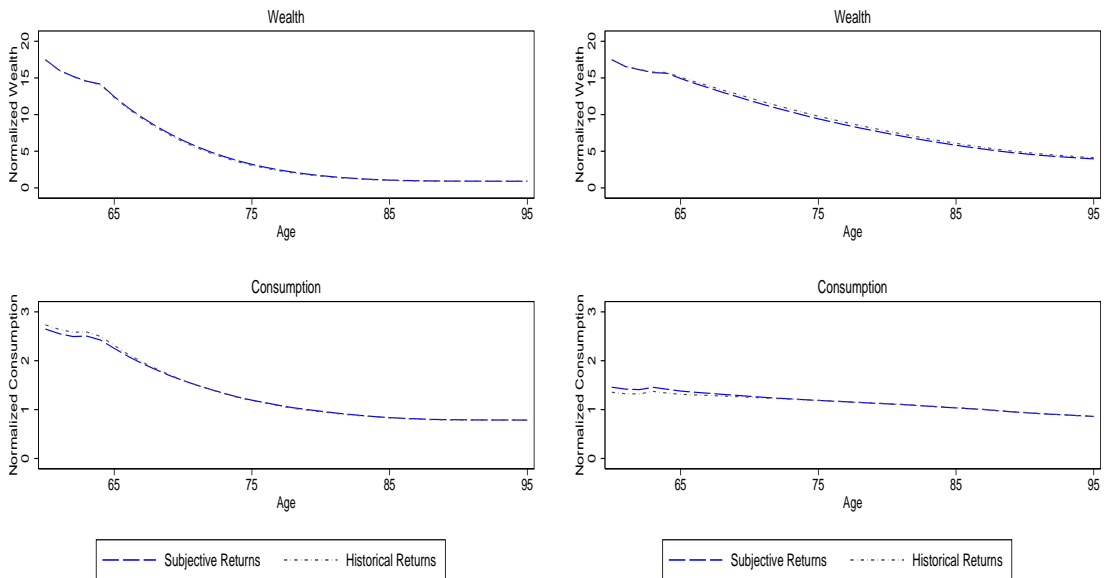
Relative risk aversion = 4.0



**Figure 12:** Simulated Wealth and Consumption Profiles under Subjective and Historical Returns, with historical returns from 2001–2011

Relative risk aversion = 1.5

Relative risk aversion = 4.0



aversion is 1.5.

## 7 Discussion

Standard life cycle models of economic behavior predict that essentially everybody should invest in risky assets to some extent, which is greatly at odds with the less than 40% of households that are observed to own stocks in the data. Moreover, conditional on stock holding, standard models with realistic values of preference parameters predict much higher fractions of wealth invested in risky assets than we observe in the data. Several explanations have been put forward in the literature, but none has been able to fully reconcile theoretical predictions with observed empirical patterns.

In this paper, we explore to what extent subjective expectations about stock market returns that differ from historical returns can address the so-called “stock-market participation puzzle”. We embed subjective expectations in an otherwise relatively standard economic life cycle model. Using data on subjective expectations from the Health and Retirement Study, we show that a large fraction of the population holds beliefs that are so pessimistic that they should never invest in stocks. Of the remaining individuals, a nonnegligible fraction is very optimistic and they should invest almost all of their wealth in risky assets. A “moderate” middle group rounds out the distribution and is predicted to keep some but not all of its wealth in risky assets. The model with these heterogeneous subjective expectations is remarkably well able to match the empirical patterns in household stock market participation. The matching of observed portfolio shares is less satisfactory. In particular, while the model predicts that portfolio shares should gradually decrease over time, the fraction of wealth invested in stocks slightly increases with age in the data. Most likely this is a selection effect due to running down different asset categories, stock holding, and mortality being correlated.

Our model simulations also show that when subjective beliefs diverge from historical returns, portfolio choices informed by the former have important welfare consequences. We estimate welfare losses ranging between 2% and 12%, depending on the degree of individual risk aversion and the way historical returns are computed. We also find that welfare losses are larger, the higher the degree of risk aversion.

