

Partner Choice and the Marital College Premium*

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Abstract

Several theoretical contributions have argued that the returns to schooling within marriage play a crucial role for human capital investments. We investigate these claims using a structural model of household decision making, in which individual preferences are of the Transferable Utility (TU) type and agents have idiosyncratic preferences for marriages that may be correlated with education. In a frictionless matching context, we derive a complete, theoretical characterization of the model. We show that if the supermodularity of the resulting surplus function is invariant over time, the model is overidentified even if the surplus function varies over time. We apply our method to US data on individuals born between 1943 and 1972. Our model fits the data very closely, showing that changes in preferences towards assortative matching are not necessary to explain changes in matching patterns. We find that the deterministic part of the surplus is indeed supermodular and that, in line with theoretical predictions, the “marital college premium” has increased for women but not for men over the period.

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1 Introduction

The joint evolution of US male and female demand for college education over the recent decades raises an interesting puzzle. During the first half of the century, college attendance increased for both genders, although at a faster pace for men. According to Claudia Goldin and Larry Katz (2008), male and female college attendance rates were about 10% for the generation born in 1900, and reached respectively 55% and 50% for men and women born in 1950. This common trend, however, broke down for the cohorts born in the 50s and later. These individuals faced a market rate of return to schooling (the “college premium”) that was substantially higher than their predecessors; therefore one would have expected their college attendance rate to keep increasing, possibly at a faster pace. This prediction is satisfied for women: 70% of the generation born in 75 attended college. On the contrary, the male college attendance rate increased at a much slower rate, if at all. As a result, in recent cohorts women are more educated than men, by an increasing margin.

To explain these strikingly asymmetric responses to seemingly identical incentives, Pierre-André Chiappori, Murat Iyigun and Yoram Weiss (2009, from now on CIW) stress the role of gender differences in the returns to schooling within marriage¹. They argue that the return to education has two distinct components. One is the standard market college premium, whereby a college degree significantly increase wages; this component has evolved in a largely similar way for men and women (see CIW for more details). Secondly, education has an impact on a person’s situation on the marriage market; it affects the probability of getting married, the characteristics of the future spouse, and the size and distribution of the surplus generated within marriage. CIW advance the hypothesis that, unlike the market college premium, this “marital college premium” may have evolved in a highly asymmetric way between genders. In their paper, agents have heterogeneous attitudes towards marriage and heterogeneous costs of human capital acquisition; their investment in education is based on their (rational) expectations about the total (standard and marital) returns, which in turns are determined at equilibrium by the distribution

¹Another, largely complementary explanation proposed by Becker, Hubbard and Murphy (2009) relies on the differences between male and female distributions of unobserved ability. Still, these authors also emphasize that educated women must have received some additional, intrahousehold return to their education. It is precisely that additional term that our approach allows to evaluate.

of education by gender. Various technological changes reducing household chores, (as in Jeremy Greenwood et al., 2005) as well as progress in birth control technologies, medical techniques and infant feeding methods (stressed by Robert Michael, 2000, Goldin and Katz, 2002, and Stefania Albanesi et al., 2009) can trigger a change in equilibrium, leading to marital returns to education that are higher for women than for men. This types of asymmetry can then generate discrepancies in the demand for higher education of women and men.

While this argument is theoretically consistent, establishing empirically its relevance is a challenging task. In contrast to the returns to schooling in the labor market, which can be recovered from observed wages data, the returns to schooling within marriage are not directly observed and can only be estimated indirectly from the marriage patterns of individuals with different levels of schooling.

Our paper provides the first such estimates. Specifically, we consider a frictionless matching framework with Transferable Utility (TU). The analysis of the marriage market as a matching process, which dates back to Gary Becker’s seminal contributions (see Becker 1973, 1974, 1991) has recently attracted renewed attention². Jeremy Fox (2010a,b) provides a nonparametric approach that does not explicitly model the stochastic structure of the joint surplus; instead, it relies on a “rank order” property, which postulates that assignments that generate more surplus in a deterministic model are more likely to be observed when stochastic aspects are introduced. In contrast, our approach is explicitly structural, in the line of the seminal contribution by Eugene Choo and Aloysius Siow (2006). They use a specification of the stochastic elements in the joint surplus that yields a very simple inversion formula, from observed matching patterns to the underlying joint surplus function. They applied this approach to study the response of the US marriage market to the legalization of abortion; see Maristella Botticini and Siow (2008) and Siow (2009) for other applications. Alfred Galichon and Bernard Salanié (2012) generalize the Choo and Siow framework to arbitrary separable stochastic distributions; they also provide a theoretical and econometric analysis of multicriterion matching under the same separability assumption.

We extend the empirical matching literature in three directions. First, we clarify the

²The reader is referred to Graham (2011) for a general presentation.

underlying theoretical structure needed for these approaches. We consider a structural model of matching on the marriage market that is close, in spirit, to that adopted by CIW. The model provides an explicit representation of household behavior based on a collective framework, with individual preferences belonging to Theodore Bergstrom and Richard Cornes’s “Generalized Quasi Linear” (GQL) family (2003). Such preferences are necessary and sufficient for a TU framework; i.e., they admit a cardinal representation in which the Pareto frontier is a straight line with slope -1, and whose intercept is an increasing convex function of the household’s total income. Agents match after choosing their education level, but before their permanent income is revealed; they therefore consider their expected surplus conditional on their educational level and that of potential partners. In addition, still following CIW, we assume that each individual has idiosyncratic preferences for marriage, which is known before investment in human capital is decided. We work out the implications of this framework for the key endogenous variables, namely individual utilities at the stable match. The theoretical structure that we use relies on separability between systematic complementary traits and additive random elements in the gains from marriage. This property that we call “separability” is sufficient to fully characterize the stochastic distribution of the endogenous variables; moreover, the matching equilibrium conditions translate into a simple discrete choice structure.

Second, we consider a general econometric specification that allows class-specific distributions of the random components - in contrast to Choo and Siow (2006), who assumed that the distributions of unobservable preferences for marriage are identical across education classes. Class-specific distributions are clearly required by our theoretical background; indeed, an immediate consequence of our model (and actually of CIW) is that selection into education cannot be independent from (unobservable) preferences for marriage. We show that, in this extended context, the “marital college premium” has a direct interpretation in terms of differences in ex ante expected utility conditional on education; moreover, our structural approach still leads to a closed form characterization of these differences.

Our third contribution is to extend the approach to a “multi-market” framework³. We consider several cohorts of men and women, which introduces variation in the proportions

³See Fox (2010a, 2010b) and Fox and Yang (2012) for different approaches to pooling data from many markets.

of men and women at all education levels. We impose a simple (and empirically testable) restriction, which posits that *preferences for assortative matching remain constant* across cohorts (although the surplus generated may vary, and possibly in an education-specific manner). We show that the model is then vastly overidentified, even without independence between preferences for marriage and educational choices. In fact, one can identify a more general structure, in which the systematic component of the surplus involves class-specific temporal drifts; this generalized model still generates strong overidentification restrictions.⁴ In summary, our framework allows us to study the evolution of matching patterns throughout time, allowing the gains from marriage and the intra-household allocation of these gains to evolve over time in a class-specific way. From this information, we can extract the time patterns of the marital education premiums of men and women.

We apply our model to the US population, for the cohorts born between 1940 and 1975. We show that the marital college premium evolved non monotonically over the period. Specifically, we find that in the beginning of that period (for cohorts born before the mid-1950s), the premium decreases for both men and women. For the following cohorts, however, the evolution is gender-specific; we find that the marital premium has increased sharply for women over the period, while they have not changed much for men. These findings are not based on a model of individual demand for education: our marital college premium is estimated exclusively from the observed marriage patterns. Yet they closely fit the argument of CIW (section II.C), which is based on a theoretical analysis of investment in higher education. This buttresses their claim that the increase in the marital component of the education premium for women could explain the spectacular increase in female demand for higher education.

Our structural approach allows us to break down the changes in the marital college premium into their components. In particular, we identify the group specific “prices” (i.e., dual variables of the matching process) that determine the division of the gains from marriage between husbands and wives of different types. We find that in couples in which both spouses have a college degree, the share of the wife in the gains from marriage has increased over time, despite the increase in the number of educated women relative to

⁴As is well known, the model of Choo and Siow is just identified with cross-sectional data; allowing for education-specific distributions would therefore lead to serious (under) identification problems in such context.

educated men. This happened because the marginal contribution of educated women to the surplus with educated men has risen over time. This in turn is mainly due to the variable component: educated women became more productive (of surplus) relative to less educated women in all marriages, irrespective of the type of the husband. In the end, educated women have gained relative to uneducated women in three ways: by marrying at higher rates and by receiving a higher share of a larger marital surplus. Our findings are fully consistent with explanations based on the “liberating effects” of new technologies; in addition, we find that these effects vary considerably with the wife’s education, but are more or less independent of the schooling of the husband.

We also find that the gains generated by marriages with equally educated partners have declined for all education levels, reflecting the general reduction in marriage over time. However, the smallest decline is in matches in which one or both partners have college education. This finding can be related to empirical work showing that such marriages are also less likely to break (see Weiss and Willis 1997 and Bruze, Svarer and Weiss 2010.)

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Our results also shed light on the evolution of marriage patterns between the 1960s and the 2000s. They have changed in complex ways. Overall, the percentage of couples in which both spouses have a college degree has significantly increased over the period; however, as women with college degree became more abundant, the proportion of educated women who marry educated men has declined, as some educated women had to “marry downwards” (with less educated men.) Many observers have nevertheless concluded that assortative matching on education is stronger now than four decades ago⁵; Burtless (1999) for instance argues that this evolution complements the increase in the labor market college premium in explaining increased interhousehold income inequality.

Part of the increase in the proportion of couples where both partners have similar educations reflects the shifts in the education of women; but some of it may also derive from changes in preferences towards assortative matching. An important advantage of our structural approach is that it allows to formally disentangle these two effects, notably by

⁵One of the difficulties in measuring changes in assortative matching is that none of the indexes that are used in the literature have a very convincing foundation.

providing a structural interpretation of the notion of ‘preferences for assortativeness’ as indicated by the supermodularity of the surplus function that stems from the individual preferences under consideration. Our identifying assumption is that preferences for assortativeness did not change over cohorts; therefore, the quality of the empirical fit of our model is a direct test of the relevance of this assumption. Based on our findings, the conclusion is clear-cut; the model fits the data remarkably well, indicating that one can explain the changing marriage patterns of the last decades *without* appealing to changes in preferences towards assortative matching.

This finding seems to contradict results in the sociological literature that argue that even after accounting for changes in the relative number of men and women in each skill group, homogamy has increased in the US and several other countries (see for instance Schwartz and Mare, 2005.) However, these conclusions were drawn from reduced-form, log-linear models with no direct economic interpretation and can therefore be quite misleading. To check this, we used our model to generate marriage data and we ran it through the type of log-linear regression that is common in the sociological literature. The results spuriously suggest that preferences for homogamy have changed, even though our model rules out such changes. These findings demonstrate the importance of a structural approach to guide the interpretation of the empirical results.

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Section 2 presents some stylized facts. Then we introduce our theoretical framework in Section 3, and section 4 describes the basic principles underlying its empirical implementation. In Section 5, we discuss identification issues and present our main theoretical results on that topic. Section 6 describes the matching patterns in the data. Our empirical findings are presented in Section 7.

2 The Data

We begin by describing our data and some stylized facts about the evolution of matching by education over the last decades in the US. We use the American Community Survey, a representative extract of the Census, which we downloaded from IPUMS (see Ruggles et al (2008).) Unlike earlier waves of the survey, the 2008 survey has information on current

marriage status, number of marriages, and year of current marriage. Of the 3,000,057 observations in our original sample, we only keep white adults (aged 18 to 70) who are out of school; the resulting sample at this stage has 1,307,465 observations and is 49.5% male. We used the “detailed education variable” of the ACS to define three subcategories:

1. High School Dropouts (HSD)
2. High School Graduates (HSG)
3. Some College (SC)—this last category includes anyone who attended college, whether they graduated or not⁶.

When studying matching patterns, we have to decide which match to consider: the current match of a couple, or earlier unions in which the current partners entered? also, do we define a single as someone who never married, or as someone who is currently not married?

It is notoriously hard to model divorce and remarriage in an empirically credible manner. Since this is not the object of this paper, we chose instead to only keep first matches, and never-married singles. Given this sample selection, in each cohort we miss:

- those individuals who died before the 2008 Survey;
- those who are single in 2008 but were married before: there are
 - 36,094 individuals who are separated from their spouse
 - 218,839 who are divorced
 - 143,963 who are widowed.
- those who are married in 2008, but not in a first marriage—more precisely, in Table 1, we only kept the top left cell.

Outcomes are truncated in our data, since young men and women who are single in 2008 may still marry; in our figures (and later in our estimates) we circumvent this difficulty by stopping at the cohort born in 1972—the first union occurs before age 35 for most men and

⁶A finer classification would be desirable, but cell sizes shrink fast.