The current model assumes that different individuals may have different expectations, but they keep the same expectations throughout their lifetimes. In practice, optimistic individuals who consume much and expect the asset returns to support their life style may notice a systematic rundown of their assets caused by lower returns than expected. They may respond to this by updating their expectations and behaving more cautiously in later life. Moreover, we have seen that in the data, subjective expectations vary between waves and may be related to the business cycle. This suggests a role for the business cycle in changing expectations as well. Therefore, extending the model with a mechanism for updating beliefs is an important topic for future research.

Our model contains risks of medical expenses as estimated from the HRS data. If individuals hold beliefs about their likelihood of being in good health or about the risks of large medical expenses conditional on health status that are different from historical distributions, these may also impact individuals’ behavior. We have performed limited experiments with subjective

distributions of health transitions, but these did not have much effect on the results. However, it is worthwhile to study this more systematically and focus on expectations about out-of-pocket medical expenses directly.

## Acknowledgments

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## Appendix

### A Model's Solution

**Normalization** Consider the model set-up of section 2 and let lowercase letters indicate the ratios of the original variables to the level of retirement income (e.g.,  $x_t = X_t/P$ ). Then expected remaining lifetime utility (8) is

$$U_t = P^{1-\gamma} \left[ \frac{c_t^{1-\gamma}}{1-\gamma} + \mathbb{E}_t \left( \sum_{j=t+1}^{T_M} \beta^{j-t} \frac{c_j^{1-\gamma}}{1-\gamma} \right) \right], \quad (\text{A.1})$$

where  $T_M$  is stochastic length of life.

Using the definition of cash on hand  $X_t$  in the text, the wealth transition (7) can be rewritten as

$$x_{t+1} = \left[ \alpha_t(1 + \tilde{r}_{t+1}^e) + (1 - \alpha_t)(1 + \tilde{r}) \right] (x_t - c_t) + (1 - \tau_t) - hc_{t+1}. \quad (\text{A.2})$$

Clearly,  $P$  does not play any role in the utility function and it is eliminated from the wealth transition equation. Thus, exploiting the scale-independence of the original maximization problem, we can rewrite all variables as ratios to the constant flow of retirement income and reduce the dimensionality of the problem from 4 state variables—cash-on-hand, retirement income, age, and health status—to 3 state variables—normalized cash-on-hand, age, and health status. This renormalization, introduced by Carroll (1992, 1997), makes the numerical problem more tractable as it significantly decreases the computational burden. Carroll (1992, 1997) shows that this normalization can also be applied to models featuring variable and uncertain income. Gomes et al. (2009) solve a model of saving and portfolio choice with taxable and tax-deferred accounts normalizing all variables with respect to the permanent component of stochastic income.

After the normalization, the individual's maximization problem can be expressed in recursive form as follows:

$$v_t(x_t, H_t, t) = \max_{c_t, \alpha_t} \left\{ \frac{c_t^{1-\gamma}}{1-\gamma} + \beta \mathbb{E}_t \left[ v_{t+1}(x_{t+1}, H_{t+1}, t+1) \right] \right\}, \quad (\text{A.3})$$

subject to (A.2) and  $w_t \geq 0, \forall t$ .

Note that both  $x_{t+1}$  and  $H_{t+1}$  are stochastic functions of  $\{x_t, H_t, c_t, \alpha_t, t\}$ , and the survival probability is also part of the expectation.

**Numerical solution** The model is solved via backward induction, from the final period  $T = 100$  to the initial period  $t_0 = 60$ . The state space is given by  $\{x_t, H_t, t\}$ . In order to implement this solution algorithm, we discretize the continuous state variable, normalized cash-on-hand, by defining an equally spaced grid,  $\{x_k\}_{k=1}^K$ . The upper bound is chosen to be nonbinding in all periods, specifically  $x_K = 99$ . The lower bound is set to 0.01. This can be interpreted as a minimum level of consumption guaranteed by government transfers in all periods. By imposing  $x_t \geq 0.01$  for all  $t$ , the model solution implicitly accounts for transfers from welfare programs equivalent to 1% of retirement income plus medical expenses minus all available resources, if that amount is positive, and zero otherwise.

The optimization procedure combines a root-finding algorithm (Brent’s method; Brent, 1973, chapter 4) and standard grid search. For given values of  $x_t$  and  $H_t$ , the value function is concave with respect to  $c_t$ . Thus, optimal normalized consumption can be computed using Brent’s method, significantly improving on computational efficiency. It is not known whether the value function is concave in portfolio share  $\alpha_t$ . Hence, to avoid the potential danger of selecting local optima, we optimize over the space of the remaining decision variable using a standard grid search. For this purpose, we discretize the share of risky assets,  $\alpha$ , to take 11 equally spaced values in the interval  $[0, 1]$ .