Number of marriages	1	2	≥ 3	Total
1	384,291	42,147	5,945	432,383
2	46,773	56,210	14,146	117,129
≥ 3	7,250	15,334	9,069	31,653
Total	438,314	113,691	29,160	581,165

Table 1: Men in rows, women in columns

women. To examine marriage patterns, we dropped the small number of couples where one partner married before age 16 or after age 35 (recall that these are first unions.)

This leaves us with 179,353 couples, 44,344 single men, and 32,985 single women. The increasing level of education of women is shown on Figure 1: in cohorts born after 1955 women are more likely than men to attend college. Not coincidentally, the proportion of marriages in which the husband is more educated than the wife has fallen quite dramatically. Figure 2 shows that while husbands used to “marry down”, husbands born after 1955 are more likely to be married to a wife with a higher level of education than theirs⁷.

Figures 3 and 4 describe changes in the level of education of the partners of married men (resp. women) between the earlier cohorts (born in the early 40s) and the most recent cohorts in our sample (born in the early 70s.) Figure 3 shows that college-educated men now find a college-educated wife much more easily; and in fact even less-educated men are now more likely to marry a college-educated woman—if they marry at all. On the other hand, the marriage patterns of women are remarkably stable, as evidenced in Figure 4.

We illustrate the decline in marriages by plotting the percentage of individuals of a given cohort who never married in Figures 5 and 6. They show that a higher education has tempered the decline in marriage, especially for women; and that high-school dropouts on the other hand have faced a very steep decline in marriage rates.

⁷Note that these results are exactly in line with the existing literature (see for instance Goldin and Katz (2008, p. 252), suggesting that the selection into our sample does not affect the main patterns under consideration.

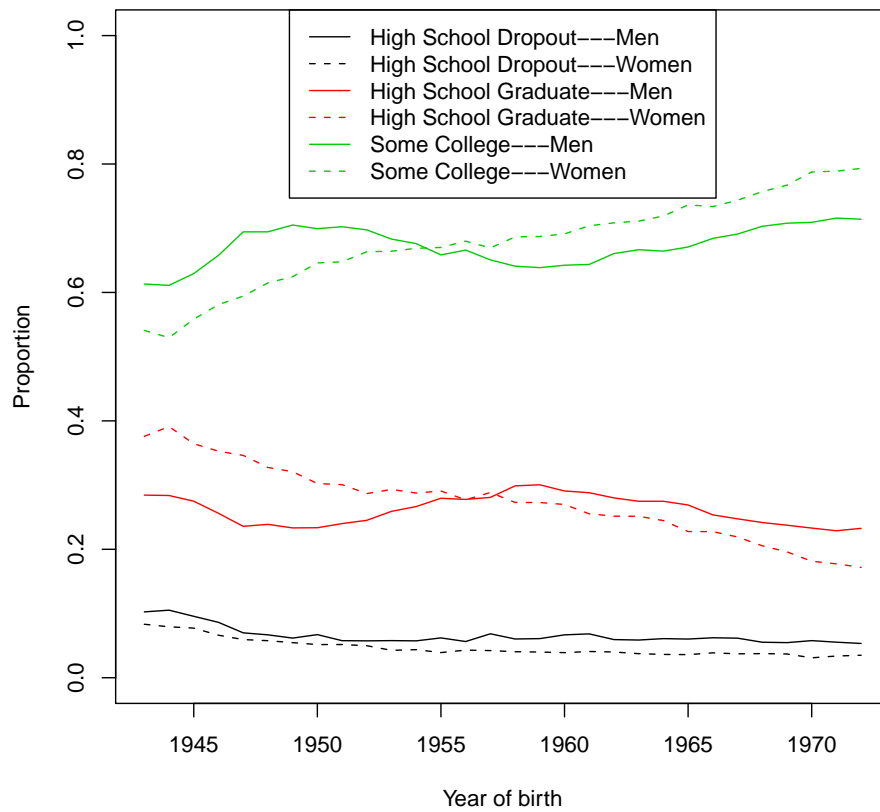


Figure 1: Education levels of men and women

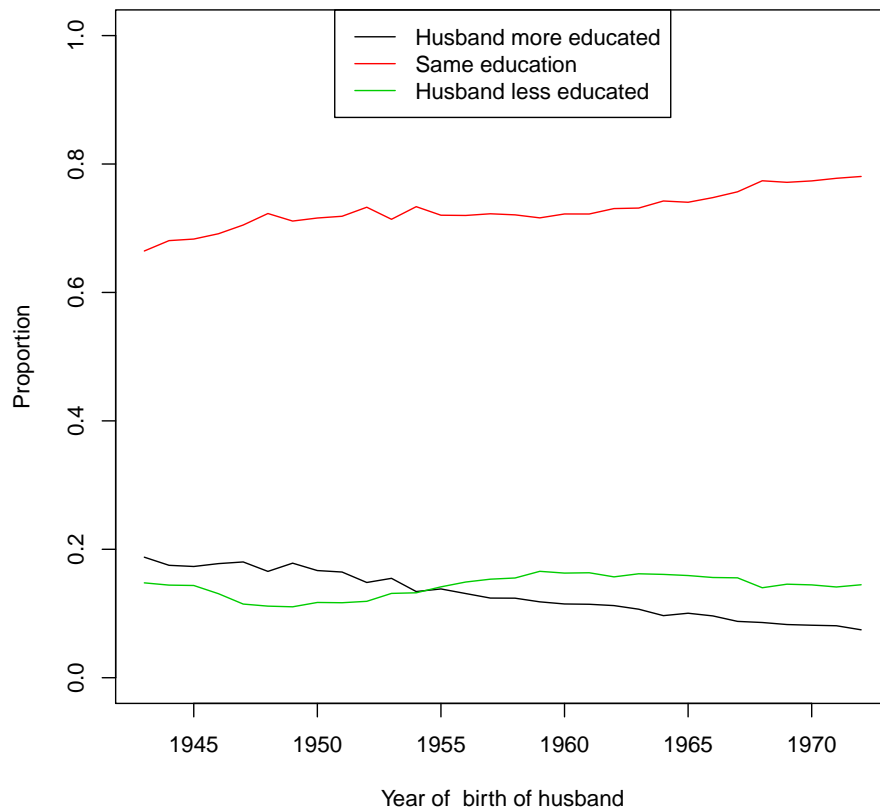


Figure 2: Relative education of partners

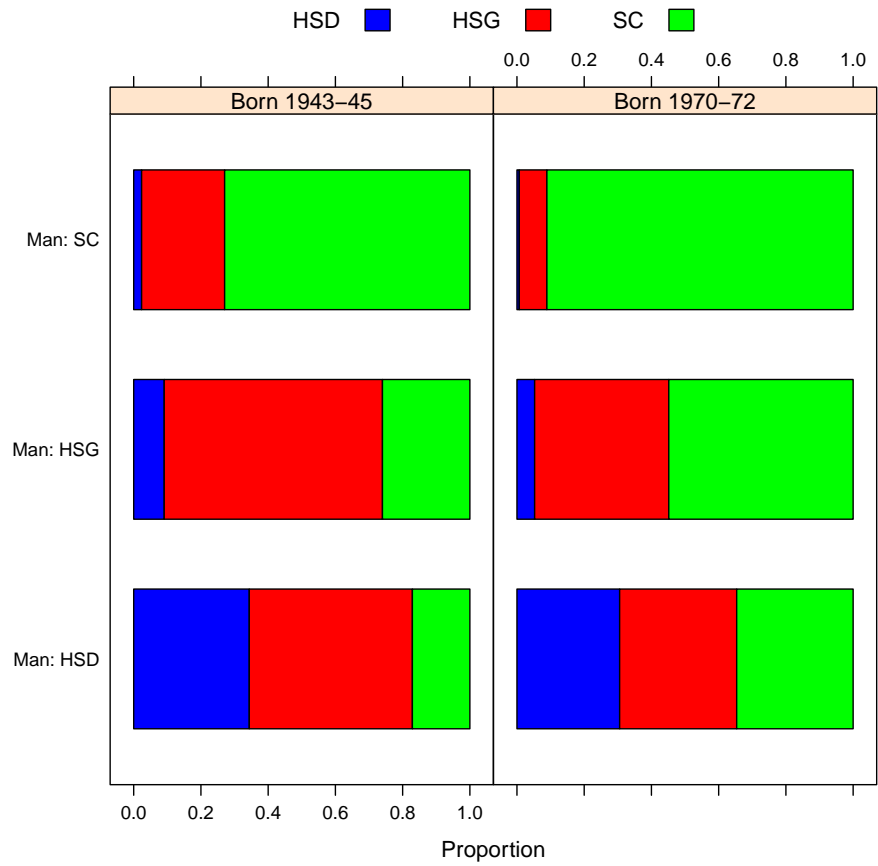


Figure 3: Marriage patterns of men who marry

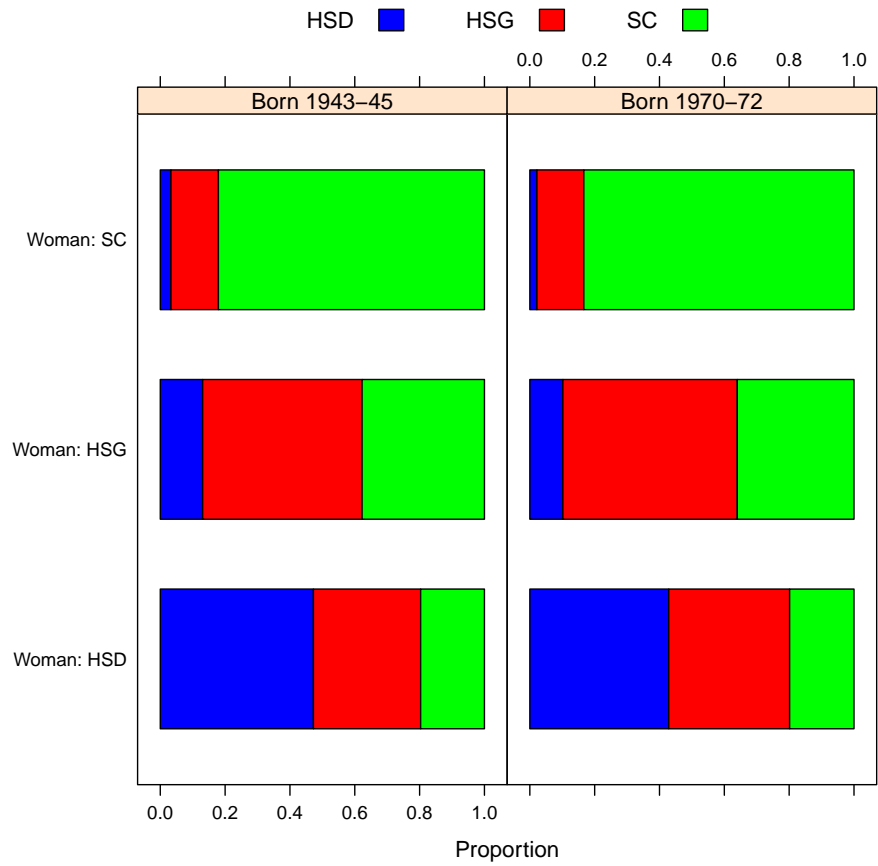


Figure 4: Marriage patterns of women who marry

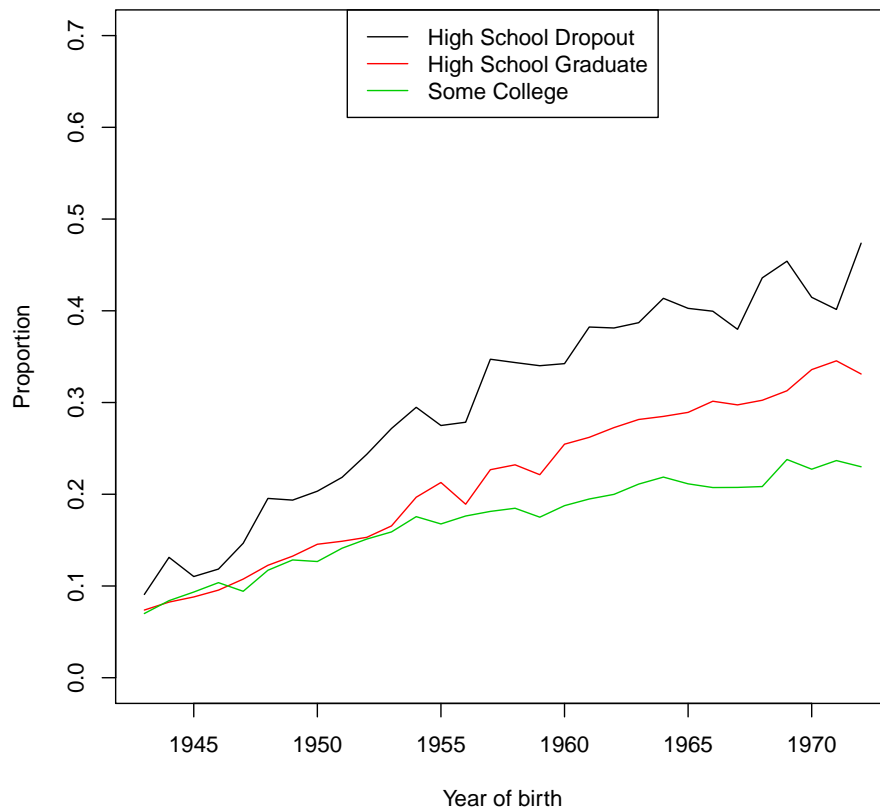


Figure 5: Proportion of men who never married

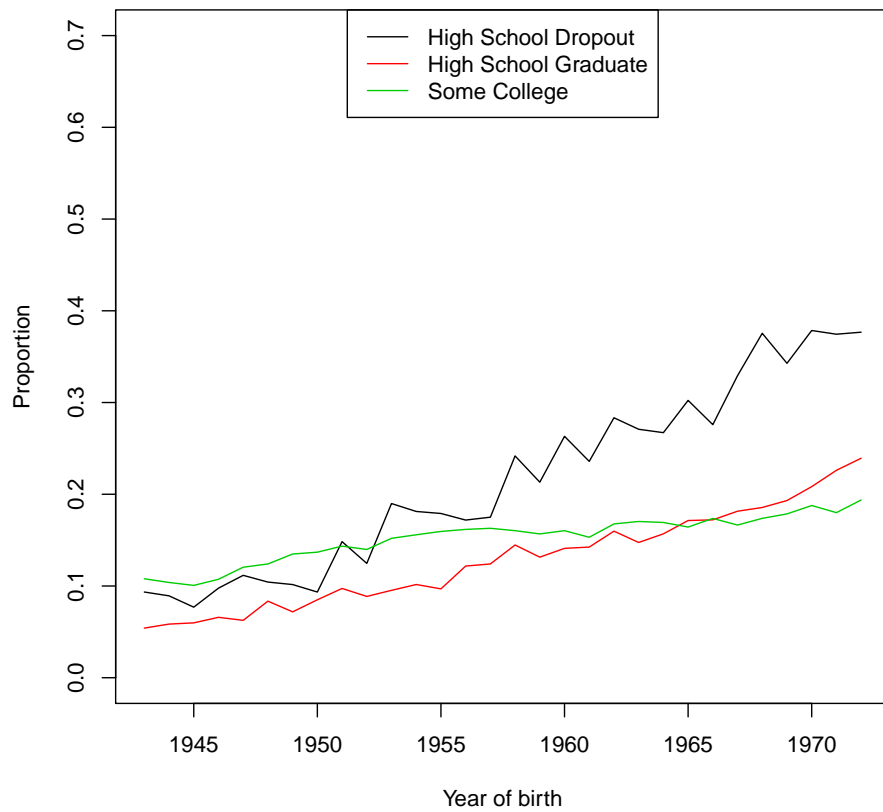


Figure 6: Proportion of women who never married

3 Theoretical framework

Our model derives from CIW. Consider an economy with two periods and large numbers of men and women. In period one, agents draw a cost of investment in human capital and (a vector of) marital preferences from some random distributions; then they invest in education by choosing from a finite set of possible educational levels (and paying the corresponding, person-specific cost). In period 2, agents match on a frictionless marriage market with transferable utility; they each receive an income, the realization of which depends on the agent’s education; and they consume, according to an allocation of resources that was part of the matching agreement.

When investing in human capital, agents must anticipate the outcome of their investment. This outcome has two distinct components. One is a larger future income. In our framework, this effect is taken to be exogenous, and it benefits the agents irrespective of their marital situation. Second, a higher educational level has an impact on marital prospects; it affects the probability of getting married, the expected income of the future spouse, the total utility generated within the household, and the intra-couple allocation of this utility. These “marital gains”, however, depend on the equilibrium reached on the marriage market; this in turn depends on the distribution of education in the two populations, and ultimately of the investment decisions made in the first period. As usual, the model can be solved backwards using a rational expectations assumption; equilibrium is reached when the marital gains resulting from given distributions of education for men and women trigger first period investment decisions that exactly generate these distributions. Note that even if marital preferences and investment cost were independent *ex ante*, education decisions made during the first period must be correlated with preferences for marriage *ex post*: agents with stronger preferences for marriage are more likely to receive the marital gain than agents who prefer to stay single, therefore have stronger incentives to invest in education.

In the present paper, we aim at estimating and testing the second period behavior described by this model. This choice is mostly dictated by available data: while private costs of human capital investment are not observable, the resulting distribution of education by gender is. In addition, concentrating on the second period allows to introduce a slightly more general framework while addressing the empirical content of the key theoretical con-

cept: the notion of a marital college premium. We therefore consider the situation at the beginning of the second period. Agents are each characterized by their level of education, which belongs to some finite set and is observable by all, and by their preferences for marriage, which is observed by their potential mates but not by the econometrician. One can then observe matching patterns; the goal is to identify the underlying structure, and in particular the marital gains associated with each educational level.

3.1 The model

3.1.1 Preferences

The economy consists of a male population \mathcal{M} , endowed with some continuous, atomless measure $d\mu_{\mathcal{M}}$, and a female population \mathcal{F} , endowed with some continuous, atomless measure $d\mu_{\mathcal{F}}$. Each population is partitioned into a finite number of classes, $I = 1, \dots, \mathcal{I}$ for men and $J = 1, \dots, \mathcal{J}$ for women, corresponding to the various education levels available.

The economy has $(n + N)$ commodities, of which n are privately consumed by each individual and N may be publicly consumed by a couple. The preferences over commodities of each individual i are of the GQL form (Bergstrom and Cornes 1983):

$$u_i(q_i, Q) = a_i(q_i^{-1}, Q) + q_i^1 B(Q)$$

where $q_i = (q_i^1, \dots, q_i^n)$ is the vector of private consumptions by agent i , $q_i^{-1} = (q_i^2, \dots, q_i^n)$, $Q = (Q^1, \dots, Q^N)$ is the vector of household's public consumption, and a_i, B are increasing functions.

Let us normalize all prices to 1 for simplicity. If the individual is single and has an income x_i , she would choose (q_i, Q) to solve

$$\max_{q_i, Q} a_i(q_i^{-1}, Q) + q_i^1 B(Q) \text{ such that } \sum_{l=1}^n q_i^l + \sum_l Q^l = x_i.$$

Denote $V_i(x_i)$ the value of this program.

In a couple (i, j) , it is well-known that with GQL preferences, any Pareto efficient consumption such that $q_i^1 q_j^1 > 0$ must maximize the sum of utilities. We therefore define:

$$S_{ij}(x_i + x_j) = \max_{q_i, q_j, Q} u_i(q_i, Q) + u_j(q_j, Q) \text{ s.t. } \sum_k (q_i^k + q_j^k) + \sum_l Q^l = x_i + x_j.$$

If commodities are normal, S is increasing and strictly convex.⁸ In particular, the second cross derivative of S_{ij} in x_i and x_j is always positive: incomes are complementary in the production of the joint surplus $S_{ij}(x_i + x_j) - V_i(x_i) - V_j(x_j)$.

In addition to preferences over commodities, each individual has marital preferences which we model by random vectors. For instance, a woman j belonging to class J has a vector of marital preferences

$$b_j^J = (b_j^{1J}, \dots, b_j^{TJ})$$

where b_j^{nJ} denotes the extra utility j derives from marrying a spouse with an education n , as opposed to staying single. For notational convenience, we define $b_j^{0J} = 0$ for all J . Similarly, man i 's idiosyncratic marital preferences are described by the vector

$$a_i^I = (a_i^{I1}, \dots, a_i^{IJ})$$

where I denotes i 's education; as above, we define $a_i^{I0} = 0$ for all I . Note that the distribution of the a_i^I and b_j^J vectors typically depend on each person's education (I and J respectively), as indicated by the superscript; this dependence reflects, among other things, the fact that the decision to invest in education was partly driven by marital preferences. To reflect this, we define

$$A^{IJ} = E(a_i^{IJ} | i \in I) \text{ and } B^{IJ} = E(b_j^{IJ} | j \in J); \quad (1)$$

A^{IJ} , for instance, represents the average preference for women of education J of men of education I , taking into account that these men considered their marriage prospects when choosing their level of education. Finally, we let

$$\alpha_i^{IJ} = a_i^{IJ} - A^{IJ} \text{ and } \beta_j^{IJ} = b_j^{IJ} - B^{IJ}$$

denote the within-education variation in marital preferences.

3.1.2 Surplus function

The gain generated by the match of man i , belonging to class I , and woman j , belonging to class J , therefore is the sum of two components. One is the expected economic gain

⁸See for instance Browning, Chiappori and Weiss (2012, ch.7.)

generated by joint consumption; the other consists of (the sum of) the spouses' idiosyncratic preferences for marriage with a spouse belonging to that particular class. Since individuals match after choosing their human capital investment but before income is realized, the first component is the expected value of the surplus $S_{ij}(x_i + y_j)$, conditional on the spouses' levels of education:

$$S^{IJ} = E[S_{ij}(x_i + y_j) \mid i \in I, j \in J]$$

The total gain is therefore:

$$\begin{aligned} g_{ij}^{IJ} &= S^{IJ} + E(a_i^{IJ} + b_j^{IJ} \mid i \in I, j \in J) \\ &= S^{IJ} + A^{IJ} + \alpha_i^{IJ} + B^{IJ} + \beta_j^{IJ} \\ &= G^{IJ} + \alpha_i^{IJ} + \beta_j^{IJ} \end{aligned}$$

where $G^{IJ} = S^{IJ} + A^{IJ} + B^{IJ}$ and $E(\alpha_i^{IJ}) = E(\beta_j^{IJ}) = 0$.

Alternatively, i and j may choose to remain single; each gain then is

$$\begin{aligned} g_i^{I0} &= E[V_i(x_i) \mid i \in I] = G^{I0} \text{ and} \\ g_j^{0J} &= E[V_j(x_j) \mid j \in J] = G^{0J} \end{aligned}$$

As always, matching patterns are driven by the *surplus* z_{ij} generated by the matching of man i and woman j , defined as the difference between the agents' total gain when married and the sum of their individual gains as singles:

$$\begin{aligned} z_{ij} &= g_{ij}^{IJ} - G^{I0} - G^{0J} \\ &= Z^{IJ} + \alpha_i^{IJ} + \beta_j^{IJ} \end{aligned}$$

where

$$Z^{IJ} = G^{IJ} - G^{I0} - G^{0J}$$

The surplus of a single agent is normalized to zero, by definition:

$$\begin{aligned} z_{i0} &= z_{0,j} = 0 \text{ for all } i, j \text{ and} \\ Z^{I0} &= Z^{0J} = 0 \text{ for all } I, J \end{aligned}$$

The matrix $Z = (Z^{IJ})$ will play a crucial role in what follows. As we shall see, the equilibrium matching will depend on preferences through the matrix Z and the distribution

of the α 's and β 's. From the definitions above,

$$Z^{IJ} = E[S_{ij}(x_i + y_j) \mid i \in I, j \in J] + E(a_i^{IJ} - V_i(x_i) - a_i^{I0} \mid i \in I) + E(b_j^{IJ} - V_j(x_j) - b_j^{0J} \mid j \in J)$$

reflects the distribution of income and preferences over commodities of spouses who chose education levels I and J (and each other), as well as the distribution of their marital preferences. It is therefore a complex object; but it is the crucial construct that determines marital patterns in our context. Our goal is to check under which conditions it is identifiable from matching patterns.

3.2 Matching

A matching consists of

- (i) a measure $d\mu$ on the set $\mathcal{M} \times \mathcal{F}$, such that the marginal of $d\mu$ over \mathcal{M} (resp. \mathcal{F}) is $d\mu_{\mathcal{M}}$ ($d\mu_{\mathcal{F}}$); and
- (ii) a set of payoffs (or imputations) $\{u_i, i \in \mathcal{M}\}$ and $\{v_j, j \in \mathcal{F}\}$ such that

$$u_i + v_j = z_{ij} \quad \text{for any } (i, j) \in \text{Supp}(d\mu)$$

In words, a matching indicates who marries whom (note that the allocation may be random, hence the measure), and how any married couple shares the gain z_{ij} generated by their match. The numbers u_i and v_j are the expected utilities man i and woman j get on the marriage market, on top of their utilities when they remain single; for any pair that marries with positive probability, they must add up to the total surplus generated by the union.

3.2.1 Stable match

A matching is stable if one can find neither a man i who is currently married but would rather be single, nor a woman j who is currently married but would rather be single, nor a woman j and a man i who are not currently married together but would both rather be married together than remain in their current situation. Formally, we must have that:

$$u_i \geq 0, v_j \geq 0 \quad \text{and} \tag{2}$$

$$u_i + v_j \geq z_{ij} \quad \text{for any } (i, j) \in \mathcal{M} \times \mathcal{F}. \tag{3}$$

The two conditions in (2) implies that married agents would not prefer remaining single; the third (condition (3)) translates the fact that for any possible match (i, j) , the realized surplus z_{ij} cannot exceed the sum of utilities respectively reached by i and j in their current situation (i.e., a violation of this condition would imply that i and j could both strictly increase their utility by matching together).