Interpolation plays a crucial role in the solution of the dynamic programming problem at hand. We use two-dimensional cubic splines interpolation to evaluate the value function between points on the cash-on-hand grid. Cubic splines have the advantage of being twice continuously differentiable with a nonzero third derivative. These properties preserve the *prudence* of the utility function, which plays an important role in precautionary savings and the effect of background risks on individual decisions (Eeckhoudt & Kimball, 1992; Kimball, 1990). The strictly positive lower bound on the grid for cash-on-hand implies that the value function is also bounded from below. This makes the spline interpolation work very well as long as the discretization of the state space is sufficiently fine. Numerical integrations are performed by Gaussian quadrature.

## B Data

**Source and years** We mainly use the RAND HRS (St.Clair et al., 2011), which is a postprocessed, more user-friendly version of the raw HRS data. We also add variables that are not included in the main RAND HRS file, such as subjective expectations about stock market returns (which are in the RAND-enhanced FAT files) and detailed pension income data (from the RAND income and wealth imputations files). We use waves 5–10, which cover the period from 2000 to 2010.



**Unit of analysis** The model as presented in section 2 is a model for the individual. In contrast with this, information on wealth and asset allocation is at the household level in the HRS. Thus, in our estimation and simulation exercises, we use the household as our unit of analysis. Wealth and income are reported by the financial respondent, who is the one deemed most knowledgeable about the household's finances. Accordingly, for some inherently individual-level variables (age, mortality), we use the variables for the financial respondent.

**Retirement income** Household retirement income includes pension benefits, annuities, and Social Security retirement received by the financial respondent and his or her spouse in each wave. For each household, we first compute retirement income in each wave and then a constant flow of retirement income by averaging retirement income over all periods when it is observed.

**Medical expenses** Out-of-pocket medical expenses comprise the costs incurred in a year for hospital, nursing home, doctor visits, dentist, outpatient surgery, average monthly prescription drug, home health care, and special facilities. Household out-of-pocket medical expenses are obtained summing the costs reported by the financial respondent and, if present, his or her spouse.

**Health** We define an individual to be in bad health if he or she reports his or her health status as "fair" or "poor", and in good health otherwise. Because health costs are borne by the household, and economic decisions are made at the household level, the risk of health costs related to the spouse's health must be taken into account. Therefore, in our analyses, we use an indicator of bad health at the household level. The household is defined to be in bad health if either the respondent or the spouse (if any), or both are in bad health.

**Assets** The wealth measure for the baseline analysis includes net housing wealth (primary and secondary residence, other real estate, minus balances of mortgages and other loans), vehicles, personal items of value, and net financial wealth (checking and savings accounts, certificates of deposit, bonds, stocks, businesses, IRAs, minus non-housing debt). In section 6.1, we present robustness checks that use only the financial assets component (without the amount of assets in businesses).

We consider stocks, held either directly or indirectly through an IRA, as the risky asset. We do not include balances in defined contribution (DC) pension plans in total wealth, nor the stock fraction of these balances in stock wealth, because these balances are not readily available. Gustman, Steinmeier, and Tabatabai (2010) construct measures of DC (and DB) pension wealth and the fraction of that invested in stocks, but the latter is only available for 2006, in which among over 6,400 fully retired financial respondents 60 and over, less than 1% had stocks in a DC account and therefore this omission does not influence our results.<sup>6</sup>

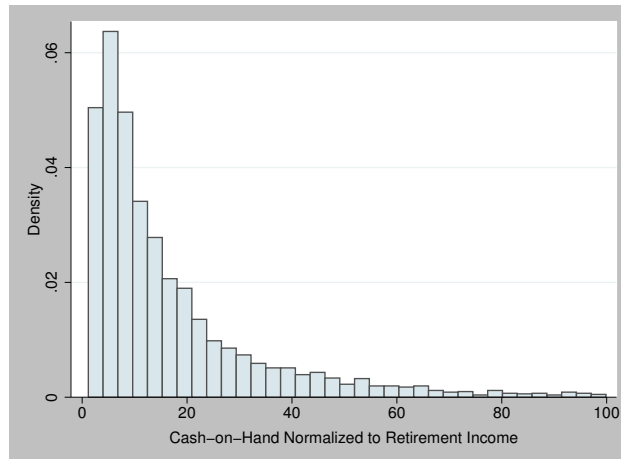
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<sup>6</sup>The reasons for this are arguably that in this age group, relatively few individuals have DC plans and that upon retirement, many of those who do have a DC plan roll over their balances into an IRA or take out the pension in the form of a lump sum.