As is well known, a stable matching of this type is equivalent to a maximization problem; specifically, a match is stable if and only if it maximizes total surplus, $\int z d\mu$, over the set of measures whose marginal over \mathcal{M} (resp. \mathcal{F}) is $d\mu_{\mathcal{M}}$ (resp. $d\mu_{\mathcal{F}}$). A first consequence is that existence is guaranteed under mild assumptions. Moreover, the dual of this maximization problem generates, for each man i (resp. woman j), a dual variable or “shadow price” u_i (resp. v_j), and the dual constraints these variables must satisfy are exactly (2): the dual variables exactly coincide with payoffs associated to the matching problem.

Finally, is the stable matching unique? With finite populations, the answer is no; in general, the payoffs u_i and v_j can be marginally altered without violating the (finite) set of inequalities (2). However, when the populations become large, the intervals within which u_i and v_j may vary typically shrink; in the limit of continuous and atomless populations, (the distributions of) individual payoffs are exactly determined. On all these issues, the reader is referred to Chiappori, McCann and Nesheim (2009) for precise statements.

3.2.2 A basic lemma

From an economic perspective, our main interest lies in the dual variables u and v . Indeed, v_j is the additional utility provided to woman j by her equilibrium marriage outcome. While this value is individual-specific (it depends on Mrs. j 's preferences for marriage), its expected value conditional of j having reached a given level of education J is directly related to the marital premium associated with education J (more on this below).

In our context, there exists a simple and powerful characterization of these dual variables; it is given by the following Lemma:

Lemma 1 *For any stable matching, there exist numbers U^{IJ} and V^{IJ} , $I = 1, \dots, M$, $J = 1, \dots, N$, with*

$$U^{IJ} + V^{IJ} = Z^{IJ} \tag{4}$$

satisfying the following property: for any matched couple (i, j) such that $i \in I$ and $j \in J$,

$$\begin{aligned} u_i &= U^{IJ} + \alpha_i^{IJ} \\ &\text{and} \\ v_j &= V^{IJ} + \beta_j^{IJ} \end{aligned} \tag{L}$$

Proof. Assume that i and i' both belong to I , and their partners j and j' both belong to J . Stability requires that:

$$u_i + v_j = Z^{IJ} + \alpha_i^{IJ} + \beta_j^{IJ} \tag{1}$$

$$u_i + v_{j'} \geq Z^{IJ} + \alpha_i^{IJ} + \beta_{j'}^{IJ} \tag{2}$$

$$u_{i'} + v_{j'} = Z^{IJ} + \alpha_{i'}^{IJ} + \beta_{j'}^{IJ} \tag{3}$$

$$u_{i'} + v_j \geq Z^{IJ} + \alpha_{i'}^{IJ} + \beta_j^{IJ} \tag{4}$$

Subtracting (1) from (2) and (4) from (3) gives

$$\beta_{j'}^{IJ} - \beta_j^{IJ} \leq v_{j'} - v_j \leq \beta_{j'}^{IJ} - \beta_j^{IJ}$$

hence

$$v_{j'} - v_j = \beta_{j'}^{IJ} - \beta_j^{IJ}$$

It follows that the difference $v_j - \beta_j^{IJ}$ does not depend on j , i.e.:

$$v_j - \beta_j^{IJ} = V^{IJ} \quad \text{for all } i \in I, j \in J$$

The proof for u_i is identical. ■

In words, Lemma 1 states that the dual utility v_j of woman j , belonging to class J and married with a husband in education class I , is the sum of two terms. One is woman j 's idiosyncratic preference for a spouse with education I , β_j^{IJ} ; the second term, V^{IJ} , only depends on the spouses' classes, not on who they are. In terms of surplus division, therefore, the U^{IJ} and V^{IJ} denote how the deterministic component of the surplus, Z^{IJ} , is divided between spouses; then a spouse's utility is the sum of their share of the common component and their own, idiosyncratic contribution. Note, incidentally, that Lemma 1 is also valid for singles if we set $U^{I0} = Z^{I0} = 0$ and $V^{0J} = Z^{0J} = 0$.

3.2.3 Stable matching: a characterization

An immediate consequence of Lemma 1 is that the stable matching has a simple characterization in terms of individual choices:

Proposition 2 *A set of necessary and sufficient conditions for stability is that*

1. *for any matched couple $(i \in I, j \in J)$ one has*

$$\alpha_i^{IJ} - \alpha_i^{IK} \geq U^{IK} - U^{IJ} \quad \text{for all } K \quad (5)$$

$$\alpha_i^{IJ} \geq -U^{IJ} \quad (6)$$

and

$$\beta_j^{IJ} - \beta_j^{KJ} \geq V^{KJ} - V^{IJ} \quad \text{for all } K \quad (7)$$

$$\beta_j^{IJ} \geq -V^{IJ} \quad (8)$$

2. *for any single man $i \in I$ one has*

$$\alpha_i^{IJ} \leq -U^{IJ} \quad \text{for all } J \quad (9)$$

3. *for any single woman $j \in J$ one has*

$$\beta_j^{IJ} \leq -V^{IJ} \quad \text{for all } J \quad (10)$$

Proof. *The proof is in several steps. Let $(i \in I, j \in J)$ be a matched couple. Then:*

1. *First, man i must be better off than being single, which gives:*

$$U^{IJ} + \alpha_i^{IJ} \geq 0$$

hence $\alpha_i^{IJ} \geq -U^{IJ}$ and the same must hold with woman j . This shows that (6), (8), (9) and (10) are necessary.

2. *Take some woman j' in J , currently married to some i' in I . Then i must be better off matched with j than j' , which gives:*

$$U^{IJ} + \alpha_i^{IJ} \geq z_{ij'} - v_{j'} = z^{IJ} + \alpha_i^{IJ} + \beta_j^{IJ} - (V^{IJ} + \beta_{j'}^{IJ})$$

and one can readily check that this inequality is always satisfied as an equality, reflecting the fact that i is indifferent between j and j' , and symmetrically j is indifferent between i and i' .

3. Take some woman k in $K \neq J$, currently married to some i' in I . Then “ i is better off matched with j than k ” gives:

$$U^{IJ} + \alpha_i^{IJ} \geq z_{ik} - v_k = z^{IK} + \alpha_i^{IK} + \beta_k^{IK} - (V^{IK} + \beta_k^{IK})$$

which is equivalent to

$$\alpha_i^{IJ} - \alpha_i^{IK} \geq U^{IK} - U^{IJ}$$

and we have proved that the conditions (5) are necessary. The proof is identical for (7).

4. We now show that these conditions are sufficient. Assume, indeed, that they are satisfied. We want to show two properties. First, take some woman j' in J , currently married to some l in $L \neq I$. Then i is better off matched with j than j' . Indeed,

$$U^{IJ} + \alpha_i^{IJ} \geq z_{ij'} - v_{j'} = z^{IJ} + \alpha_i^{IJ} + \beta_{j'}^{IJ} - (V^{LJ} + \beta_{j'}^{LJ})$$

is a direct consequence of (7) applied to l . Finally, take some woman k in $K \neq J$, currently married to some l in $L \neq I$. Then i is better off matched with j than j' . Indeed, it is sufficient to show that

$$U^{IJ} + \alpha_i^{IJ} \geq z_{ik} - v_k = z^{IK} + \alpha_i^{IK} + \beta_j^{IK} - (V^{LK} + \beta_k^{LK})$$

But from (7) applied to k we have that:

$$\beta_k^{LK} - \beta_k^{IK} \geq V^{IK} - V^{LK}$$

and from (5) applied to i :

$$\alpha_i^{IJ} - \alpha_i^{IK} \geq U^{IK} - U^{IJ}$$

and the required inequality is just the sum of the previous two.

■

Stability thus readily translates into a set of inequalities in our framework; and each of these inequalities relates to *one* agent only. This property is crucial, because it implies that the model can be estimated using standard statistical procedures applied at the individual level, without considering conditions on couples. This separation is possible because the endogenous factors U^{IJ} and V^{IJ} adjust to make the separate individual choices consistent with each other.

3.3 Interpretation

The notion of surplus is crucial in analyzing matching patterns. It also has an important economic interpretation that goes back to the theoretical background provided by CIW. Labor economics defines the “college premium” as the percentage increase in expected wage warranted by a college education (as opposed to, say, a high school diploma). This wage premium can readily be measured using available data (and controlling for selection into college); existing empirical work suggests that, at the first order, it is similar for singles and married persons and for men and women (although, clearly, the number of hours, and therefore the resulting gain in labor income, may markedly differ across these populations). The point made by CIW is that, in addition to the standard wage premium, there exists a *marital college premium*, whereby a college education enhances an individual’s marital prospects, including not only the probability of being married and the expected education (or income) of the spouse, but also the size of the surplus generated and its division within the couple. In other words, it is well-understood that college education provides benefits to individuals in terms of higher wages, better career prospects, etc. What we explore here is whether college-educated individuals receive *additional* benefits from their education on the marriage market); and whether this can contribute to explain the observed asymmetry between men and women in terms of demand for education.

An obvious problem is that the marital college premium is quite difficult to estimate empirically, because intrahousehold allocation cannot be directly observed. The main purpose of the present paper is to provide a methodology for such estimation. The crucial remark, at this point, is that the notions previously defined allow a clear definition of the college premium. Indeed, the surplus is computed as the difference between the total utility generated within the couple and the sum of individual utilities of the spouses if single, thus

capturing exactly the additional gains from education that only benefit married people.

Regarding individual well-being, an intuitive interpretation of U^{IJ} (or equivalently of V^{IJ}) would be the following. Assume that a man randomly picked in class I is forced to marry a woman belonging to class J (assuming that the populations are large, so that this small deviation from stability does not affect the equilibrium payoffs). Then his expected utility is exactly U^{IJ} (the expectation being taken over the random choice of the individual—therefore of his preference vector—within the class).

Note, however, that this value does *not* coincide with the average utility of men in class I who end up being married to women J *at a stable matching*; the latter value is larger than U^{IJ} , reflecting the fact that an agent’s choice of his spouse’s class is endogenous. Formally, the expected surplus of an agent with education I is in fact:

$$\bar{u}^I = E \max_{J=0,1,\dots,\mathcal{J}} (U^{IJ} + \alpha^{IJ})$$

where the expectation is taken upon the distribution of the preference shock α . In particular, this expected surplus depends on the distribution of the preference shocks; it will be computed below under a specific assumption on this distribution⁹.

Finally, the difference $\bar{u}^I - \bar{u}^J$ denotes the difference in expected surplus obtained by reaching the education level I instead of J . It therefore represents exactly the marital premium generated by that change in education level—that is, the gain that accrues to married people, in addition to the benefits received by singles.

4 Empirical implementation

4.1 Probabilities

Having transformed the problem into a standard discrete choice problem, it is natural to make the following assumption¹⁰:

⁹With a continuum of agents, while the α ’s and β ’s are random, the U^{IJ} and V^{IJ} are not.

¹⁰This distribution is also referred to as “type-I extreme value distribution”. It was first introduced into economics by Daniel McFadden (1973) to deal with “statistical inference on a model of individual choice behavior from data obtained by sampling from a population” and where “the individual choice depends on unobserved characteristics”. Dagsvik (200) and Choo and Siow (2006) were the first to apply this specification in marriage market analysis.

Assumption 1 *HG (Heteroskedastic Gumbel): The random terms α and β are such that*

$$\begin{aligned}\alpha_i^{IJ} &= \sigma^I \tilde{\alpha}_i^{IJ} \\ \beta_i^{IJ} &= \mu^J \tilde{\beta}_i^{IJ}\end{aligned}$$

where the $\tilde{\alpha}_i^{IJ}$ and $\tilde{\beta}_j^{IJ}$ follow independent Gumbel distributions $G(-k, 1)$, with $k \simeq 0.5772$ the Euler constant.

In particular, the $\tilde{\alpha}_i^{IJ}$ and $\tilde{\beta}_j^{IJ}$ have mean zero and variance $\frac{\pi^2}{6}$, therefore the α_i^{IJ} and β_j^{IJ} have mean zero and respective variance $\frac{\pi^2}{6} (\sigma^I)^2$ and $\frac{\pi^2}{6} (\mu^J)^2$.

A direct consequence of Proposition 2 is that, for any I and any $i \in I$:

$$\begin{aligned}\gamma^{IJ} &\equiv \Pr(i \text{ matched with a woman in } J) \\ &= \frac{\exp(U^{IJ}/\sigma^I)}{\sum_K \exp(U^{IK}/\sigma^I) + 1}\end{aligned}$$

and

$$\begin{aligned}\gamma^{I0} &\equiv \Pr(i \text{ single}) \\ &= \frac{1}{\sum_K \exp(U^{IK}/\sigma^I) + 1}\end{aligned}$$

where we normalize U^{I0} to 0. Similarly, for any J and any woman $j \in J$:

$$\delta^{IJ} \equiv P(j \text{ matched with a man in } I) \tag{11}$$

$$= \frac{\exp(V^{IJ}/\mu^J)}{\sum_K \exp(V^{KJ}/\mu^J) + \exp(V^{0J}/\mu^J)} \text{ and} \tag{12}$$

$$\delta^{0J} \equiv P(j \text{ single}) = \frac{1}{\sum_K \exp(V^{KJ}/\mu^J) + 1}$$

where $V^{0J} = 0$.