**Table B.1:** Initial age and health distributions in the simulation sample

Age		Health	
Value	Fraction (%)	Value	Fraction (%)
60	9.95	Good	64.07
61	13.72	Bad	35.93
62	17.60		
63	27.03		
64	31.69		

**Figure B.1:** Distribution of initial normalized wealth in the simulation sample



**Sample selection** We restrict our analysis to financial respondents aged 60 and older who are fully retired. We exclude households whose reported retirement income in a given wave is less than 5,000 or more than 1,000,000 dollars per year or whose total wealth is less than 10,000 or more than 3,000,000 dollars.

The simulation sample is obtained by first selecting fully retired individuals 60–64 years of age (3,607 retirees) and then sampling 10,000 times with replacement. Table B.1 shows the age and health distributions in the simulation sample, where, as mentioned above, health status is defined at the household level. Hence, about 36% of the selected individuals belong to couples where at least one of the members reports being either in fair or poor health. Figure B.1 presents the distribution of the ratio of cash-on-hand to constant retirement income. This distribution is highly skewed with roughly 75% of the individuals living in households with wealth less than 20 times their retirement income.

## C Auxiliary Models

### C.1 Survival and health transition probabilities

The model has two interrelated transition processes: mortality and health transitions. We follow De Nardi et al. (2010) and others and assume that survival from age  $t-1$  to age  $t$ , and health status at age  $t$  conditional on survival, depend on health status at age  $t-1$ . We cannot directly estimate the one-year survival and health transition probabilities (1)–(2) from the HRS, because the HRS interviews individuals every two years and thus we do not observe one-year transitions. This is only a minor nuisance, because probabilities of two-year transitions follow from the probabilities of one-year transitions:

$$\begin{aligned}\Phi_t(\text{good} | h) &\equiv \Pr(\text{alive}_t, H_t = \text{good} | \text{alive}_{t-2}, H_{t-2} = h) \\ &= s_{t-1}^h \left[ \phi_{t-1}^h s_t^{\text{good}} \phi_t^{\text{good}} + (1 - \phi_{t-1}^h) s_t^{\text{bad}} \phi_t^{\text{bad}} \right]\end{aligned}\quad (\text{C.1a})$$

$$\begin{aligned}\Phi_t(\text{bad} | h) &\equiv \Pr(\text{alive}_t, H_t = \text{bad} | \text{alive}_{t-2}, H_{t-2} = h) \\ &= s_{t-1}^h \left[ \phi_{t-1}^h s_t^{\text{good}} (1 - \phi_t^{\text{good}}) + (1 - \phi_{t-1}^h) s_t^{\text{bad}} (1 - \phi_t^{\text{bad}}) \right]\end{aligned}\quad (\text{C.1b})$$

$$\begin{aligned}\Phi_t(\text{dead} | h) &\equiv \Pr(\text{dead}_t | \text{alive}_{t-2}, H_{t-2} = h) \\ &= 1 - \Phi_t(\text{good} | h) - \Phi_t(\text{bad} | h).\end{aligned}\quad (\text{C.1c})$$

The left-hand sides of these expressions are observable in the HRS and thus can be used to estimate the one-year transition probabilities. However, the number of deaths in the HRS is insufficient to reliably estimate conditional survival rates directly. Therefore, as in De Nardi et al. (2010), we combine information from the data with unconditional survival probabilities from life tables and Bayes' rule to estimate conditional survival probabilities.