These formulas can be inverted to give:

$$\exp(U^{IJ}/\sigma^I) = \frac{\gamma^{IJ}}{1 - \sum_K \gamma^{IK}} \tag{13}$$

and

$$\exp(V^{IJ}/\mu^J) = \frac{\delta^{IJ}}{1 - \sum \delta^{KJ}} \tag{14}$$

therefore:

$$\begin{aligned} U^{IJ} &= \sigma^I \ln \left(\frac{\gamma^{IJ}}{1 - \sum_K \gamma^{IK}} \right) \\ V^{IJ} &= \mu^J \ln \left(\frac{\delta^{IJ}}{1 - \sum \delta^{KJ}} \right) \end{aligned}$$

In what follows, we assume that there are singles in each class: $\gamma_{I0} > 0$ and $\delta_{0J} > 0$ for each I, J , implying that $\sum_K \gamma^{IK} < 1$ and $\sum_K \delta^{KJ} < 1$ for all I, J . Note that a direct consequence of these results is that, knowing the Z^{IJ} and the population sizes, we can algebraically compute U^{IJ}/σ^I and V^{IJ}/μ^J for all (I, J) .

Finally, we can readily compute the class-specific expected utilities described above:

$$\bar{u}^I = E \left[\max_J (U^{IJ} + \sigma^I \tilde{\alpha}_i^{IJ}) \right]$$

In words, \bar{u}^I is the expected utility of an agent in class I , given that this agent will chose a spouse in his preferred class. From the properties of Gumbel distributions, we have that:

$$\begin{aligned} \bar{u}^I &= \sigma^I E \left[\max_J (U^{IJ}/\sigma^I + \tilde{\alpha}_i^{IJ}) \right] \\ &= \sigma^I \ln \left(\sum_J \exp (U^{IJ}/\sigma^I) + 1 \right) = -\sigma^I \ln (\gamma^{I0}) \end{aligned} \quad (15)$$

and similarly

$$\bar{v}^J = \mu^J \ln \left(\sum_I \exp (V^{IJ}/\mu^J) + 1 \right) = -\mu^J \ln (\delta^{0J}) \quad (16)$$

4.2 Heteroskedasticity: a short discussion

An important property of the model just presented is heteroskedasticity: the variance of the unobserved heterogeneity parameters is class-specific. This is a key difference with the framework adopted by Choo and Siow (2006), in which shocks are assumed homoskedastic. In our context, heteroskedasticity is a direct consequence of the theoretical background described above. An agent's investment in human capital depends, among other things, of their marital preferences; even if the distribution of preferences *ex ante* (i.e. before the agent chooses an education level) is iid, the conditional distribution given the chosen

level of education will depend on this level, because of the selection operated on these preferences.

Heteroskedasticity, in turn, has several implications. First, the expected utility of an arbitrary agent in class I , as given by (15), is directly proportional to the standard deviation of the random shock. Indeed, remember that the agent chooses the class of his spouses so as to maximize his utility; and the expectation of the maximum increases with the variance. It follows that the utility generated by the access to the marriage market cannot be exclusively measured by the probability of remaining single (reflected in the $-\ln(\gamma^{I0})$ term).

This remark, in turn, has important consequences for measuring the marital college premium. To see that, start from a model in which the random component of the marital gain is homoskedastically distributed (i.e., the variance is the same across categories: $\sigma^I = \mu^J = 1$ for all I, J). The marital college premium is measured by the difference $\bar{u}^I - \bar{u}^K$, where I is the college education class whereas K is the high school graduate one. Condition (15) then implies that

$$\bar{u}^I - \bar{u}^K = \ln\left(\frac{\gamma^{K0}}{\gamma^{I0}}\right)$$

In words, the gain can directly be measured by the (log) ratio of singlehood probabilities in the two classes. The intuition is that people marry if and only if their (idiosyncratic) gain is larger than some threshold. If these random gains are homoskedastically distributed, then there is a one-to-one correspondence between the mean of the distribution for a particular class and the percentage of that class that is below the threshold and remains single. The higher the mean, the smaller is the proportion (see Figure 7). For instance, if one sees that college graduates are more likely to remain single than high school graduates ($\gamma^{I0} > \gamma^{K0}$, implying that $\ln(\gamma^{K0}/\gamma^{I0}) < 0$), we would then conclude that the expected marital gain is smaller for college graduates ($\bar{u}^I < \bar{u}^K$), therefore that the marital college premium is negative.

Consider, now, the heteroskedastic version. Things are different here, because the percentage of single depends on both the mean and the variance. If educated women are more likely to remain single, it may be because the gain is on average smaller, but it may also be that the variance is larger (even with a higher mean), as illustrated in Figure 8. The one-to-one relationship no longer holds and a higher percentage does not necessarily

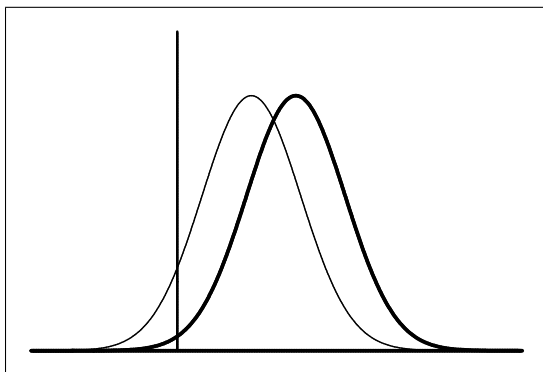


Figure 7: Homoskedastic random gains

imply a smaller mean. One has to compute the respective variances—which, in turn, may affect the computation of the marital college premium. Specifically, we now have that:

$$\bar{u}^I - \bar{u}^K = \sigma^K \ln(\gamma^{K0}) - \sigma^I \ln(\gamma^{I0}) \quad (17)$$

If $\gamma^{I0} > \gamma^{K0}$ and $\sigma^I \leq \sigma^K$, one can conclude that $\bar{u}^I - \bar{u}^K < 0$; but whenever $\sigma^I > \sigma^K$ the conclusion is not granted, and depends on the precise estimates.

Generally, education influences marital prospects through four different channels: it increases marriage probabilities; it changes the potential “quality” (here education) of the future spouses; and it affects the size and the distribution of surplus within the household. In the special homoskedastic version of the model, these three channels are intrinsically mixed, and the expected utility of each spouse is fully determined by the percentage of persons in the same education class that remains single. The heteroskedastic version is much richer; welfare impacts go beyond the sole probability of marriage, and involve other considerations. Consequently, the conclusions drawn from the model significantly depend on the assumptions made regarding its homoskedasticity properties. It is therefore important that these assumptions be testable rather than ad hoc—i.e., that homoskedasticity be

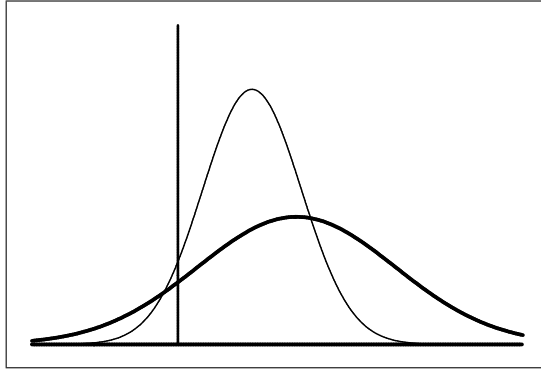


Figure 8: Heteroskedastic random gains

imposed by the data (or at least compatible with them) rather than assumed a priori. In that sense, the estimation of the variances is a crucial part of the identification process.¹¹

There are of course many ways to go beyond the original model of Choo and Siow (2006). Heteroskedasticity is quite natural in our framework: education is partly determined by marital preferences, so that the distribution of marital preferences depends on education in a complex way. Their conditional expectation is subsumed in the matrix Z ; but even if all α 's and β 's were ex ante i.i.d, no reasonable assumption would make their conditional variances constant across education levels. A more complex statistical model would also allow for correlation across the conditional distributions of marital preference shocks; the framework of Galichon and Salanié (2012) shows how such models can be analyzed and estimated, provided that the data is rich enough and/or enough identifying assumptions are imposed to recover the primitives of the model.

¹¹Note however from equations (15) and (16) that if the variances are assumed to be constant across time, then the *variations* in singlehood probability must still reflect similar changes in the expected gains from marriage. In other words, if we find that the percentage of, say, unskilled women remaining single has increased between two cohorts c and c' , we can unambiguously conclude that the gains from marriage have diminished for these women over the period.

4.3 Extension: Covariates

The basic framework just described can be extended to the presence of covariates; i.e., we may specify the α 's and β 's as functions of observed individual characteristics (other than their class). Let X_i be a vector of such characteristics of man i , and Y_j of woman j . We may use the following stochastic structure (where, for simplicity, we disregard heteroskedasticity):

$$\begin{aligned}\tilde{\alpha}_i^{IJ} &= X_i \cdot \zeta_m^{IJ} + \tilde{\alpha}_i^{IJ} \\ \alpha_i^{I0} &= X_i \cdot \zeta_m^{I0} + \tilde{\alpha}_i^{I0} \\ \beta_j^{IJ} &= Y_j \cdot \zeta_f^{IJ} + \tilde{\beta}_j^{IJ} \\ \beta_j^{0J} &= Y_j \cdot \zeta_f^{0J} + \tilde{\beta}_j^{0J}\end{aligned}$$

where $\zeta_m^{IJ}, \zeta_f^{IJ}$ are vector parameters, with the normalization $U^{I0} = \zeta_m^{I0} = 0$ and $V^{0J} = \zeta_f^{0J} = 0$, and where as above the $\tilde{\alpha}_i^{IJ}$ (resp. $\tilde{\beta}_j^{IJ}$) follow independent, type 1 extreme values distributions $G(-k, 1)$. Then the computations are as above, and we can estimate for $i \in I$:

$$\begin{aligned}\gamma^{IJ} &= \Pr(i \text{ matched with a woman in } J) = \frac{\exp(U^{IJ} + X_i \cdot \zeta_m^{IJ})}{\sum_K \exp(U^{IK} + X_i \cdot \zeta_m^{IK}) + \exp(U^{I0} + X_i \cdot \zeta_m^{I0})} \\ \gamma^{I0} &= \Pr(i \text{ single}) = \frac{\exp(U^{I0} + X_i \cdot \zeta_m^{I0})}{\sum_K \exp(U^{IK} + X_i \cdot \zeta_m^{IK}) + \exp(U^{I0} + X_i \cdot \zeta_m^{I0})}\end{aligned}$$

and the conclusions follow. In particular, these models can be estimated running standard (multinomial) logits.

5 Identification

We now consider the identification problem. In practice, we observe realized matching—i.e., populations in each classes and the corresponding marital patterns. To what extent can one recover the fundamentals—i.e., the surplus matrix Z and the heteroskedasticity parameters σ and μ —crucially depends on the type of data available.

We first consider a static context, in which population sizes are fixed. We show that in that case, the model is exactly identified if we assume complete homoskedasticity, and not identified otherwise. Much more interesting is the situation in which population sizes vary over time while (some of) the structural parameters remain constant. Then one can identify both the surplus matrix Z and the heteroskedasticity parameters σ and μ , provided that they remain constant over time; actually, one can even introduce either time varying heteroskedasticity or a drift in the surplus matrix without losing identifiability; and finally, the model generates strong overidentifying restrictions. We consider the two cases successively.

5.1 The static framework

We start with a purely static framework. Define a model M as a set $(Z^{IJ}, \sigma^I, \mu^J)$ such that

$$z_{ij} = Z^{IJ} + \sigma^I \tilde{\alpha}_i^{IJ} + \mu^J \tilde{\beta}_j^{IJ}$$

where the $\tilde{\alpha}_i^{IJ}$ and $\tilde{\beta}_j^{IJ}$ follow independent Gumbel distributions $G(-k, 1)$. Note that the model is clearly invariant when the $(Z^{IJ}, \sigma^I, \mu^J)$ are all multiplied by a common, positive constant; for that reason, in what follows we normalize σ^1 to be 1.

The following result is valid for static (cross-sectional) data:

Proposition 3 *Assume that a model $M = (Z^{IJ}, \sigma^I, \mu^J)$ generates some matching probabilities $(\gamma^{IJ}, \delta^{IJ})$, and let U^{IJ}, V^{IJ} denote the corresponding dual variables. Then*

$$U^{IJ} = \sigma^I \log \frac{\gamma^{IJ}}{1 - \sum_K \gamma^{IK}} \quad (18)$$

and

$$V^{IJ} = \mu^J \log \frac{\delta^{IJ}}{1 - \sum_K \delta^{KJ}} \quad (19)$$

therefore

$$Z^{IJ} = \sigma^I \log \frac{\gamma^{IJ}}{1 - \sum_K \gamma^{IK}} + \mu^J \log \frac{\delta^{IJ}}{1 - \sum_K \delta^{KJ}}$$

Moreover, for any $(\bar{\sigma}^I, \bar{\mu}^J) \in R^+$, the model $N = (\bar{Z}^{IJ}, \bar{\sigma}^I, \bar{\mu}^J)$ where

$$\bar{Z}^{IJ} = \frac{\bar{\sigma}^I}{\sigma^I} U^{IJ} + \frac{\bar{\mu}^J}{\mu^J} V^{IJ} \quad (20)$$

generates the same matching probabilities, and the corresponding, dual variables are

$$\bar{U}^{IJ} = \frac{\bar{\sigma}^I}{\sigma^I} U^{IJ} \quad (21)$$

$$\bar{V}^{IJ} = \frac{\bar{\mu}^J}{\mu^J} V^{IJ} \quad (22)$$

Conversely, if two models $M = (Z^{IJ}, \sigma^I, \mu^J)$ and $N = (\bar{Z}^{IJ}, \bar{\sigma}^I, \bar{\mu}^J)$ generate the same matching probabilities, then the conditions (20), (21) and (22) must hold.

Proof. From the previous calculations, there is a one-to-one relationship between the γ^{IJ} and the v^{IJ} ; the result follows. ■

The previous result is essentially negative; it states that in a static context, the heteroskedastic version of the model is not identified. The heteroskedasticity parameters (σ^I, μ^J) can be chosen arbitrarily; for any value of these parameters, one can find values $\{Z^{IJ}, I = 1, \dots, N, J = 1, \dots, M\}$ that exactly rationalize the data. An interpretation of the non identifiability result is in terms of utility scales. The unit in which the U s and V s are measured is not determined unless we make assumptions on the variances of the α s and β s. This negative result is important, in particular, for welfare comparisons. In a cross-sectional setting, comparing welfare between men and women or between individuals belonging to different classes is highly problematic, since it can only rely on arbitrary choices of the units.

5.2 Changes in population sizes

Much more promising is a situation in which one can observe marriage market outcomes over different periods (or for different cohorts), when the various populations change in respective sizes over the periods. Then a richer model can actually be estimated. We start with the benchmark case, then consider the generalized version that will be taken to data later.

5.2.1 The benchmark version

Let us consider a stable heteroskedastic structural model $M = (Z^{IJ}, \sigma^I, \mu^J)$ that holds for different cohorts of agents, $c = 1, \dots, T$, with varying class compositions. The basic

structure becomes:

$$z_{ij,c} = Z^{IJ} + \sigma^I \alpha_{i,c}^{IJ} + \mu^J \beta_{j,c}^{IJ}$$

Also, assume for the time being that each man marries a woman within his cohort.¹² The model then defines, for each cohort, a matching problem associated with shadow prices that are cohort specific, leading to the definition of U_c^{IJ} and V_c^{IJ} . Then

$$\begin{aligned} \gamma_c^{IJ} &= \Pr(i \in I \text{ matched with a woman in } J \text{ in cohort } c) = \frac{\exp(U_c^{IJ}/\sigma_I)}{1 + \sum_K \exp(U_c^{KJ}/\sigma_K)} \\ \gamma_c^{I0} &= \Pr(i \in I \text{ single}) = \frac{1}{1 + \sum_K \exp(U_c^{KJ}/\sigma_K)} \end{aligned}$$

therefore

$$\exp(U_c^{IJ}/\sigma_I) = \frac{\gamma_c^{IJ}}{1 - \sum_K \gamma_c^{IK}} \quad (23)$$

and similarly:

$$\begin{aligned} \delta_c^{IJ} &= \Pr(j \in J \text{ matched with a woman in } I \text{ in cohort } c) = \frac{\exp(V_c^{IJ}/\mu_J)}{1 + \sum_K \exp(V_c^{IK}/\mu_K)} \\ \delta_c^{I0} &= \Pr(j \in J \text{ single}) = \frac{1}{1 + \sum_K \exp(V_c^{IK}/\mu_K)} \end{aligned}$$

implying that

$$\exp(V_c^{IJ}/\mu_J) = \frac{\delta_c^{IJ}}{1 - \sum_K \delta_c^{IK}} \quad (24)$$

Moreover, we have

$$U_c^{IJ} + V_c^{IJ} = Z^{IJ} \quad (25)$$

Now, let $p_c^{IJ} = U_c^{IJ}/\sigma_I$ and $q_c^{IJ} = V_c^{IJ}/\mu_J$. The crucial remark is that from (23) and (24), the p_c^{IJ} and q_c^{IJ} are directly observable from the data. It follows that (25) has a direct, testable implication. Indeed, define the vectors:

¹²Empirically, this is not exactly right; women tend to marry slightly older men, so that in the application the wife of a man in cohort c typically belongs to cohort $(c+1)$ —a fact that will be taken into account in the empirical application, but can be ignored for the time being.

$$\begin{aligned}
\mathbf{p}^{IJ} &= (p_1^{IJ}, \dots, p_T^{IJ}) \\
\mathbf{q}^{IJ} &= (q_1^{IJ}, \dots, q_T^{IJ}) \\
&\text{and} \\
\mathbf{1} &= (1, \dots, 1)
\end{aligned}$$

Then for each pair (I, J) , the vectors \mathbf{p}^{IJ} , \mathbf{q}^{IJ} and $\mathbf{1}$ must be colinear:

$$\sigma^I \mathbf{p}^{IJ} + \mu^J \mathbf{q}^{IJ} - Z^{IJ} \mathbf{1} = 0 \quad (26)$$

If $T \geq 3$, this generates a first testable restriction—namely that for each (I, J) , any 3×3 determinant extracted from the matrix $M_{IJ} = (\mathbf{p}^{IJ}, \mathbf{q}^{IJ}, \mathbf{1})$ must be zero.

If that restriction is satisfied, assume that either p^{IJ} or q^{IJ} is not constant over the cohorts. Then the vectors p^{IJ} and $\mathbf{1}$ (or q^{IJ} and $\mathbf{1}$) are linearly independent, so that the linear combination in (26) is unique up to a common multiplicative constant. Since, in our case, the constant is pinned down by the normalization $\sigma^1 = 1$, we conclude that for each pair (I, J) , the regression exactly identifies σ^I , μ^J and Z^{IJ} . Finally, since each σ^I but σ^1 (resp. each μ^J) is identified from N (M) different regressions, the model generates a second set of overidentifying restrictions.

We conclude that whenever the populations are not constant across cohorts, the heteroskedastic version of the benchmark structural model is (vastly) overidentified.

5.2.2 Extension: category-specific drifts

The previous overidentification results suggest that a more general version of the model may actually be identified. Specifically, we now relax the assumption that the Z_c^{IJ} are constant across cohorts and introduce category-specific drifts, whereby the Z^{IJ} s vary according to:

$$Z_c^{IJ} = \zeta_c^I + \xi_c^J + Z^{IJ} \quad (27)$$

This is equivalent to assuming that, for all (I, J) and (K, L) , the second difference:

$$Z_c^{IJ} - Z_c^{IL} - Z_c^{KJ} + Z_c^{KL} = Z^{IJ} - Z^{IL} - Z^{KJ} + Z^{KL}$$

is independent of c . That is, we assume that the supermodularity properties of the marital gains are constant over time.

There are several reasons to expect that the surplus generated by marriage would vary across time. One is that technological innovations have drastically altered the technology of domestic production, therefore the respective gender roles within the household (see Greenwood et al 2005.) Other important factors were the evolution of fertility control, as emphasized by Michael (2000) and Goldin and Katz (2002) among others; and improvements in medical techniques and in infant feeding (Albanesi and Olivetti 2009.) Finally, remember that in our framework, the systematic part of the surplus, Z^{IJ} , can be interpreted as a reduced form for more dynamic interactions, including divorce and remarriage; as a consequence, changes in divorce laws or remarriage probabilities may affect the surplus. It is therefore important to stress what the proposed extension allows and what it rules out. Under (27), the benefits of marriage may evolve over time (although the variances do not); and these evolutions may be both gender- and education- specific. In words, we allow, for instance, the gains generated by marriage to decrease less for an educated women than for an unskilled man. However, the components reflecting complementarity (or supermodularity) between education classes—the second differences $(Z^{IJ} - Z^{IL}) - (Z^{KJ} - Z^{KL})$ —are left invariant. In particular, the forces driving the assortativeness of the match are supposed to be constant for the various cohorts. Our challenge is precisely to test whether this hypothesis is compatible with the evolutions in marital patterns observed over the last decades.

Normalizations The form (27) requires additional normalizations. We normalize $\zeta_1^I = \xi_1^J = 0$ so that $Z^{IJ} = Z_1^{IJ}$. Also, note that for any $c > 1$, the ζ_c^I and ξ_c^J are only defined up to a (common) additive constant; i.e. for any given scalar k , one can replace (ζ_c^I, ξ_c^J) with $(\zeta_c^I + k, \xi_c^J - k)$ for all (I, J) without changing (27). We can therefore normalize ξ_c^1 to be zero for all c .

Testing the framework Under (27), equation (25) becomes:

$$\sigma^I p_c^{IJ} + \mu^J q_c^{IJ} = \zeta_c^I + \xi_c^J + Z^{IJ} \quad \forall I, J, c \quad (28)$$

This implies that for all I and all $J \geq 2$, we have:

$$\sigma^I (p_c^{IJ} - p_c^{I1}) + \mu^J q_c^{IJ} - \mu^1 q_c^{I1} = \xi_c^J + Z^{IJ} - Z^{I1} \quad (29)$$

Computing this expression for $I = 1$ and differencing:

$$\sigma^I (p_c^{IJ} - p_c^{I1}) - \sigma_1 (p_c^{1J} - p_c^{11}) + \mu^J (q_c^{IJ} - q_c^{1J}) - \mu^1 (q_c^{I1} - q_c^{11}) = Z^{IJ} - Z^{I1} - Z^{1J} + Z^{11}. \quad (30)$$

This requires a normalization since all terms can be multiplied by the same factor. We could choose for instance $\sigma_1 = 1$, so that

$$p_c^{1J} - p_c^{11} = \sigma^I (p_c^{IJ} - p_c^{I1}) + \mu^J (q_c^{IJ} - q_c^{1J}) - \mu^1 (q_c^{I1} - q_c^{11}) - (Z^{IJ} - Z^{I1} - Z^{1J} + Z^{11})$$

From this, we derive a first testable restriction. To simplify notation, denote

$$D_2 Z^{IJ} = Z^{IJ} - Z^{I1} - Z^{1J} + Z^{11}$$

the second difference of the mean surplus; and define the vectors:

$$\begin{aligned} \mathbf{P}^{IJ} &= (p_1^{IJ} - p_1^{I1}, \dots, p_T^{IJ} - p_T^{I1}) \\ \mathbf{Q}^{IJ} &= (q_1^{IJ} - q_1^{1J}, \dots, q_T^{IJ} - q_T^{1J}) \\ \mathbf{R}^{IJ} &= (p_1^{1J} - p_1^{11}, \dots, p_T^{1J} - p_T^{11}) \\ &\text{and} \\ \mathbf{1} &= (1, \dots, 1) \end{aligned}$$

Then for each pair $(I > 1, J > 1)$:

$$\mathbf{R}^{IJ} = \sigma^I \mathbf{P}^{IJ} + \mu^J \mathbf{Q}^{IJ} - \mu^1 \mathbf{Q}^{I1} - D_2 Z^{IJ} \mathbf{1} \quad (31)$$

and R^{IJ} belongs to the subspace generated by $\{\mathbf{P}^{IJ}, \mathbf{Q}^{IJ}, \mathbf{Q}^{I1}, \mathbf{1}\}$, a first testable restriction for each $(I > 1, J > 1)$. A second set of testable restrictions comes from the fact that when we decompose R^{IJ} over the basis $\{\mathbf{P}^{IJ}, \mathbf{Q}^{IJ}, \mathbf{Q}^{I1}, \mathbf{1}\}$, the coefficient of P^{IJ} (resp. Q^{IJ} , resp. Q^{I1}) does not depend on J (resp. I , resp. is constant).

In practice, we first estimate the probabilities of the various marital outcomes directly from the data, and we use them to construct estimates of the vectors P, Q and R ; then

we choose the heterogeneity parameters $((\sigma^I), (\mu^J))$ and the second differences $(D_2 Z^{IJ})$ so as to minimize the deviations from the conditions in (31). This minimum distance estimation technique also allows us to test the model by evaluating the distance function at its minimum. In our application there are 116 conditions in (31), and only 9 free parameters; this is quite a stringent test since the probabilities of the various matches are estimated from a large sample and thus very precisely.

Once we have estimated the heterogeneity parameters σ^I and μ^J we can also reconstruct the left-hand side of equation (28):

$$\hat{A}_c^{IJ} = \hat{\sigma}^I p_c^{IJ} + \hat{\mu}^J q_c^{IJ}. \quad (32)$$

Our theory (and more specifically equation (28)) implies that in an ANOVA regression of this \hat{A}_c^{IJ} , only 1- way and 2-way effects should appear. To put this in terms more familiar to applied econometricians: a regression of \hat{A}_c^{IJ} on fixed effects for I , for J , and for c (the 1-way effects) and on fixed effects for the interactions (I, J) , (I, c) and (J, c) (the 2-way effects) should have an R^2 of one. This is an alternative way of evaluating departures from the theory, based more on economic significance than on statistical significance.