Specifically, we start with the 2007 Actuarial Life Table from the Social Security Administration (<http://www.ssa.gov/oact/STATS/table4c6.html>). We weight the male and female columns by the fraction of males and females among 60-year olds in the HRS, and then add them to obtain a combined life table from which we compute the unconditional one-year survival probabilities,  $s_t = \Pr(\text{alive}_t | \text{alive}_{t-1})$ . Conditional survival probabilities can now be expressed using Bayes' rule as

$$\begin{aligned}s_t^{\text{good}} &= \Pr(\text{alive}_t | H_{t-1} = \text{good}; \text{alive}_{t-1}) = \frac{\Pr(H_{t-1} = \text{good} | \text{alive}_t)}{\Pr(H_{t-1} = \text{good} | \text{alive}_{t-1})} \times \Pr(\text{alive}_t | \text{alive}_{t-1}) \\ &= \frac{\Pr(H_{t-1} = \text{good} | \text{alive}_t)}{\Pr(H_{t-1} = \text{good} | \text{alive}_{t-1})} \times s_t,\end{aligned}\quad (\text{C.2})$$

and analogously for survival conditional on bad health. Thus, we estimate the numerator and denominator probabilities in (C.2) and compute  $s_t^{\text{good}}$  and  $s_t^{\text{bad}}$  using (C.2) and the unconditional survival probabilities from the life table. It follows that we estimate four probabilities: the health transition probabilities  $\phi_t^{\text{good}}$  and  $\phi_t^{\text{bad}}$  from (2) and the health status probabilities  $\Pr(H_{t-1} = \text{good} | \text{alive}_t)$  and  $\Pr(H_{t-1} = \text{good} | \text{alive}_{t-1})$  from (C.2).

To obtain smooth estimates, we specify each of these four probabilities as a logit model with a cubic polynomial in age as covariates. We insert the resulting expressions (2) and (C.2) into (C.1)

**Table C.1:** Parameter estimates for the transition models

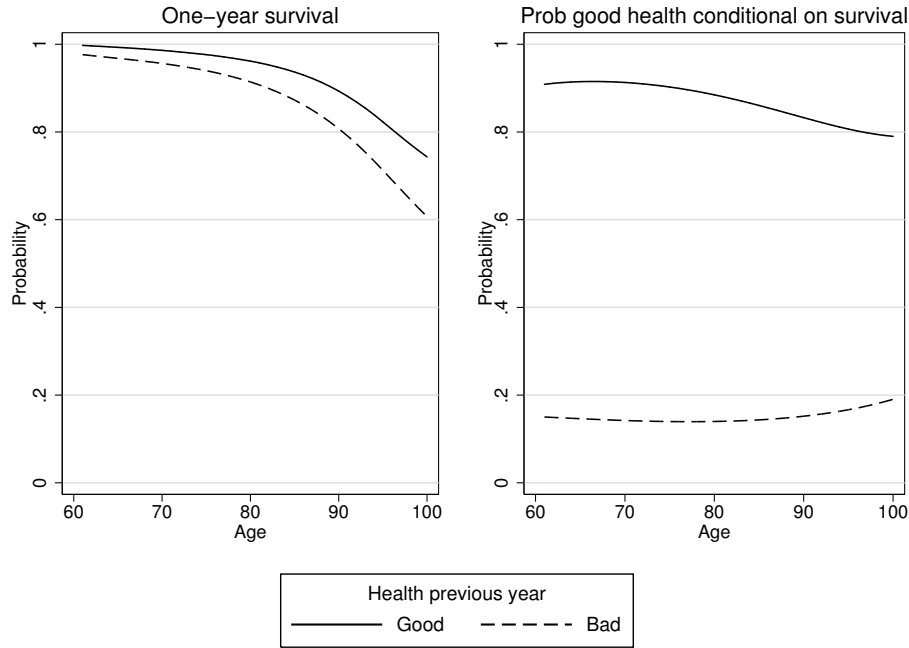
Covariate	Coef.	s.e.
<i>Prob. good health(t)</i>		
$a = (\text{Age} - 80)/10$	-.5861***	(.1619)
$a^2$	-.1296	(.0856)
$a^3$	.1331**	(.0662)
Constant	1.2143***	(.1107)
<i>Prob. good health(t) given survival until t + 1</i>		
$a$	-.5514***	(.1622)
$a^2$	-.1133	(.0858)
$a^3$	.1366**	(.0663)
Constant	1.2683***	(.1109)
<i>Prob. good health(t + 1) given survival and good health(t)</i>		
$a$	-.4289***	(.0476)
$a^2$	-.0472**	(.0216)
$a^3$	.0459**	(.0183)
Constant	1.9953***	(.0280)
<i>Prob. good health(t + 1) given survival and bad health(t)</i>		
$a$	.0405	(.0608)
$a^2$	.0605**	(.0245)
$a^3$	.0102	(.0232)
Constant	-1.8134***	(.0332)
Sample size	46,654	

\*\*  $p < .05$ ; \*\*\*  $p < .01$ .

and estimate all four probabilities jointly by maximum likelihood. Finally, to increase sample size we do not impose the sample restrictions that we use in the main analysis but include all respondents 60 and over in the HRS 2000–2010. The estimation results are given in Table C.1.

Figure C.1 plots the resulting one-year conditional survival probabilities  $s_t^h$  and health transition probabilities  $\phi_t^h$ . As expected, survival probabilities drop with age, especially after age 80 or so (note that these are one-year survival probabilities, not cumulative survival probabilities), and survival probabilities are lower for bad health than for good health. The probability of staying in good health decreases a little as well, and the probability of being in good health is much lower if the individual (or household, rather) was in bad health in the previous year. Remarkably, the probability of transitioning from bad health to good health increases a little at higher ages. We suspect that this is due to relatively healthier individuals (within the “bad health” category) living

**Figure C.1:** Conditional one-year survival probabilities and health transitions



longer.

## C.2 Out-of-Pocket Medical Expenses

As the structural model is solved in terms of normalized variables, we subtract the logarithm of retirement income from both sides of (3) and estimate a process for the logarithm of the ratio of household medical expenses to household retirement income:  $\ln hc_t = \ln(HC_t/P)$ . By assuming  $\delta_1 = 1$ ,  $P$  drops out of the right-hand side of the model. We specify the function  $f(\cdot)$  in (3) to be a third-order polynomial in age and an indicator for bad health status interacted with age. We also allow the variance of the unobserved component  $\eta_t$  to depend on health status in order to capture differential risks for households with different health.

We include fixed effects in the model for the medical costs process to control for the potentially biasing effect of changes in sample composition with age due to differential mortality. Suppose, for example, that only wealthier households are observed at very old ages and that their ratio of medical expenses to retirement income is relatively low because of more generous pension allowances. Estimating the age profile of medical costs by pooled OLS would understate the extent to which the fraction of resources spent in health care increases with age. The fixed-effects estimator overcomes this problem. By using the time variation within each cross section, in fact, it allows us to correctly infer the age profile of household medical expenses. This also has the advantage of capturing cohort effects, which may be important for the variable of interest. If

not appropriately accounted for, systematic differences in health care behavior among generations may lead to biased assessments of the age effect on household medical costs. The regressions also include year dummies, which pick up variations in medical prices that differ from inflation and variations in medical costs due to other unmodeled causes, such as the business cycle.

The estimates are presented in Table C.2. Figure C.2 presents the estimated age profiles for the ratio of medical expenses to retirement income by health status. This is the prediction from the model except for the fixed effects and the year dummies, and reflects the predicted median health costs in 2000 by age and health status. Median health costs amount to less than 20% of income among individuals in their early 60s. They reach 30% for individuals in their late 80s and rise sharply thereafter. The effect of health status on medical costs is modest until 85, but becomes substantial in the later part of the life span. These results are in line with what has been found in other studies. For example, De Nardi et al. (2010) estimate that single individuals at the median permanent income incur medical costs of roughly \$1,700 (in 2000 dollars) at age 70 and \$4,500 at age 90. Assuming that the typical household has two members and knowing that the median (constant) household retirement income is \$21,000, the implied ratios of medical expenses to permanent income are 16% at age 70 and 42% at age 90. The estimates in the last two rows of Table C.2 indicate that households in bad health face a significantly higher risk of incurring high medical costs.