Identification: the main result Finally, should we fail to reject, the model is identified. To see why, note that the decomposition of R^{IJ} over $\{\mathbf{P}^{IJ}, \mathbf{Q}^{IJ}, \mathbf{Q}^{I1}, \mathbf{1}\}$ is generically unique; the σ^I and μ^J are therefore (over) identified as the respective coefficients of the first two vectors in the decomposition, and μ^1 as minus the coefficient of the third. Rewriting (28) for $c = 1$ gives

$$\sigma^I p_1^{IJ} + \mu^J q_1^{IJ} = Z^{IJ}$$

which shows that the Z^{IJ} are identified. Last, applying (28) identifies ζ_c^I for all I since we set $\xi_c^1 \equiv 0$; and (29) then identifies ξ_c^J for all $J \geq 2$.

A more parsimonious version Coming back to the parsimonious version introduced above ($\sigma^I = \sigma$ for all I and $\mu^J = \mu$ for all J), condition (30) becomes (with the same notations as above):

$$\sigma ((p_c^{IJ} - p_c^{I1}) - (p_c^{1J} - p_c^{11})) + \mu ((q_c^{IJ} - q_c^{1J}) - (q_c^{I1} - q_c^{11})) = Z^{IJ} - Z^{I1} - Z^{1J} + Z^{11}$$

In this case, the computation of μ has a simple and intuitive interpretation. For any ($I \geq 1, J \geq 1$), let $\Delta_2\gamma_c^{IJ}$ denote the second difference of the log probability γ_c^{IJ} that a man in I marries a woman in J , taking for instance the first category as a benchmark for both genders:

$$\Delta_2\gamma_c^{IJ} = \ln \gamma_c^{IJ} - \ln \gamma_c^{I1} - \ln \gamma_c^{1J} + \ln \gamma_c^{11}$$

Clearly, the use of such second differences refers to the supermodularity properties of the (log) probabilities. In particular, if $\ln \gamma_c^{IJ}$ is additively separable:

$$\ln \gamma_c^{IJ} = s_c^I + t_c^J$$

then $\Delta_2\gamma_c^{IJ} = 0$ for all (I, J, c) .

Now, let $\Delta_3\gamma_c^{IJ}$ denote the variation of this second difference over cohorts:

$$\Delta_3\gamma_c^{IJ} = \Delta_2\gamma_{c+1}^{IJ} - \Delta_2\gamma_c^{IJ}$$

We can similarly define $\Delta_2\delta_c^{IJ}$ and $\Delta_3\delta_c^{IJ}$ for women. Then our model implies that:

$$\frac{\Delta_3\gamma_c^{IJ}}{\Delta_3\delta_c^{IJ}} = -\frac{\mu}{\sigma}$$

Therefore the ratio $\Delta_3\gamma_c^{IJ}/\Delta_3\delta_c^{IJ}$ should not depend on the classes I and J nor on the cohort—and the ratio μ/σ then has a natural interpretation in terms of minus this ratio. For instance, the ratio is close to zero if the second difference Δ_2 varies much less for men than for women.¹³

Actually, more complex models can in principle be tested and estimated in this framework. For instance, one may assume a uniform drift in the Z s but allow for cohort-specific variances; the model would then become:

$$g_{ij,c} = Z^{IJ} + \zeta_c + \sigma_c^I \alpha_{i,c}^{IJ} + \mu_c^J \beta_{j,c}^{IJ}$$

Again, one can show that this model (i) generates testable restrictions and (ii) is identified up to simple normalizations (a formal proof is available from the authors).

¹³This property could in principle be used to construct both a specification test and a non parametric estimator of the ratio. In our data, however, the power of the test is quite weak, due to insufficient variations in the second difference across cohorts.

6 Results

We estimate the $\Pr(J|I, c)$ and $\Pr(I|J, c)$ probabilities by the obvious nonparametric technique of counting numbers of marriages in cells, assuming that a man of cohort c marries a woman of cohort $(c + 1)$ (the one-year gap is both the mode and the median age difference at marriage.) We ran the analysis for cohorts of men born between 1943 to 1971.

Then we reconstitute the p and q terms and we run the minimum distance procedure, taking I and $J = 3$ rather than 1 as reference, since category 1 (high-school dropouts) becomes less numerous over time. We also found it more convenient to normalize estimates using the restriction

$$Z^{33} + Z^{11} - Z^{13} - Z^{31} = 1,$$

that scales the constant part of the joint surplus by making the largest cross-difference term equal to one. This allows us to maintain the symmetry between men and women.

Minimum distance estimation amounts to choosing the heterogeneity parameters and the second difference so as to minimize the length of the residuals in (31). As usual, the optimal choice of a norm is the inverse of the variance-covariance matrix of the residuals. Since we use 29 cohorts and we have three categories, the vector of residuals has dimension $29 * (3 - 1) * (3 - 1) = 116$, and its variance-covariance matrix is rather unwieldy. To avoid relying too much on imprecise estimates of some off-diagonal elements of the variance matrix, we only used its diagonal elements¹⁴. Using the full matrix does not materially alter our results.

6.1 Tests

There are two different ways of evaluating the empirical fit of the model. First, we can use the value of the distance function at its minimum; as explained in section 5.2.2, this generates a χ^2 specification test. The value of the statistic is 189.8, for 109 degrees of freedom, which strongly rejects the model. While this sounds like a disappointing outcome, the ANOVA procedure described in section 5.2.2 gives much more positive results.

¹⁴Our estimator of these diagonal elements relies on a first-step minimum distance estimator based on weighting the residuals by the observed number of marriages. In computing it, we neglect the correlation between the estimated \mathbf{P} and \mathbf{Q} .

Remember that the main implication of our framework is that, according to equation (28), $\sigma^I p_c^{IJ} + \mu^J q_c^{IJ}$ can be expressed as a linear combination of terms involving either 1-way effects (i.e., fixed effects for the husband’s education, the wife’s education and the cohort) or 2-way effects (i.e., interactions of any two of the previous terms). Since we can reconstruct the estimator $\hat{A}_c^{IJ} = \hat{\sigma}^I p_c^{IJ} + \hat{\mu}^J q_c^{IJ}$, this property can directly be tested. We find that in a direct ANOVA regression¹⁵ the main effects were the 1-way effects on I and J (for a total of 46.2% of the variance), the 1-way cohort effect (for 13.8%), and the 2-way (I, J) effect (for 37.4%). More strikingly, the residual, which in practice measures the deviation from our theory (i.e. the part of the observed patterns that cannot be explained by 1- or 2-ways effects), accounts for only 0.5% of the variance of \hat{A}_c^{IJ} . This is a remarkably small number, since the 3-way interaction terms comprise 104 degrees of freedom, for $3 * 3 * 29 = 261$ observations. As an illustration, we generated randomly 1,000 samples of 261 such observations (drawn from iid $N(0, 1)$ distributions); performing the same ANOVA regressions on these random samples, we find that the 3-way interaction accounts for no less than 43% of the variance on average (with a dispersion of 3%).

These apparently divergent results are a striking illustration of the difference between statistical significance and economic significance. Since we use rather large samples of men and women, the odds ratios p_c^{IJ} and q_c^{IJ} are very precisely estimated, and any small deviation from the theory (the 0.5% of the variance above) results in a very large value of the test statistic, and thus a spectacular statistical rejection. Thus the statistical rejection of our theory is a minor distraction, and we pursue our analysis of the 99.5% of the variance in marriage patterns that we manage to explain.

6.2 Estimated Heterogeneities

Table 2 gives our estimates of the σ^I and μ^J terms. The model in Choo-Siow (2006) imposes that they all be equal; on the contrary, we find clear and significant variations across our estimates. Indeed, the hypothesis that each σ equals the corresponding μ is strongly rejected. In each schooling class, the estimated variance of unobservable tastes among women is larger than the corresponding variance among men. There also appears to be much less heterogeneity among high-school graduates than for the other two categories;

¹⁵We weighted each (I, J, c) observation by the corresponding number of marriages in the data.

given the discussion of section 4.2, this will play an important role in what follows.

Group	σ^I	μ^J
HSD	0.089 (0.017)	0.148 (0.027)
HSG	0.060 (0.017)	0.071 (0.018)
SC	0.087 (0.014)	0.137 (0.017)

Table 2: σ^I in rows, μ^J in columns

6.3 Estimated Surpluses

The reconstructed values¹⁶ of the Z^{IJ} (the cohort-independent part of the joint surplus) are in Table 3. We ran “supermodularity tests” by evaluating the 9 cross-difference terms

$$Z^{KL} + Z^{IJ} - Z^{IL} - Z^{KJ}$$

with $K > I$ and $L > J$. Rather strikingly, they were all positive. Since the joint surplus

$$Z^{IJ} + \xi_c^I + \zeta_c^J$$

adds to Z a part which is additively separable in I and J , and which therefore cannot alter its supermodularity properties, we can conclude that the joint surplus is supermodular in educations.

Group	HSD	HSG	SC
HSD	0.331	0.193	-0.128
HSG	0.195	0.272	0.098
SC	-0.028	0.233	0.468

Table 3: Z values: men in rows, women in columns

¹⁶The estimated standard errors are between 0.01 and 0.04.

Our method also yield estimates of the ξ and ζ terms, so that for any value of (I, J) we can reconstruct changes in the joint surplus across cohorts. Figure 9 focuses on “diagonal” matches $I = J$. The dashed horizontal lines give the values of Z^{II} , and the curves add $\xi_c^I + \zeta_c^I$. The differences that prevailed for the older cohorts are dwarfed by the evolutions since then: while all categories of matches have become less attractive (relative to staying single), the fall is much steeper for high-school dropouts.

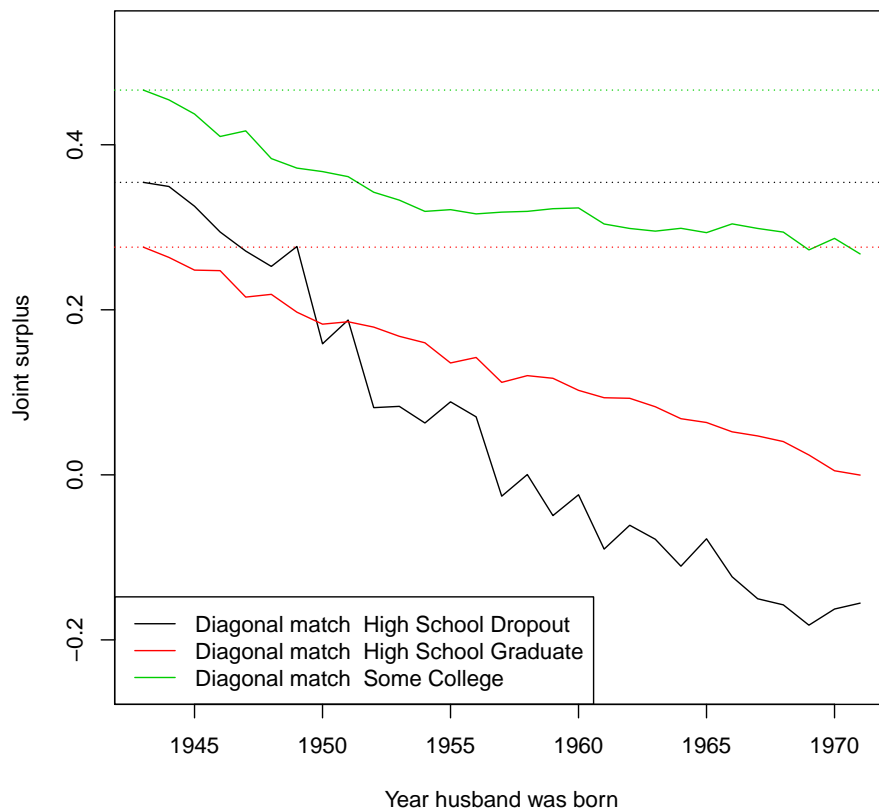


Figure 9: Joint surplus of diagonal matches

6.4 Interpretation

The marital college premium can be decomposed into several components. First, education affects the probability of being married. Second, conditional on being married, it also affects the education of the spouse (or more exactly its distribution); intuitively, we expect educated women to find a “better” husband, at least in terms of education, and conversely. Third, the impact on the total surplus generated by marriage is twofold. Take women for instance. A wife’s education has a direct impact on the surplus; this impact can be measured, for college education, by the difference $(Z_c^{J3} - Z_c^{J2})$. In addition, since a more-educated woman is more likely to marry a more educated husband, the husband’s higher expected education further boosts the surplus, by the average of these $(Z_c^{I3} - Z_c^{I2})$ terms weighted by the difference in probability of marrying a college-educated husband instead of a high school graduate.