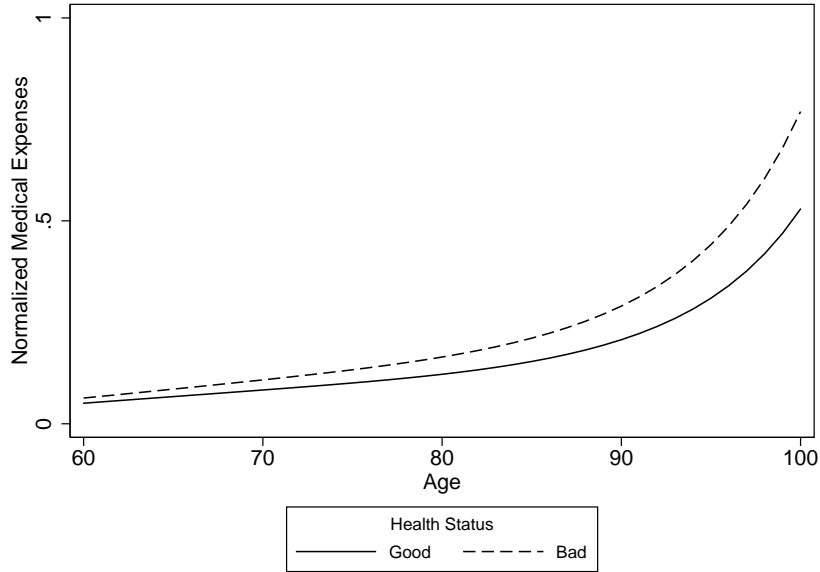
**Table C.2:** Estimates of the medical expenses process

Covariate	Coef.	s.e.
Age	0.761***	(0.255)
Age <sup>2</sup> /100	-0.973***	(0.332)
Age <sup>3</sup> /1000	0.044***	(0.014)
Bad Health × Age/100	0.374***	(0.029)
Year 2002	0.177***	(0.058)
Year 2004	0.211**	(0.102)
Year 2006	-0.017	(0.149)
Year 2008	-0.218	(0.197)
Year 2010	-0.168	(0.254)
Constant	-23.017***	(6.676)
Residual s.d. ( $\sigma_{\eta}^2$ ):		
Good health	1.033***	(0.027) <sup>a</sup>
Bad health	1.383***	(0.031) <sup>a</sup>
Sample size	26,050	
F-statistic fixed effects	3.15***	

\*\*  $p < .05$ ; \*\*\*  $p < .01$ .

<sup>a</sup> Standard errors computed by bootstrap using 1000 replications.

**Figure C.2:** Medical expenses process by health status



## D Taxes, asset returns, and preference parameters

**Tax rates** We assume that the tax rate on retirement income ( $\tau_r$ ) is 20%. The U.S. income tax code is progressive, and thus not every household faces the same average and marginal tax rates. Furthermore, tax rates change over time. Hence, the constant rate in our model is a simplification. The 20% rate is close to the median and mean federal income tax rates in our sample period (CBO, 2010; Piketty & Saez, 2007). In our model, we use a tax rate on nominal asset returns ( $\tau_a$ ) of 15%, which is the maximum rate on capital gains and dividends (Piketty & Saez, 2007).

**Asset returns and inflation** We base our parameters for asset returns and inflation on the outcomes of the past 50 years (01/01/1961–12/31/2011). From the CPI (all urban consumers, series; <http://www.bls.gov/cpi/>), we derive that the average inflation rate between January 1961 and December 2011 was about 4%, so in the model we take  $\pi = 4\%$ . For the risk-free nominal asset return, we use the returns on 1-year Treasury Bills (<http://www.federalreserve.gov/releases/H15/data.htm>), which had an average real return of about  $r = 1.57\%$ . For the risky asset returns, we use the total returns—including dividends—of the S&P 500 (data available at <http://www.econ.yale.edu/~shiller/data.htm>), which had an average real return of 6.8% and, hence, a real risk premium of 5.23%. In our model, we use  $\mu = 5\%$  as opposed to the historical 5.23% to take into account transaction costs such as mutual fund and brokerage fees (Gomes et al., 2009). The standard deviation of the risk premium is  $\sigma_\epsilon = 0.170$ , which comprises the observed historical variation in both the real risk-free and risky assets returns.

**Preference parameters** Preference parameters are taken from previous studies using life-cycle models of saving and portfolio choice. In particular, the discount factor,  $\beta$ , is set to 0.96, as in Cocco et al. (2005), and the coefficient of relative risk aversion,  $\gamma$ , is set to 4, as in De Nardi et al. (2010). In the text we also present results with  $\gamma = 1.5$ , which is closer to what Hurd (1989) and Attanasio and Weber (1993, 1995) find.

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