Finally, the share of the surplus going to the wife in any given match is also affected by her education. Consider the average surplus from a match between an I -man of cohort c and a J -woman of cohort $(c + 1)$ —recall that we assumed a fixed age difference. This average surplus is the expected value of

$$E(Z_c^{IJ} + \sigma^I \alpha_{i,c}^{IJ} + \mu^J \beta_{j,c}^{IJ}),$$

conditional on i and j marrying each other in equilibrium. Given the additive structure of our theory, it can also be rewritten as the sum of two conditional expectations: that of

$$\max_K (U_c^{IK} + \sigma^I \alpha_{i,c}^{IK})$$

conditional on the maximum being achieved on $K = J$; and that of

$$\max_K (V_c^{KJ} + \mu^J \beta_{j,c}^{KJ})$$

conditional on the maximum being achieved on $K = I$. But given the peculiar nature of type-I extreme value errors, the first expectation is \bar{u}_c^I , independently of the value of J ; and the second one is \bar{v}_c^J , independently of the value of I . Therefore the ratio

$$\frac{\bar{v}_c^J}{\bar{u}_c^I + \bar{v}_c^J}$$

measures the share of the surplus that goes to the wife in an (I, J) marriage, in expected terms.

All these components can readily be computed from our estimates. Focus on women who are either high-school graduates or have some college education.. Table 4 presents some marital outcomes for such women. We record the percentage of women who are married, how many of the married women have a college-educated husband, the total surplus in such a marriage, and the wife’s share of the marital surplus.

Cohort born	1944-46		1970-72	
Education	HSG	SC	HSG	SC
Married	0.933	0.896	0.791	0.818
College-educated husband	0.380	0.833	0.376	0.841
Marital surplus	0.191	0.464	-0.041	0.330
Wife’s share	0.419	0.570	0.404	0.625

Table 4: Marital outcomes for women in early and in recent cohorts

For the early cohort, we see that:

1. College education *reduced* the probability of marrying: it was 93.9% for a high school graduate, but only 89.6% after college.
2. It allowed women who did marry to get a better-educated partner: for instance, the conditional probability of marrying a college-educated man jumped from 38.0% for a high school graduate to 83.3% for a college-educated woman.
3. The marriage of a college-educated husband with a college-educated wife generated a total surplus that was 0.464 on average, as opposed to only 0.191 if the wife did not attend college.
4. Finally, still in the case of a college-educated husband, the wife’s share of total surplus was 57.0% on average if she was college-educated, while a high school graduate received only 41.9% of the smaller surplus.

Note that in contrast to items 1 and 2, which are directly observed, an explicit model is needed to infer items 3 and 4 from the data.

With the passage of time, we find some marked changes in these estimated patterns:

1. College education now increases the probability of marrying (it is 79.1% for a high school graduate and 81.8% for a college graduate)
2. Its impact on the husband's education is pretty much unchanged: the conditional probabilities of marrying a college-educated man are 37.6% for a high school graduate and 84.1% for a college graduate.
3. Regarding the direct impact of female education on total surplus, the marriage of a high school graduate wife with a college-educated man generates negative total surplus on average (-0.041); if the wife attended college, the total surplus is 0.289 . At 0.330 , the difference is larger than it was for early cohorts (0.273).
4. The wife's share of the total surplus in a marriage with a college-educated man has decreased for high school graduates, at 40.4% now; and it has markedly increased for college-educated women—it is now 62.5%.

All in all, the impact of education on a person's marital situation is quite complex: it involves changes in the marriage probabilities, but also in the “quality” of the spouse, in the size of the surplus generated by marriage and ultimately in the distribution of this surplus between spouses. These various components may not evolve in the same direction. A spouse's expected gain, on which the definition of the marital college premium is based, must take all these elements into account; as a result, even the direction of its evolution may in principle be quite difficult to figure out. An obvious advantage of our structural model is that it provides explicit expressions for these components that can readily be evaluated from our estimates. For instance, the marital college premium for any generation c is given by equation (17) above:

$$MCP_c^m = \bar{u}_c^3 - \bar{u}_c^2 = \sigma^2 \ln(\gamma_c^{20}) - \sigma^3 \ln(\gamma_c^{30})$$

for men and

$$MCP_c^w = \bar{v}_c^3 - \bar{v}_c^2 = \mu^2 \ln(\delta_c^{20}) - \mu^3 \ln(\delta_c^{30})$$

for women.

Figures 10 and 11 plot the evolution over cohorts of the expected gains \bar{u}_c^I and \bar{v}_c^J for the various education classes under consideration. One sees, in particular, that the fate of high-school dropouts has deteriorated for both genders, while that of college-educated women has improved. The latter point is illustrated in Figure 12, which plots the evolution of the “marital college premium” ($\bar{u}_c^3 - \bar{u}_c^2$) and ($\bar{v}_c^3 - \bar{v}_c^2$) over cohorts for both genders. Beyond the year-to-year changes, the nonparametric smoothers in dashed lines tell a clear story: the marital college premium of women started to increase sharply for cohorts born around 1955, who graduated from college around 1980; and it has crept upwards ever since. No such change can be seen for men: their marital college premium has remained remarkably flat over the period.

7 Conclusion

It has long been recognized (at least since Becker’s 1973 seminal contributions) that the division of the surplus generated by marriage should be analyzed as an equilibrium phenomenon. As such, it responds to changes in the economic environment; conversely, investments made before marriage are partly driven by agents’ current expectations about the division of surplus that will prevail after marriage. Theory shows that such considerations may explain the considerable differences in male and female demand for higher education. In a nutshell, when deciding whether to go to college, agents take into account not only the market college premium (i.e., the wage differential resulting from a college education) but also the “marital college premium”, which represents the impact of education on marital prospects; the latter includes not only marriage probabilities, but also the expected “quality” of the future spouse and the resulting distribution of marital surplus. Our first contribution is to provide a simple but rich model in which these components can be econometrically identified. Our framework generalizes a previous contribution by Choo and Siow (2006); we show, in particular, that it can be (over)identified using temporal variations in the compositions of the populations at stake. Applying the model to US data, we first study how our main identifying assumption—that the gains from assortative matching, as measured by level of supermodularity in the marital surplus, remain constant over time—fits the data. We show that our model does a remarkably good job at explaining observed evolutions; in fact, we explain no less than 99.5% of total variance—although, due to the

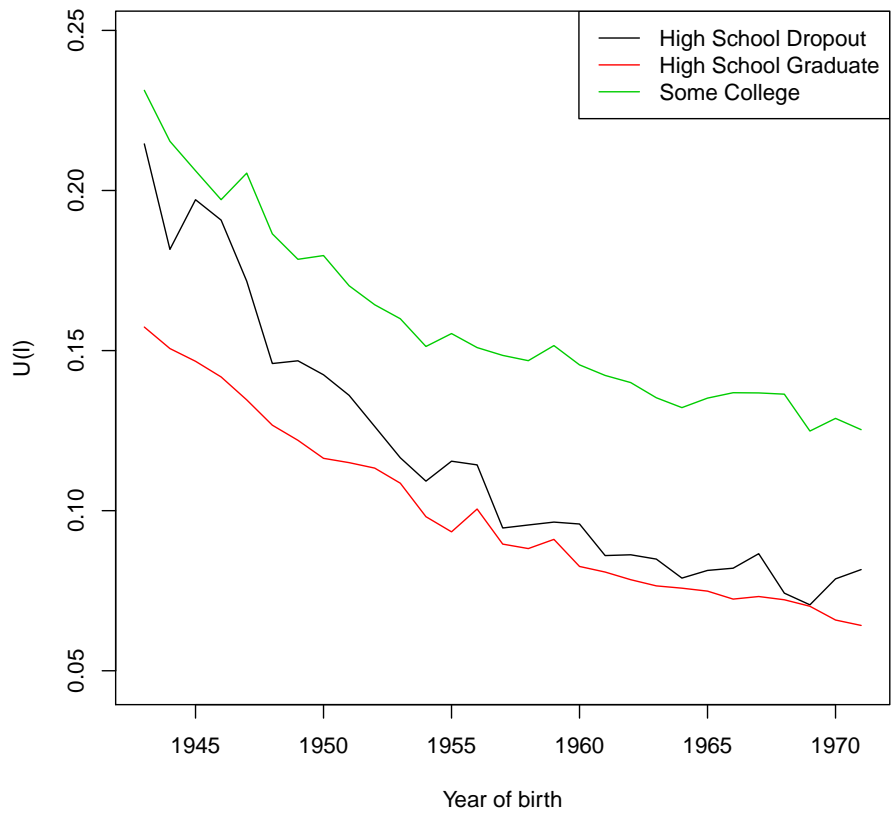


Figure 10: Gains from marriage for men \bar{u}_c^I

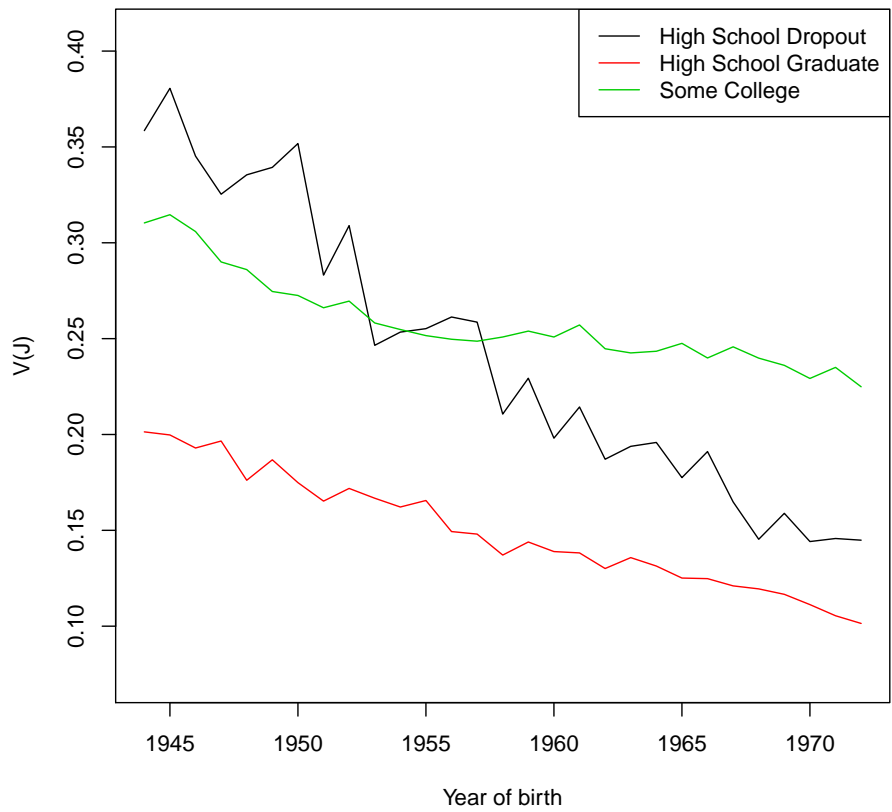


Figure 11: Gains from marriage for women \bar{v}_c^J

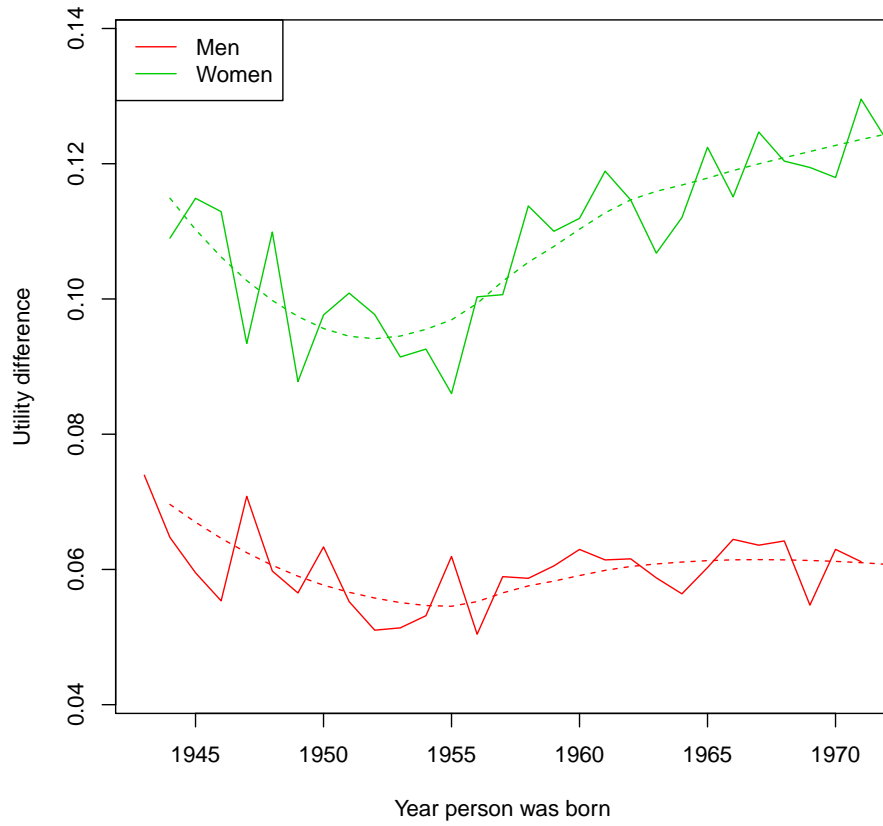


Figure 12: The marital college premium

very large size of the sample, the remaining .5% discrepancy is sufficient to statistically reject.

We find that while the gains from marriage have declined over the period, the decline has been smaller for educated agents. In particular, the “marital college premium” has markedly increased for women in cohorts born after 1955, while remaining stable for men, which confirms the theoretical predictions discussed above.

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